MathQuest: Series

Geometric Series

1. What will we get if we add up the infinite series of numbers: $16 + 8 + 4 + 2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$?

   (a) This infinite sum will reach a number less than 32.

   (b) This infinite sum is equal to 32.

   (c) This infinite sum will reach a number greater than 32.

   (d) Because we're adding up an infinite number of numbers which are all greater than zero, the sum diverges to infinity.

   Answer: (b). This infinite sum = 32. This question is designed to be asked with no introduction to infinite series at all. Just from logical reasoning we can see that each time we add another term, we get halfway closer to 32, and so the limit of an infinite sum should be equal to 32.

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   SER.00.01.010

   CC KC MA232 S07: 75/**10**/5/10 time 2:00
   CC HZ MA232 S08: 54/**33**/4/8 time 2:00
   HHS JG MA232 S08: 55/**27**/18/0
   CC JS MA232 S09: 39/**48**/4/9
   CC HZ MA232 S10: 57/**28**/10/6 time 2:30

2. What will we get if we add up the infinite series of numbers: $12 + 4 + \frac{4}{3} + \frac{4}{9} + \frac{4}{27} + \cdots$?

   (a) This infinite sum will converge to a number less than 18.

   (b) This infinite sum is equal to 18.

   (c) This infinite sum will converge a number between 18 and 19.

   (d) This infinite sum will converge a number greater than 19.

   (e) This infinite sum diverges to infinity.

   Answer: (b). This infinite sum = 18. This sum is a bit harder than the previous one, because this time each time we add a term we get 2/3 of the way closer to 18. Students should be able to figure out that it converges to 18, just by looking at partial sums, but to explain why is harder. This question motivates the derivation of the formula for geometric series $S = a + ax + ax^2 + ax^3 + \cdots = a/(1-x)$ if $|x| < 1$.

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SER.00.01.020

CC KC MA232 S07: 5/**85**/10/0/0 time 2:00
CC HZ MA232 S08: 0/**71**/21/8 time 2:30
HHS JG MA232 S08: 0/**100**/0/0/0
CC JS MA232 S09: 11/**84**/5/0/0
CC HZ MA232 S10: 14/**64**/7/11/4 time 3:00

3. What will we get if we add up the infinite series of numbers: $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} + \cdots$?

   (a) This infinite sum will converge to 1/2.

   (b) This infinite sum will converge to 2/3.

   (c) This infinite sum will converge to 2.

   (d) This is not a geometric series.

   Answer: (b). Here we have the series starting with $a = 1$ and the successive ratio $x = -1/2$, so the infinite series is $1/(1 - (-\frac{1}{2})) = 2/3$.

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   SER.00.01.030

   CC KC MA232 S07: 0/**95**/0/5 time 3:00
   CC HZ MA232 S08: 0/**78**/21/4 time 2:00
   HHS JG MA232 S08: 25/**75**/0/0
   CC JS MA232 S09: 0/**75**/4/21
   CC HZ MA232 S10: 0/**80**/10/7 time 2:30

4. What will we get if we add up the first 10 terms in the series: $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} + \cdots$?

   (a) 0.663

   (b) 0.664

   (c) 0.666

   (d) 0.667

   (e) 0.668

   Answer: (c). This is a straight forward check of the partial sum formula. Here we have the series starting with $a = 1$ and the successive ratio $x = -1/2$, so the partial sum is $S_n = 1(1 - (-0.5)^n)/(1 - (-0.5))$, so $S_{10} = 0.666$.

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SER.00.01.040

CC KC MA232 S07: 0/0/**14**/86/0 time 2:00
CC JS MA232 S09: 0/0/**62**/38/0

5. What is $\sum_{j=1}^{5} 4j$?

   (a) 15
   (b) 20
   (c) 40
   (d) 60

   Answer: (d). This is a quick check to see if students understand summation notation.

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   SER.00.01.050

   CC KC MA232 S07: 0/5/0/**95** time 1:00
   CC HZ MA232 S08: 0/0/0/**100** time 2:00
   HHS JG MA232 S08: 0/18/0/**82**
   CC JS MA232 S09: 0/0/0/**100**
   CC HZ MA232 S10: 0/0/10/**90** time 2:30

6. What will we get if we add up the infinite series: $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} \cdots$?

   (a) 2
   (b) A number between 2 and 3.
   (c) A number between 3 and 4.
   (d) A number between 4 and 5.
   (e) A number between 5 and 10.
   (f) This infinite series diverges to infinity.

   Answer: (f). This is the harmonic series, and we introduce it to point out that not all series are geometric, and not all series converge, even if $\lim_{n\to\infty} a_n = 0$.

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   SER.00.01.060

   CC KC MA232 S07: 29/29/19/5/5/**14** time 4:00
   CC HZ MA232 S08: 25/46/13/4/**13** time 4:00
   HHS JG MA232 S08: 0/33/8/8/8/**42**
   CC JS MA232 S09: 0/5/0/0/0/**95**

7. Which of the following series is not geometric?

   (a) $\sum_{n=0}^{\infty} \frac{15}{3^n}$
   (b) $\sum_{n=5}^{\infty} 12^{2n+4}$
   (c) $\sum_{n=1}^{\infty} 9^{-n}$
   (d) $\sum_{n=1}^{\infty} 4^{1/n}$
   (e) $\sum_{n=0}^{\infty} \frac{5 \cdot 3^n}{7^{3n}}$
   (f) More than one of these is not geometric.

   Answer: (d). This series is $4 + 4^{1/2} + 4^{1/3} + 4^{1/4} + \cdots$ which cannot be written in the form $\sum ax^n$.

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   SER.00.01.070

   CC HZ MA232 S08: 0/46/0/**8**/4/42 time 2:40

8. Which of the following geometric series converge?

   (a) $\sum_{n=0}^{\infty} \frac{8}{(-2)^n}$
   (b) $\sum_{n=5}^{\infty} 6^{3n+2}$
   (c) $\sum_{n=1}^{\infty} (-4)^{-n}$
   (d) $\sum_{n=0}^{\infty} \frac{6 \cdot 2^n}{6^{3n}}$
   (e) Exactly two of these converge.
   (f) Exactly three of these converge.

   Answer: (f). Only the series in (b) does not converge. For all the other series if we write them in the form $\sum ax^n$ we have $|x| < 1$.

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   SER.00.01.080