

MathQuest: Series

Convergence Tests

1. For what values of p does the series $\sum_{n=1}^{\infty} 1/n^p$ converge?

- (a) This series converges for all values of p .
- (b) This series converges only if $p > 2$.
- (c) This series converges only if $p > 1$.
- (d) This series converges only if $p > 0$.
- (e) This series does not converge for any values of p .

Answer: (c). This question is designed to be asked with little or no introduction to convergence tests. It should be quickly apparent that the series diverges if $p \leq 0$, however it is not usually obvious exactly how large p must be to make the series converge. After this question we can show that the series converges if $p > 1$.

CC KC MA232 S07: 0/19/71/10 time 2:30

by Carroll College MathQuest

SER.00.02.010

2. Does the series $\sum_{n=1}^{\infty} \frac{100}{n^2+2}$ converge?

- (a) Yes, this series converges.
- (b) No, this series does not converge.
- (c) It is impossible to tell.

Answer: (a). In the post vote discussion, two points need to be made: First, the numerator is irrelevant. We can multiply any series by any constant and it does not affect its convergence. Thus all we are really interested in is whether the series $\sum 1/(n^2 + 2)$ converges. Second, $1/(n^2 + 2) < 1/n^2$, and we know that $\sum 1/n^2$ converges. This leads us to the idea of the comparison test.

CC KC MA232 S07: 68/32/0 time 2:00

by Carroll College MathQuest

SER.00.02.020

3. Does the series $\sum_{n=1}^{\infty} \frac{\ln(n)}{n}$ converge?

- (a) This series converges.

- (b) This series diverges.
- (c) It is impossible to tell.

Answer: (b). This can be understood with another application of the comparison test. We know that the harmonic series $\sum 1/n$ diverges, and $\ln(n) > 1$ for $n \geq 3$, thus this series must diverge.

CC KC MA232 S07: 18/77/5 time 3:00

by Carroll College MathQuest
SER.00.02.030

4. Does the series $\sum_{n=1}^{\infty} \frac{n^3}{3^n}$ converge?
- (a) This series converges.
 - (b) This series diverges.
 - (c) It is impossible to tell.

Answer: (a). This is not as easy to do with the comparison test. However the students should be able to see that the exponential term grows much faster than the cubic term. This leads us to the ratio test: For a series $\sum a_n$ we find the limit $L = \lim_{n \rightarrow \infty} |a_{n+1}|/|a_n|$. If $L < 1$ the series converges. If $L > 1$ the series diverges. If $L = 1$ we learn nothing.

CC KC MA232 S07: 81/19/0 time 2:30

by Carroll College MathQuest
SER.00.02.040

5. Does the series $\sum_{n=1}^{\infty} \frac{n!}{(2n)!}$ converge?
- (a) This series converges.
 - (b) This series diverges.
 - (c) It is impossible to tell.

Answer: (a). There are several ways to argue this one: The ratio test is fairly straight forward: $|a_{n+1}|/|a_n| = 1/(4n + 2)$ and so as $n \rightarrow \infty$, $L \rightarrow 0$. This question provides some experience dealing with factorials. At this point the students should have some good intuition about series, so the important thing to emphasize in the discussion is not *whether* the series converges, but exactly why it does or doesn't and how to make a logical argument demonstrating this.

CC KC MA232 S07: 27/18/5 time 2:00

by Carroll College MathQuest
SER.00.02.050

6. Does the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converge?

- (a) This series converges.
- (b) This series diverges.
- (c) It is impossible to tell.

Answer: (a). The students should recognize that this is a variation on the harmonic series, which we know diverges, so it may be surprising that this converges. This leads to the alternating series test: Any series of the form $\sum(-1)^n a_n$ converges if $a_n > 0$ and $\lim_{n \rightarrow \infty} a_n = 0$.

CC KC MA232 S07: 61/17/22 time 2:45

by Carroll College MathQuest

SER.00.02.060