Classroom Voting Questions: Precalculus

Polynomials, Synthetic Division, and Rational Functions

1. Let $f(x) = (x-1)^3(x+4)^4(x+7)^2$. What is the degree of this polynomial?

   (a) 3
   (b) 4
   (c) 9
   (d) 24

2. Let $f(x) = (x-1)^3(x+4)^4(x+7)^2$. Where does the graph of this function cross the x-axis?

   (a) 1
   (b) -4 and -7
   (c) 1, -4, and -7

3. Find the poynomial $f(x)$ with smallest degree that has zeros at $x = 1$, $x = 2$, and $x = 3$ such that $f(5) = 8$.

   (a) $f(x) = (x-1)(x-2)(x-3)$
   (b) $f(x) = (x-1)(x-2)(x-3)(x-5)$
   (c) $f(x) = 8(x-1)(x-2)(x-3)$
   (d) $f(x) = 8(x-1)(x-2)(x-3)(x-5)$
   (e) $f(x) = \frac{1}{3}(x-1)(x-2)(x-3)$
   (f) $f(x) = \frac{1}{42}(x-1)(x-2)(x-3)$

4. Find the zeros of $f(x) = (x^2-3)^4(x^5+x^4-12x^3)$ and find the multiplicity of each zero.

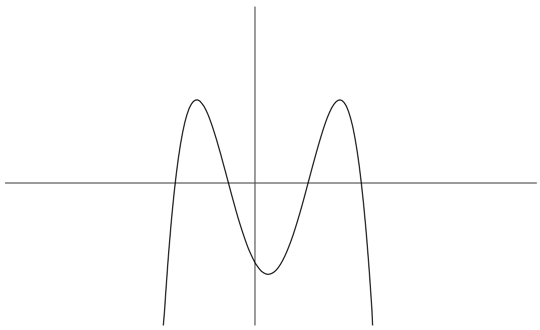
   (a) $x = \sqrt{3}$, mult. 8
   (b) $x = \sqrt{3}$, mult. 4; $x = -\sqrt{3}$, mult. 4

(c) $x = 0$, mult. 3; $x = -4$, mult. 1; $x = 3$, mult. 1

(d) $x = 0$, mult. 3; $x = -4$, mult. 1; $x = 3$, mult. 1; $x = \sqrt{3}$, mult. 4; $x = -\sqrt{3}$, mult. 4

(e) $f(x)$ has no zeros.

5. Which description matches the graph of the polynomial below?



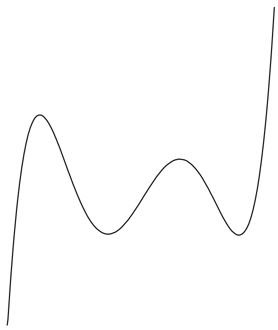
(a) odd degree, lead coefficient negative

(b) even degree, lead coefficient negative

(c) odd degree, lead coefficient positive

(d) even degree, lead coefficient positive

6. Which description matches the graph of the polynomial below?



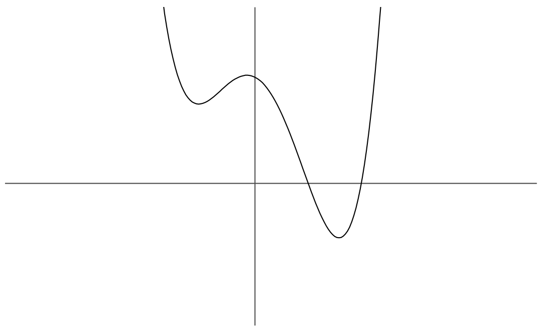
(a) odd degree, lead coefficient negative

(b) even degree, lead coefficient negative

(c) odd degree, lead coefficient positive

(d) even degree, lead coefficient positive

7. Which description matches the graph of the polynomial below?



(a) odd degree, lead coefficient negative
(b) even degree, lead coefficient negative
(c) odd degree, lead coefficient positive
(d) even degree, lead coefficient positive

8. Use synthetic division to find the remainder of $(4x^4 - 16x^3 + 7x^2 + 20) \div (x + 2)$.

   (a) 2
   (b) -16
   (c) -90
   (d) 240

9. What is the remainder of $(x^4 - 23x^2 - 18x + 40) \div (x + 4)$?

   (a) 0
   (b) -144
   (c) -320
   (d) -336

10. You are doing polynomial long-division to find the result of dividing a degree 7 polynomial $f(x)$ by a degree 3 polynomial $h(x)$. What will be the degree of the quotient polynomial $q(x)$ that you get at the conclusion of the division?

   (a) 3
   (b) 4
   (c) 5
   (d) 6
   (e) Not enough information to say

11. If $f(x)$ is a polynomial such that $f(c) = 0$ for a real number c, then $f(x)$ can be written as $(x - c)g(x)$ for some polynomial $g(x)$.

   (a) True, and I am very confident.
   (b) True, but I am not very confident.
   (c) False, but I am not very confident.
   (d) False, and I am very confident.

12. Find a quadratic polynomial $f(x)$ with all real coefficients, leading coefficient 1, and zero $2 + i$.

   (a) $f(x) = x^2 - 4x + 5$
   (b) $f(x) = (x - 2 - i)(x - 2 + i)$
   (c) $f(x) = x(x - 2 - i)$
   (d) All of the above.
   (e) Two of the above.

13. Determine the possible rational zeros of $f(x) = 7x^3 + 4x^2 - 45x + 18$.

   (a) $\pm 1,\ \pm\frac{1}{2},\ \pm\frac{1}{3},\ \pm\frac{1}{6},\ \pm\frac{1}{9},\ \pm\frac{1}{18},\ \pm 7,\ \pm\frac{7}{2},\ \pm\frac{7}{3},\ \pm\frac{7}{6},\ \pm\frac{7}{9},\ \pm\frac{7}{18}$
   (b) $\pm 1,\ \pm 2,\ \pm 3,\ \pm 6,\ \pm 9,\ \pm 18,\ \pm\frac{1}{7},\ \pm\frac{2}{7},\ \pm\frac{3}{7},\ \pm\frac{6}{7},\ \pm\frac{9}{7},\ \pm\frac{18}{7}$
   (c) $\pm 1,\ \pm 7$
   (d) $\pm 1,\ \pm 2,\ \pm 3,\ \pm 6,\ \pm 9,\ \pm 18$

14. Find all zeros of $f(x) = 5x^3 - 22x^2 + 33x - 10$. [Hint: Note that $f(0) = -10$ and $f(1) = 6$. Use the Intermediate Value Theorem. (It so happens that the zero the IVT helps locate here is rational.)]

   (a) $x = 0,\ x = 1,\ x = \frac{1}{5}$
   (b) $x = 0,\ x = 1,\ x = \frac{2}{5}$
   (c) $x = -4,\ x = 2,\ x = \frac{2}{5}$
   (d) $x = -2 + i,\ x = -2 - i,\ x = \frac{2}{5}$

15. The function $f(x) = \dfrac{x^2 + 2x}{x^2 - x - 6}$ has

    (a) a hole at $x = -2$
    (b) a vertical asymptote at $x = 3$
    (c) a horizontal asymptote at $y = 1$
    (d) all of the above

16. A polynomial function may have a horizontal asymptote.

    (a) True, and I am very confident.
    (b) True, but I am not very confident.
    (c) False, but I am not very confident.
    (d) False, and I am very confident.

17. A polynomial function may have a vertical asymptote.

    (a) True, and I am very confident.
    (b) True, but I am not very confident.
    (c) False, but I am not very confident.
    (d) False, and I am very confident.

18. Suppose $g(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_o$ and $h(x) = b_m x^m + b_{m-1}x^{m-1} + \ldots + b_o$ are polynomials, and define the rational function $f(x) = g(x)/h(x)$. Then $f(x)$ has a vertical asymptote at every number x where $h(x) = 0$.

    (a) True, and I am very confident.
    (b) True, but I am not very confident.
    (c) False, but I am not very confident.
    (d) False, and I am very confident.

19. Suppose $g(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_o$ and $h(x) = b_m x^m + b_{m-1}x^{m-1} + \ldots + b_o$, and define the rational function $f(x) = g(x)/h(x)$. Let c be a real number such that $g(c) = 0$ and $h(c) = 0$. Then $f(x)$ has a hole at $x = c$.

    (a) True, and I am very confident.
    (b) True, but I am not very confident.

(c) False, but I am not very confident.

(d) False, and I am very confident.

20. Suppose $g(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_o$ and $h(x) = b_m x^m + b_{m-1}x^{m-1} + \ldots + b_o$, and define the rational function $f(x) = g(x)/h(x)$. Under what conditions could $f(x)$ have a vertical asymptote at $x = c$?

(a) $g(c) = 0$ and $h(c) \neq 0$.

(b) $g(c) \neq 0$ and $h(c) = 0$.

(c) $g(c) = 0$ and $h(c) = 0$.

(d) (a) and (b).

(e) (a) and (c).

(f) (b) and (c).

(g) (a), (b), and (c).

21. Identify all of the functions below which are positive only on the domain described by

$$x < -2 \text{ and } x > 4.$$

(a) $f(x) = x^2 - 2x - 8$.

(b) $f(x) = x^2 - 6x + 8$.

(c) $f(x) = (x^2 - 6x + 8)/(x^2 - 4)$.

(d) (a) and (b).

(e) (a) and (c).

(f) (b) and (c).

(g) (a), (b), and (c).

22. For what values of x is

$$2x^3 - x^2 - 15x > 0?$$

(a) $x < -5/2$ and $0 < x < 3$

(b) $-5/2 < x < 0$ and $x > 3$

(c) $-5/2 < x < 0$ and $0 < x < 3$

(d) $x < -5/2$ and $x > 3$

23. Suppose $g(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_o$ and $h(x) = b_m x^m + b_{m-1}x^{m-1} + \ldots + b_o$, and define the rational function $f(x) = g(x)/h(x)$. Under what condition does $f(x)$ have a horizontal asymptote at $y = 0$?

(a) $m > n$.
(b) $m \geq n$.
(c) $m = n$.
(d) $m \leq n$.
(e) $m < n$.
(f) Never.

24. Suppose $g(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_o$ and $h(x) = b_m x^m + b_{m-1}x^{m-1} + \ldots + b_o$, and define the rational function $f(x) = g(x)/h(x)$. Under what condition does $f(x)$ have a horizontal asymptote at $y = a_n/b_m$?

(a) $m > n$.
(b) $m \geq n$.
(c) $m = n$.
(d) $m \leq n$.
(e) $m < n$.
(f) Never.

25. Suppose $g(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_o$ and $h(x) = b_m x^m + b_{m-1}x^{m-1} + \ldots + b_o$, and define the rational function $f(x) = g(x)/h(x)$. Under what condition does $f(x)$ have a horizontal asymptote at $y = b_m/a_n$?

(a) $m > n$.
(b) $m \geq n$.
(c) $m = n$.
(d) $m \leq n$.
(e) $m < n$.
(f) None of the above.

26. Suppose $g(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_o$ and $h(x) = b_m x^m + b_{m-1}x^{m-1} + \ldots + b_o$, and define the rational function $f(x) = g(x)/h(x)$. Under what condition does $f(x)$ have no horizontal asymptotes?

(a) $m > n$.

(b) $m \geq n$.

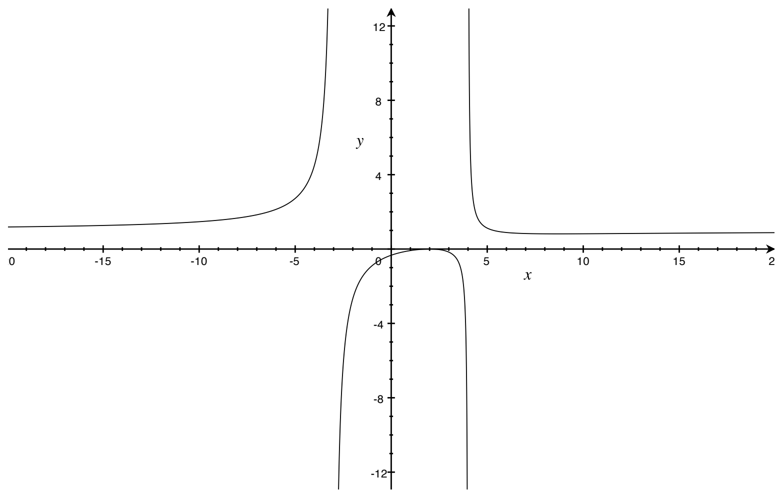
(c) $m = n$.

(d) $m \leq n$.

(e) $m < n$.

(f) None of the above.

27. Based upon the graph of the rational function below, what can you conclude about the degrees of the polynomials in the numerator and the denominator?



(a) The degree of the numerator is strictly greater than the degree of the denominator.

(b) The degree of the denominator is strictly greater than the degree of the numerator.

(c) The degrees of the numerator and denominator are the same.

(d) Not enough information to draw a conclusion.