

Classroom Voting Questions: Precalculus

Polynomials, Synthetic Division, and Rational Functions

1. Let $f(x) = (x - 1)^3(x + 4)^4(x + 7)^2$. What is the degree of this polynomial?
 - (a) 3
 - (b) 4
 - (c) 9
 - (d) 24

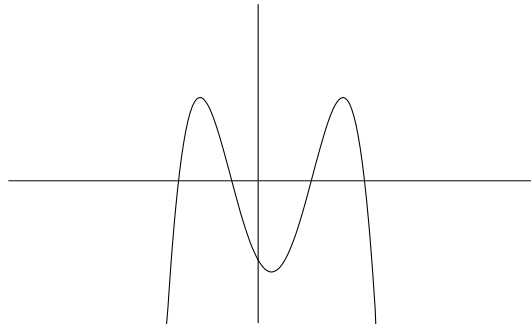
2. Let $f(x) = (x - 1)^3(x + 4)^4(x + 7)^2$. Where does the graph of this function cross the x-axis?
 - (a) 1
 - (b) -4 and -7
 - (c) 1, -4, and -7

3. Find the polynomial $f(x)$ with smallest degree that has zeros at $x = 1$, $x = 2$, and $x = 3$ such that $f(5) = 8$.
 - (a) $f(x) = (x - 1)(x - 2)(x - 3)$
 - (b) $f(x) = (x - 1)(x - 2)(x - 3)(x - 5)$
 - (c) $f(x) = 8(x - 1)(x - 2)(x - 3)$
 - (d) $f(x) = 8(x - 1)(x - 2)(x - 3)(x - 5)$
 - (e) $f(x) = \frac{1}{3}(x - 1)(x - 2)(x - 3)$
 - (f) $f(x) = \frac{1}{42}(x - 1)(x - 2)(x - 3)$

4. Find the zeros of $f(x) = (x^2 - 3)^4(x^5 + x^4 - 12x^3)$ and find the multiplicity of each zero.
 - (a) $x = \sqrt{3}$, mult. 8
 - (b) $x = \sqrt{3}$, mult. 4; $x = -\sqrt{3}$, mult. 4

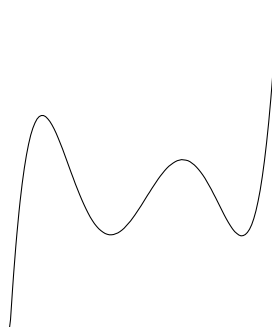
- (c) $x = 0$, mult. 3; $x = -4$, mult. 1; $x = 3$, mult. 1
- (d) $x = 0$, mult. 3; $x = -4$, mult. 1; $x = 3$, mult. 1; $x = \sqrt{3}$, mult. 4; $x = -\sqrt{3}$, mult. 4
- (e) $f(x)$ has no zeros.

5. Which description matches the graph of the polynomial below?



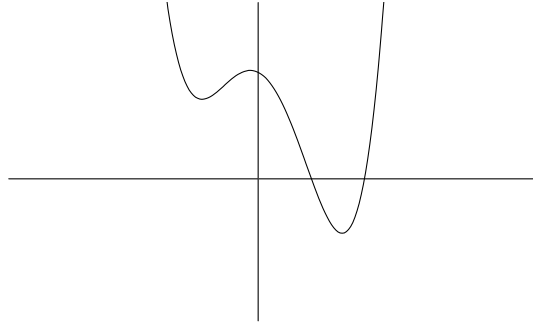
- (a) odd degree, lead coefficient negative
- (b) even degree, lead coefficient negative
- (c) odd degree, lead coefficient positive
- (d) even degree, lead coefficient positive

6. Which description matches the graph of the polynomial below?



- (a) odd degree, lead coefficient negative
- (b) even degree, lead coefficient negative
- (c) odd degree, lead coefficient positive
- (d) even degree, lead coefficient positive

7. Which description matches the graph of the polynomial below?



- (a) odd degree, lead coefficient negative
 - (b) even degree, lead coefficient negative
 - (c) odd degree, lead coefficient positive
 - (d) even degree, lead coefficient positive
8. Use synthetic division to find the remainder of $(4x^4 - 16x^3 + 7x^2 + 20) \div (x + 2)$.
- (a) 2
 - (b) -16
 - (c) -90
 - (d) 240
9. What is the remainder of $(x^4 - 23x^2 - 18x + 40) \div (x + 4)$?
- (a) 0
 - (b) -144
 - (c) -320
 - (d) -336
10. You are doing polynomial long-division to find the result of dividing a degree 7 polynomial $f(x)$ by a degree 3 polynomial $h(x)$. What will be the degree of the quotient polynomial $q(x)$ that you get at the conclusion of the division?
- (a) 3
 - (b) 4
 - (c) 5
 - (d) 6
 - (e) Not enough information to say

11. If $f(x)$ is a polynomial such that $f(c) = 0$ for a real number c , then $f(x)$ can be written as $(x - c)g(x)$ for some polynomial $g(x)$.
- True, and I am very confident.
 - True, but I am not very confident.
 - False, but I am not very confident.
 - False, and I am very confident.
12. Find a quadratic polynomial $f(x)$ with all real coefficients, leading coefficient 1, and zero $2 + i$.
- $f(x) = x^2 - 4x + 5$
 - $f(x) = (x - 2 - i)(x - 2 + i)$
 - $f(x) = x(x - 2 - i)$
 - All of the above.
 - Two of the above.
13. Determine the possible rational zeros of $f(x) = 7x^3 + 4x^2 - 45x + 18$.
- $\pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{6}, \pm \frac{1}{9}, \pm \frac{1}{18}, \pm 7, \pm \frac{7}{2}, \pm \frac{7}{3}, \pm \frac{7}{6}, \pm \frac{7}{9}, \pm \frac{7}{18}$
 - $\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18, \pm \frac{1}{7}, \pm \frac{2}{7}, \pm \frac{3}{7}, \pm \frac{6}{7}, \pm \frac{9}{7}, \pm \frac{18}{7}$
 - $\pm 1, \pm 7$
 - $\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$
14. Find all zeros of $f(x) = 5x^3 - 22x^2 + 33x - 10$. [Hint: Note that $f(0) = -10$ and $f(1) = 6$. Use the Intermediate Value Theorem. (It so happens that the zero the IVT helps locate here is rational.)]
- $x = 0, x = 1, x = \frac{1}{5}$
 - $x = 0, x = 1, x = \frac{2}{5}$
 - $x = -4, x = 2, x = \frac{2}{5}$
 - $x = -2 + i, x = -2 - i, x = \frac{2}{5}$

15. The function $f(x) = \frac{x^2 + 2x}{x^2 - x - 6}$ has
- (a) a hole at $x = -2$
 - (b) a vertical asymptote at $x = 3$
 - (c) a horizontal asymptote at $y = 1$
 - (d) all of the above
16. A polynomial function may have a horizontal asymptote.
- (a) True, and I am very confident.
 - (b) True, but I am not very confident.
 - (c) False, but I am not very confident.
 - (d) False, and I am very confident.
17. A polynomial function may have a vertical asymptote.
- (a) True, and I am very confident.
 - (b) True, but I am not very confident.
 - (c) False, but I am not very confident.
 - (d) False, and I am very confident.
18. Suppose $g(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_0$ and $h(x) = b_m x^m + b_{m-1} x^{m-1} + \dots + b_0$ are polynomials, and define the rational function $f(x) = g(x)/h(x)$. Then $f(x)$ has a vertical asymptote at every number x where $h(x) = 0$.
- (a) True, and I am very confident.
 - (b) True, but I am not very confident.
 - (c) False, but I am not very confident.
 - (d) False, and I am very confident.
19. Suppose $g(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_0$ and $h(x) = b_m x^m + b_{m-1} x^{m-1} + \dots + b_0$, and define the rational function $f(x) = g(x)/h(x)$. Let c be a real number such that $g(c) = 0$ and $h(c) = 0$. Then $f(x)$ has a hole at $x = c$.
- (a) True, and I am very confident.
 - (b) True, but I am not very confident.

- (c) False, but I am not very confident.
- (d) False, and I am very confident.

20. Suppose $g(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_0$ and $h(x) = b_m x^m + b_{m-1} x^{m-1} + \dots + b_0$, and define the rational function $f(x) = g(x)/h(x)$. Under what conditions could $f(x)$ have a vertical asymptote at $x = c$?

- (a) $g(c) = 0$ and $h(c) \neq 0$.
- (b) $g(c) \neq 0$ and $h(c) = 0$.
- (c) $g(c) = 0$ and $h(c) = 0$.
- (d) (a) and (b).
- (e) (a) and (c).
- (f) (b) and (c).
- (g) (a), (b), and (c).

21. Identify all of the functions below which are positive only on the domain described by

$$x < -2 \text{ and } x > 4.$$

- (a) $f(x) = x^2 - 2x - 8$.
- (b) $f(x) = x^2 - 6x + 8$.
- (c) $f(x) = (x^2 - 6x + 8)/(x^2 - 4)$.
- (d) (a) and (b).
- (e) (a) and (c).
- (f) (b) and (c).
- (g) (a), (b), and (c).

22. For what values of x is

$$2x^3 - x^2 - 15x > 0?$$

- (a) $x < -5/2$ and $0 < x < 3$
- (b) $-5/2 < x < 0$ and $x > 3$
- (c) $-5/2 < x < 0$ and $0 < x < 3$
- (d) $x < -5/2$ and $x > 3$

23. Suppose $g(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_0$ and $h(x) = b_m x^m + b_{m-1} x^{m-1} + \dots + b_0$, and define the rational function $f(x) = g(x)/h(x)$. Under what condition does $f(x)$ have a horizontal asymptote at $y = 0$?

- (a) $m > n$.
- (b) $m \geq n$.
- (c) $m = n$.
- (d) $m \leq n$.
- (e) $m < n$.
- (f) Never.

24. Suppose $g(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_0$ and $h(x) = b_m x^m + b_{m-1} x^{m-1} + \dots + b_0$, and define the rational function $f(x) = g(x)/h(x)$. Under what condition does $f(x)$ have a horizontal asymptote at $y = a_n/b_m$?

- (a) $m > n$.
- (b) $m \geq n$.
- (c) $m = n$.
- (d) $m \leq n$.
- (e) $m < n$.
- (f) Never.

25. Suppose $g(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_0$ and $h(x) = b_m x^m + b_{m-1} x^{m-1} + \dots + b_0$, and define the rational function $f(x) = g(x)/h(x)$. Under what condition does $f(x)$ have a horizontal asymptote at $y = b_m/a_n$?

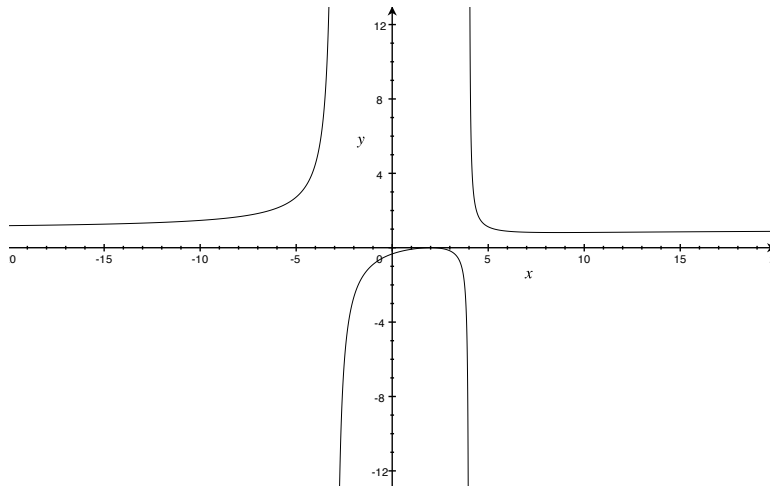
- (a) $m > n$.
- (b) $m \geq n$.
- (c) $m = n$.
- (d) $m \leq n$.
- (e) $m < n$.
- (f) None of the above.

26. Suppose $g(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_0$ and $h(x) = b_m x^m + b_{m-1} x^{m-1} + \dots + b_0$, and define the rational function $f(x) = g(x)/h(x)$. Under what condition does $f(x)$ have no horizontal asymptotes?

- (a) $m > n$.

- (b) $m \geq n$.
- (c) $m = n$.
- (d) $m \leq n$.
- (e) $m < n$.
- (f) None of the above.

27. Based upon the graph of the rational function below, what can you conclude about the degrees of the polynomials in the numerator and the denominator?



- (a) The degree of the numerator is strictly greater than the degree of the denominator.
- (b) The degree of the denominator is strictly greater than the degree of the numerator.
- (c) The degrees of the numerator and denominator are the same.
- (d) Not enough information to draw a conclusion.