

# MathQuest: Series

## Taylor Series

- Find the Taylor series for the function  $\ln(x)$  at the point  $a = 1$ .
  - $(x - 1) - \frac{1}{2}(x - 1)^2 + \frac{1}{3}(x - 1)^3 - \frac{1}{4}(x - 1)^4 + \dots$
  - $(x - 1) - (x - 1)^2 + 2(x - 1)^3 - 6(x - 1)^4 + \dots$
  - $\ln(x) + \frac{1}{x}(x - 1) - \frac{1}{x^2}(x - 1)^2 + \frac{2}{x^3}(x - 1)^3 - \frac{6}{x^4}(x - 1)^4 + \dots$
  - $\ln(x) + \frac{1}{x}(x - 1) - \frac{1}{2x^2}(x - 1)^2 + \frac{1}{3x^3}(x - 1)^3 - \frac{1}{4x^4}(x - 1)^4 + \dots$
  - This is not possible.
- If  $a = 0$ , what function is represented by the Taylor series  $1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \dots$ ?
  - $\exp(x)$
  - $\sin(x)$
  - $\cos(x)$
  - This is not a Taylor series.
- A Taylor series converges when  $x = 12, 13$  and  $15$ , but diverges when  $x = 9, 16$  and  $18$ . Which of the following could be  $a$ , the point where the Taylor series is centered?
  - $a = 9$
  - $a = 11$
  - $a = 13$
  - $a = 15$
  - All of the above are possible.
  - None of the above are possible.
- Suppose we find a Taylor series for the function  $f(x)$  centered at the point  $a = 5$ . Where would we expect this Taylor series to probably give us a better estimate?
  - $x = 0$
  - $x = 3$
  - $x = 8$

- (d) There is no way to tell.
5. A Taylor series for a function  $f(x)$  at  $a = 10$  has a radius of convergence of 3. If we use the first 10 terms of this series to estimate  $f(15)$  we will probably get
- (a) an infinite result.
  - (b) a result which is closer to the real value of  $f(15)$  than if we used 5 terms.
  - (c) a result which is farther from the real value of  $f(15)$  than if we used 25 terms.
  - (d) a result which is closer to the real value of  $f(15)$  than if we used 15 terms.
  - (e) More than one of the above are true.
6. We are given a Taylor series for a function  $g(x)$  at  $a = -5$ , with a radius of convergence of 6. Which would give the best estimate of  $g(-5)$ ?
- (a) The first term of the Taylor series.
  - (b) The first 5 terms of the Taylor series.
  - (c) The first 10 terms of the Taylor series.
  - (d) The first 100 terms of the Taylor series.
  - (e) All would give the same result.