Classroom Voting Questions: Precalculus

Limits

1. Consider the function:

\[ f(x) = \begin{cases} 
6 & \text{if } x > 9 \\
2 & \text{if } x = 9 \\
-x + 14 & \text{if } -7 \leq x < 9 \\
21 & \text{if } x < -7 
\end{cases} \]

(a) \( \lim_{x \to 9^-} f(x) = 2 \)
(b) \( \lim_{x \to 9^-} f(x) = 5 \)
(c) \( \lim_{x \to 9^-} f(x) = 6 \)
(d) \( \lim_{x \to 9^-} f(x) = 14 \)
(e) \( \lim_{x \to 9^-} f(x) = 21 \)

2. True or False: As \( x \) increases to 100, \( f(x) = \frac{1}{x} \) gets closer and closer to 0, so the limit as \( x \) goes to 100 of \( f(x) \) is 0. Be prepared to justify your answer.

   (a) True, and I am very confident
   (b) True, but I am not very confident
   (c) False, but I am not very confident
   (d) False, and I am very confident

3. True or False: \( \lim_{x \to a} f(x) = L \) means that if \( x_1 \) is closer to \( a \) than \( x_2 \) is, then \( f(x_1) \) will be closer to \( L \) than \( f(x_2) \) is. Be prepared to justify your answer with an argument or counterexample.

   (a) True, and I am very confident
   (b) True, but I am not very confident
   (c) False, but I am not very confident
   (d) False, and I am very confident

4. The reason that \( \lim_{x \to 0} \sin \left( \frac{1}{x} \right) \) does not exist is:
(a) because no matter how close \( x \) gets to 0, there are \( x \)'s near 0 for which \( \sin \left( \frac{1}{x} \right) = 1 \), and some for which \( \sin \left( \frac{1}{x} \right) = -1 \).

(b) because the function values oscillate around 0.

(c) because \( \frac{1}{0} \) is undefined.

(d) all of the above

5. \( \lim_{x \to 0} x^2 \sin \left( \frac{1}{x} \right) \)

(a) does not exist because no matter how close \( x \) gets to 0, there are \( x \)'s near 0 for which \( \sin \left( \frac{1}{x} \right) = 1 \), and some for which \( \sin \left( \frac{1}{x} \right) = -1 \).

(b) does not exist because the function values oscillate around 0.

(c) does not exist because \( \frac{1}{0} \) is undefined.

(d) equals 0

(e) equals 1

6. You’re trying to guess \( \lim_{x \to 0} f(x) \). You plug in \( x = 0.1, 0.01, 0.001, \ldots \) and get \( f(x) = 0 \) for all of these values. In fact you’re told that for all \( n = 1, 2, \ldots \), \( f \left( \frac{1}{10^n} \right) = 0 \). True or False: Since the sequence \( f(0.1), f(0.01), f(0.001), \ldots \) goes to 0, we know that \( \lim_{x \to 0} f(x) = 0 \).

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

7. If \( \lim_{x \to a} f(x) = 0 \) and \( \lim_{x \to a} g(x) = 0 \), then \( \lim_{x \to a} \frac{f(x)}{g(x)} \)

(a) does not exist.

(b) must exist.

(c) can’t be determined. Not enough information is given.

8. True or False: Consider a function \( f(x) \) with the property that \( \lim_{x \to a} f(x) = 0 \). Now consider another function \( g(x) \) also defined near \( a \). Then \( \lim_{x \to a} [f(x)g(x)] = 0 \)

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident
9. If a function \( f \) is not defined at \( x = a \),

(a) \( \lim_{x \to a} \) cannot exist.
(b) \( \lim_{x \to a} \) could be 0.
(c) \( \lim_{x \to a} \) must approach \( \infty \).
(d) none of the above

10. Possible criteria for continuity at a point: *If the limit of the function exists at a point, the function is continuous at that point.* Which of the following examples fits the above criteria but is not continuous at \( x = 0 \)?

(a) \( f(x) = x \)
(b) \( f(x) = x^2/x \)
(c) \( f(x) = |x|/x \)
(d) None of these show a problem with this criteria.

11. Let \( f(x) = 5x^4 + 18x^3 - 2x + 3 \). As \( x \) gets really big, what becomes the most important (dominant) term in this function?

(a) \( 5x^4 \)
(b) \( 18x^3 \)
(c) \( -2x \)
(d) \( 3 \)

12. What is

\[
\lim_{x \to \infty} \frac{6x^2 - 5x}{2x^2 + 3}
\]

(a) 0
(b) 2
(c) 3
(d) 6
(e) infinity
13. What is \( \lim_{x \to \infty} \frac{3x^2 + 5x^3 - 2x + 4}{4x^3 - 5x + 6} \) ?

(a) 0  
(b) 2/3  
(c) 3/4  
(d) 5/4  
(e) infinity

14. What is \( \lim_{x \to \infty} \frac{100x^5 - 15x}{x^6 + 3} \) ?

(a) 0  
(b) 5/6  
(c) 85  
(d) 100  
(e) infinity

15. What is \( \lim_{x \to \infty} \frac{x^2 + 2x + 3}{25x - 7} \) ?

(a) 0  
(b) 1/25  
(c) 3/7  
(d) 2  
(e) infinity

16. Let \( f(x) = \frac{x^2 - 4x + 3}{x^2 - 1} \). Evaluate \( \lim_{x \to -1^+} f(x) \).

(a) −1  
(b) ∞  
(c) −∞

17. Find \( \lim_{x \to 1} \frac{x^2 + x - 2}{x-1} \).
18. Find \( \lim_{x \to -2} \frac{x^2 + x + 1}{x + 2} \).

(a) \( \infty \)
(b) \(-\infty\)
(c) 0
(d) 1
(e) The limit does not exist
(f) None of the above