MathQuest: Series

Taylor Series

1. Find the Taylor series for the function $\ln(x)$ at the point $a = 1$.

   (a) $(x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4 + \cdots$
   (b) $(x-1) - (x-1)^2 + 2(x-1)^3 - 6(x-1)^4 + \cdots$
   (c) $\ln(x) + \frac{1}{x}(x-1) - \frac{1}{x^2}(x-1)^2 + \frac{2}{x^3}(x-1)^3 - \frac{6}{x^4}(x-1)^4 + \cdots$
   (d) $\ln(x) + \frac{1}{x}(x-1) - \frac{1}{2x^2}(x-1)^2 + \frac{1}{3x^3}(x-1)^3 - \frac{1}{4x^4}(x-1)^4 + \cdots$
   (e) This is not possible.

   Answer: (a). The common misconceptions probed by this question include not evaluating the derivatives and neglecting the factorial terms.

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   CC KC MA334 S08: **27**/0/73/0/0 time 4:00
   CC KC MA334 S09: **16**/24/12/44/4 time 4:00
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2. If $a = 0$, what function is represented by the Taylor series $1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \cdots$?

   (a) $\exp(x)$
   (b) $\sin(x)$
   (c) $\cos(x)$
   (d) This is not a Taylor series.

   Answer: (c). This is a Taylor series for $\cos(x)$ centered at $x = 0$. For this problem, we expect the students to generate the Taylor series of these functions to see which matches the given series

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3. A Taylor series converges when $x = 12, 13$ and 15, but diverges when $x = 9, 16$ and 18. Which of the following could be a, the point where the Taylor series is centered?

   (a) $a = 9$
   (b) $a = 11$
   (c) $a = 13$
   (d) $a = 15$
   (e) All of the above are possible.
   (f) None of the above are possible.

   Answer: (c). If we had a radius of convergence of 3, then a Taylor series centered at $a = 13$ would converge and diverge at the specified values.

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4. Suppose we find a Taylor series for the function $f(x)$ centered at the point $a = 5$. Where would we expect this Taylor series to probably give us a better estimate?

   (a) $x = 0$
   (b) $x = 3$
   (c) $x = 8$
   (d) There is no way to tell.

   Answer: (b). Usually a Taylor series gives better estimates at point closer to the point at which the Taylor series is centered.

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   CC KC MA334 S09: 0/**84**/0/16 time 2:00
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5. A Taylor series for a function $f(x)$ at $a = 10$ has a radius of convergence of 3. If we use the first 10 terms of this series to estimate $f(15)$ we will probably get

(a) an infinite result.

(b) a result which is closer to the real value of $f(15)$ than if we used 5 terms.

(c) a result which is farther from the real value of $f(15)$ than if we used 25 terms.

(d) a result which is closer to the real value of $f(15)$ than if we used 15 terms.

(e) More than one of the above are true.

Answer: (d). As we add up more and more terms, our results will diverge, taking us farther from the true value of $f(15)$, thus fewer terms will leave us closer to $f(15)$.

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CC KC MA334 S08: 9/0/0/**18**/73 time 2:40
CC KC MA334 S09: 12/8/4/**46**/29 time 3:00
CC HZ MA232 S10: 4/4/4/**12**/76 time 3:00
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6. We are given a Taylor series for a function $g(x)$ at $a = -5$, with a radius of convergence of 6. Which would give the best estimate of $g(-5)$?

(a) The first term of the Taylor series.

(b) The first 5 terms of the Taylor series.

(c) The first 10 terms of the Taylor series.

(d) The first 100 terms of the Taylor series.

(e) All would give the same result.

Answer: (e). If we evaluate a Taylor series at $x = a$, the point where it is centered, then the first term is simply $f(a)$ and all the other terms are zero.

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