MathQuest: Series

Taylor Series

1. Find the Taylor series for the function \( \ln(x) \) at the point \( a = 1 \). (No calculators allowed.)

(a) \((x - 1) - \frac{1}{2}(x - 1)^2 + \frac{1}{3}(x - 1)^3 - \frac{1}{4}(x - 1)^4 + \cdots\)
(b) \((x - 1) - (x - 1)^2 + 2(x - 1)^3 - 6(x - 1)^4 + \cdots\)
(c) \(\ln(x) + \frac{1}{2}(x - 1) - \frac{1}{3^2}(x - 1)^2 + \frac{2}{3^3}(x - 1)^3 - \frac{6}{3^4}(x - 1)^4 + \cdots\)
(d) \(\ln(x) + \frac{1}{2}(x - 1) - \frac{1}{3^2}(x - 1)^2 + \frac{1}{3^3}(x - 1)^3 - \frac{1}{4^4}(x - 1)^4 + \cdots\)
(e) This is not possible.

2. If \( a = 0 \), what function is represented by the Taylor series \( 1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \cdots \)? (No calculators allowed.)

(a) \(e^x\)
(b) \(\sin x\)
(c) \(\cos x\)
(d) This is not a Taylor series.

3. A Taylor series converges when \( x = 12, 13 \) and \( 15 \), but diverges when \( x = 9, 16 \) and \( 18 \). Which of the following could be \( a \), the point where the Taylor series is centered?

(a) \(a = 9\)
(b) \(a = 11\)
(c) \(a = 13\)
(d) \(a = 15\)
(e) All of the above are possible.
(f) None of the above are possible.

4. Suppose we find a Taylor series for the function \( f(x) \) centered at the point \( a = 5 \). Where would we expect a finite number of terms from this Taylor series to probably give us a better estimate?

(a) \(x = 0\)
5. A Taylor series for a function $f(x)$ at $a = 10$ has a radius of convergence of 3. If we use the first 10 terms of this series to estimate $f(15)$ we will probably get

(a) an infinite result.
(b) a result which is closer to the real value of $f(15)$ than if we used 5 terms.
(c) a result which is farther from the real value of $f(15)$ than if we used 25 terms.
(d) a result which is closer to the real value of $f(15)$ than if we used 15 terms.
(e) More than one of the above are true.

6. We are given a Taylor series for a function $g(x)$ at $a = -5$, with a radius of convergence of 6. Which would give the best estimate of $g(-5)$?

(a) The first term of the Taylor series.
(b) The first 5 terms of the Taylor series.
(c) The first 10 terms of the Taylor series.
(d) The first 100 terms of the Taylor series.
(e) All would give the same result.