MathQuest: Series

Power Series

1. Consider the power series $\sum_{n=1}^{\infty} \frac{(x-4)^n}{4^n}$. What values of x will make this series converge?

   (a) This series converges for all values of x.

   (b) This series converges for all values of x between 0 and 8.

   (c) This series converges for all values of x between -4 and 4.

   (d) This series converges for all values of x between -8 and 0.

   (e) This series diverges for all values of x.

   Answer: (b). This question is designed to be asked with little or no introduction to power series. Even if they don't know how to proceed, they can try some things out numerically, picking a few values of x to see what happens. To work this out fully, the students first must recognize that this series is geometric with a successive ratio of $(x-4)/4$ and so to make the series converge the absolute value of this must be less than one. This leads us to find that the series converges if x is between 0 and 8.

   CC KC MA232 S07: 0/**74**/21/5/0 time 3:30
   CC HZ MA232 S08: 20/**44**/28/0/8 time 3:15
   CC JS MA232 S09: 0/**75**/15/5/5

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   SER.00.03.010

2. Consider the power series $\sum_{n=1}^{\infty} \frac{(x-4)^n}{4^n}$. Will this series converge if $x = 0$ or if $x = 8$?

   (a) This series converges for both $x = 0$ and $x = 8$.

   (b) This series does not converge for either $x = 0$ or $x = 8$.

   (c) This series converges for $x = 8$ but does not converge for $x = 0$.

   (d) This series converges for $x = 0$ but does not converge for $x = 8$.

   Answer: (b). Here we learn to examine the endpoints of the interval of convergence. If $x = 0$ the series is $\sum(-1)^n$ which never converges, and if $x = 8$ the series is $\sum(1)^n$ which diverges.

   CC KC MA232 S07: 23/**0**/33/44 time 3:15

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3. Consider the power series $\sum_{n=1}^{\infty} \frac{(3x)^n}{n^8}$. What values of x will make this series converge?

   (a) This series converges for all values of x.

   (b) This series converges for all values of x between -3 and 3.

   (c) This series converges for all values of x between 0 and 3.

   (d) This series converges for all values of x between -1/3 and 1/3.

   (e) This series diverges for all values of x.

   Answer: (d). We can analyze this power series with the ratio test, finding that the limit of the ratio of successive terms is $3x$. Thus the series converges if this is less than one.

   CC KC MA232 S07: 5/0/0/**90**/5 time 3:30
   CC JS MA232 S09: 15/5/5/**45**/30

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4. Consider the power series $\sum_{n=1}^{\infty} \frac{(2x)^n}{n^7}$. Will this series converge if $x = -1/2$ or if $x = +1/2$?

   (a) This series converges for both $x = -1/2$ and $x = +1/2$.

   (b) This series does not converge for either $x = -1/2$ or $x = +1/2$.

   (c) This series converges for $x = -1/2$ but does not converge for $x = +1/2$.

   (d) This series converges for $x = +1/2$ but does not converge for $x = -1/2$.

   Answer: (a). This series converges for x between $-\frac{1}{2}$ and $\frac{1}{2}$. If $x = +1/2$ we have a p series $\sum 1/x^7$ and we know this converges. If $x = -1/2$ we have an alternating series that passes the alternative series test and converges.

   CC KC MA232 S07: **72**/0/11/16 time 2:30

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5. Consider the power series $\sum_{n=1}^{\infty} \frac{(x-8)^n}{n(-6)^n}$. What values of x will make this series converge?

   (a) This series converges for all values of x.

   (b) This series converges for all values of x between 2 and 14.

   (c) This series converges for all values of x between -8 and 8.

   (d) This series converges for all values of x between 0 and 16.

(e) This series diverges for all values of x.

Answer: (b). We can analyze this power series with the ratio test, finding that the limit of the ratio of successive terms is $(8 - x)/6$. Thus the series converges if this is less than one, so it converges if x is between 2 and 14.

CC KC MA232 S07: 30/**55**/15/0/0 time 4:00

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6. Consider the power series $\sum_{n=1}^{\infty} \frac{(x-5)^n}{n(-3)^n}$. Will this series converge if $x = 2$ or if $x = 8$?

   (a) This series converges for both $x = 2$ and $x = 8$.

   (b) This series does not converge for either $x = 2$ or $x = 8$.

   (c) This series converges for $x = 2$ but does not converge for $x = 8$.

   (d) This series converges for $x = 8$ but does not converge for $x = 2$.

   Answer: (d). This series converges for x between 2 and 8. If $x = 2$ we have the harmonic series which does not converge. If $x = 8$ we have an alternating series that passes the alternative series test and converges.

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   SER.00.03.060

7. A power series converges when $x = 2.5, 2.7$ and 2.8, but diverges when $x = 2.1, 2.2$ and 2.9. Which of the following could be the point where the power series is centered?

   (a) 2.3

   (b) 2.6

   (c) 2.7

   (d) 2.8

   (e) All of the above are possible.

   (f) More than one but not all of the above are possible.

   Answer: (f). Both (b) 2.6 and (c) 2.7 are possible. 2.7 works if the interval of convergence is $2.5 \leq x < 2.9$.

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