

MathQuest: Series

Taylor Series

1. Find the Taylor series for the function $\ln(x)$ at the point $a = 1$.

- (a) $(x - 1) - \frac{1}{2}(x - 1)^2 + \frac{1}{3}(x - 1)^3 - \frac{1}{4}(x - 1)^4 + \dots$
- (b) $(x - 1) - (x - 1)^2 + 2(x - 1)^3 - 6(x - 1)^4 + \dots$
- (c) $\ln(x) + \frac{1}{x}(x - 1) - \frac{1}{x^2}(x - 1)^2 + \frac{2}{x^3}(x - 1)^3 - \frac{6}{x^4}(x - 1)^4 + \dots$
- (d) $\ln(x) + \frac{1}{x}(x - 1) - \frac{1}{2x^2}(x - 1)^2 + \frac{1}{3x^3}(x - 1)^3 - \frac{1}{4x^4}(x - 1)^4 + \dots$
- (e) This is not possible.

Answer: (a). The common misconceptions probed by this question include not evaluating the derivatives and neglecting the factorial terms.

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2. If $a = 0$, what function is represented by the Taylor series $1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \dots$?

- (a) $\exp(x)$
- (b) $\sin(x)$
- (c) $\cos(x)$
- (d) This is not a Taylor series.

Answer: (c). This is a Taylor series for $\cos(x)$ centered at $x = 0$. For this problem, we expect the students to generate the Taylor series of these functions to see which matches the given series

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3. A Taylor series converges when $x = 12, 13$ and 15 , but diverges when $x = 9, 16$ and 18 . Which of the following could be a , the point where the Taylor series is centered?

- (a) $a = 9$
- (b) $a = 11$
- (c) $a = 13$
- (d) $a = 15$
- (e) All of the above are possible.
- (f) None of the above are possible.

Answer: (c). If we had a radius of convergence of 3 , then a Taylor series centered at $a = 13$ would converge and diverge at the specified values.

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4. Suppose we find a Taylor series for the function $f(x)$ centered at the point $a = 5$. Where would we expect this Taylor series to probably give us a better estimate?

- (a) $x = 0$
- (b) $x = 3$
- (c) $x = 8$
- (d) There is no way to tell.

Answer: (b). Usually a Taylor series gives better estimates at point closer to the point at which the Taylor series is centered.

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CC HZ MA232 S08: 4/**89**/0/7 time 1:20

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CC KC MA334 S09: 0/**84**/0/16 time 2:00

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5. A Taylor series for a function $f(x)$ at $a = 10$ has a radius of convergence of 3 . If we use the first 10 terms of this series to estimate $f(15)$ we will probably get

- (a) an infinite result.
- (b) a result which is closer to the real value of $f(15)$ than if we used 5 terms.
- (c) a result which is farther from the real value of $f(15)$ than if we used 25 terms.
- (d) a result which is closer to the real value of $f(15)$ than if we used 15 terms.
- (e) More than one of the above are true.

Answer: (d). As we add up more and more terms, our results will diverge, taking us farther from the true value of $f(15)$, thus fewer terms will leave us closer to $f(15)$.

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CC HZ MA232 S08: 7/0/4/43/43 time 3:05

CC KC MA334 S08: 9/0/0/18/73 time 2:40

CC KC MA334 S09: 12/8/4/46/29 time 3:00

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6. We are given a Taylor series for a function $g(x)$ at $a = -5$, with a radius of convergence of 6. Which would give the best estimate of $g(-5)$?
- (a) The first term of the Taylor series.
 - (b) The first 5 terms of the Taylor series.
 - (c) The first 10 terms of the Taylor series.
 - (d) The first 100 terms of the Taylor series.
 - (e) All would give the same result.

Answer: (e). If we evaluate a Taylor series at $x = a$, the point where it is centered, then the first term is simply $f(a)$ and all the other terms are zero.

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CC HZ MA232 S08: 7/14/4/21/54 time 1:55

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