Classroom Voting Questions: Calculus I

2.1 How do we measure speed?

1. The speedometer in my car is broken. In order to find my average velocity on a trip from Helena to Missoula, I need

    i. the distance between Helena and Missoula

    ii. the time spent traveling

    iii. the number of stops I made during the trip

    iv. a friend with a stopwatch

    v. a working odometer

    vi. none of the above

    Select the best combination:

    (a) i, ii, & iii only

    (b) i & ii only

    (c) iv & v only

    (d) vi

    (e) a combination that is not listed here

2. The speedometer in my car is broken. In order to find my velocity at the instant I hit a speed trap, I need

   i. the distance between Helena and Missoula

   ii. the time spent traveling

   iii. the number of stops I made during the trip

   iv. a friend with a stopwatch

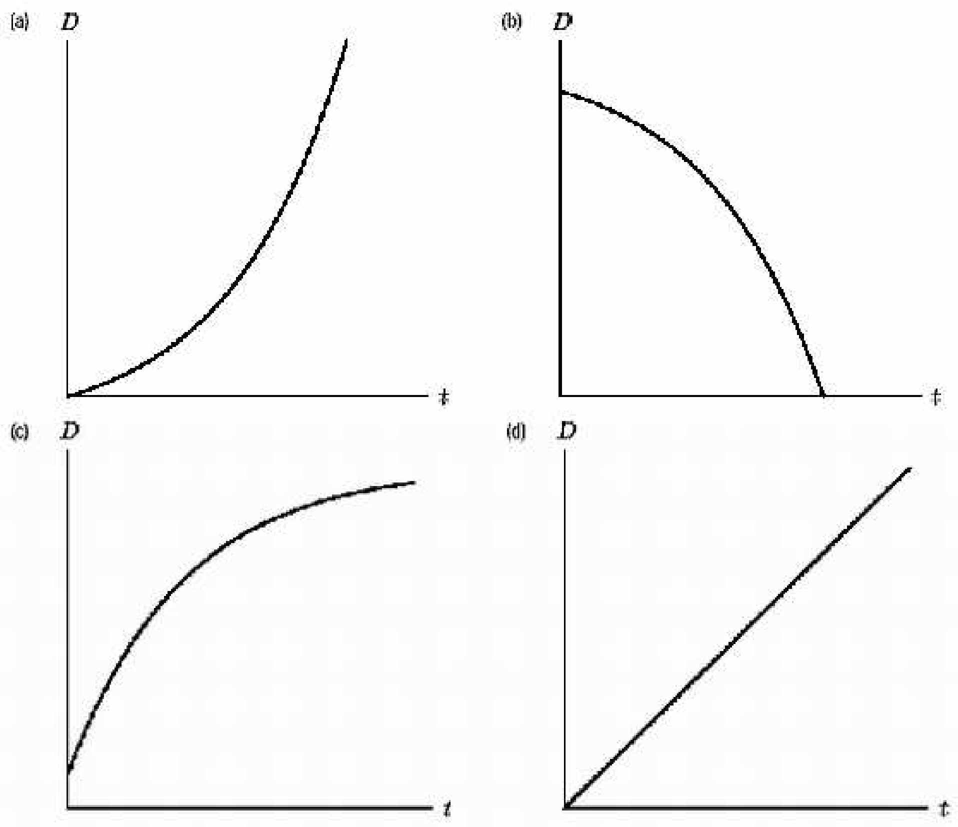
   v. a working odometer

   vi. none of the above

   Select the best combination:

   (a) i, ii, & iii only
   (b) i & ii only
   (c) iv & v only
   (d) vi
   (e) a combination that is not listed here

3. Which graph represents an object slowing down, where D is distance, and t is time? Assume that the units are the same for all graphs.



4. **True or False:** If a car is going 50 miles per hour at 2 pm and 60 miles per hour at 3 pm, then it travels between 50 and 60 miles during the hour between 2 pm and 3 pm.

   (a) True, and I am very confident
   (b) True, but I am not very confident
   (c) False, but I am not very confident
   (d) False, and I am very confident

5. **True or False:** If a car travels 80 miles between 2 and 4 pm, then its velocity is close to 40 mph at 2 pm.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

6. **True or False:** If the time interval is short enough, then the average velocity of a car over the time interval and the instantaneous velocity at a time in the interval can be expected to be close.

   (a) True, and I am very confident
   (b) True, but I am not very confident
   (c) False, but I am not very confident
   (d) False, and I am very confident

7. **True or False:** If an object moves with the same average velocity over every time interval, then its average velocity equals its instantaneous velocity at any time.

   (a) True, and I am very confident
   (b) True, but I am not very confident
   (c) False, but I am not very confident
   (d) False, and I am very confident

2.2 The Derivative at a Point

8. We want to find how the volume of a balloon changes as it is filled with air. We know $V(r) = \frac{4}{3}\pi r^3$, where r is the radius in inches and $V(r)$ is the volume is in cubic inches. The expression $\frac{V(3)-V(1)}{3-1}$ represents the

   (a) Average rate of change of the radius with respect to the volume when the radius changes from 1 inch to 3 inches.

   (b) Average rate of change of the radius with respect to the volume when the volume changes from 1 cubic inch to 3 cubic inches.

   (c) Average rate of change of the volume with respect to the radius when the radius changes from 1 inch to 3 inches.

   (d) Average rate of change of the volume with respect to the radius when the volume changes from 1 cubic inch to 3 cubic inches.

9. We want to find how the volume of a balloon changes as it is filled with air. We know $V(r) = \frac{4}{3}\pi r^3$, where r is the radius in inches and $V(r)$ is the volume is in cubic inches. Which of the following represents the rate at which the volume is changing when the radius is 1 inch?

(a) $\frac{V(1.01)-V(1)}{0.01} \approx 12.69$

(b) $\frac{V(0.99)-V(1)}{-0.01} \approx 12.44$

(c) $\lim_{h\to 0} \frac{V(1+h)-V(1)}{h}$

(d) All of the above

10. Which of the following represents the slope of a line drawn between the two points marked in the figure?

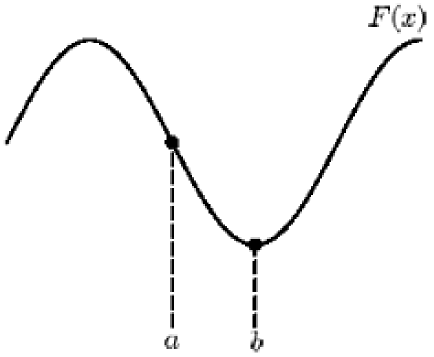


Figure 2.4

(a) $m = \frac{F(a)+F(b)}{a+b}$

(b) $m = \frac{F(b)-F(a)}{b-a}$

(c) $m = \frac{a}{b}$

(d) $m = \frac{F(a)-F(b)}{b-a}$

11. The line tangent to the graph of $f(x) = x$ at (0,0)

(a) is $y = 0$

(b) is $y = x$

(c) does not exist

(d) is not unique. There are infinitely many tangent lines.

12. Suppose that $f(x)$ is a function with $f(2) = 15$ and $f'(2) = 3$. Estimate $f(2.5)$.

(a) 10.5

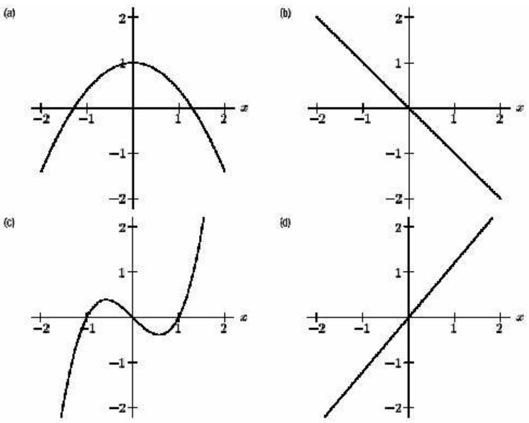
(b) 15

(c) 16.5

(d) 18

2.3 The Derivative Function

13. Which of the following graphs is the graph of the derivative of the function shown in Figure 2.6?



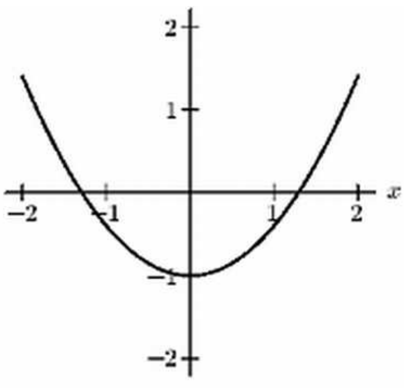


Figure 2.6

14. Which of the following graphs is the graph of the derivative of the function shown in Figure 2.8?

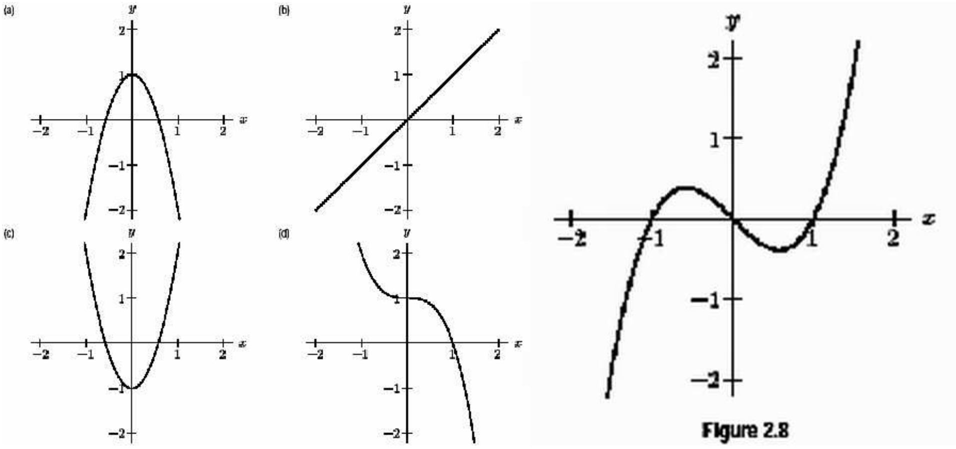


Figure 2.8

15. Which of the following graphs is the graph of the derivative of the function shown in Figure 2.9?

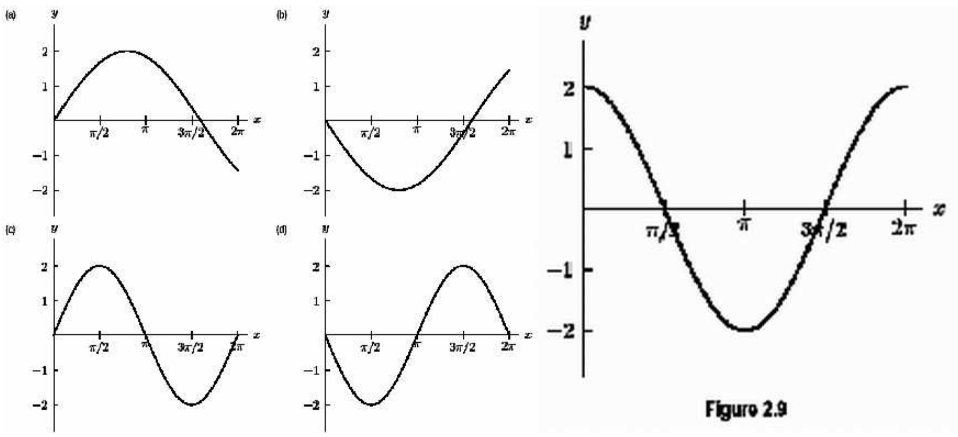


Figure 2.9

16. Which of the following graphs is the graph of the derivative of the function shown in Figure 2.10?

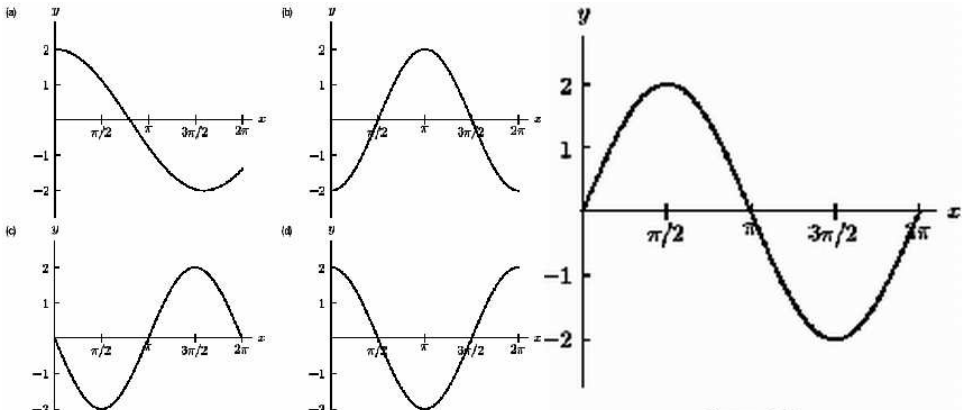


Figure 2.10

17. Which of the following graphs is the graph of the derivative of the function shown in Figure 2.11?

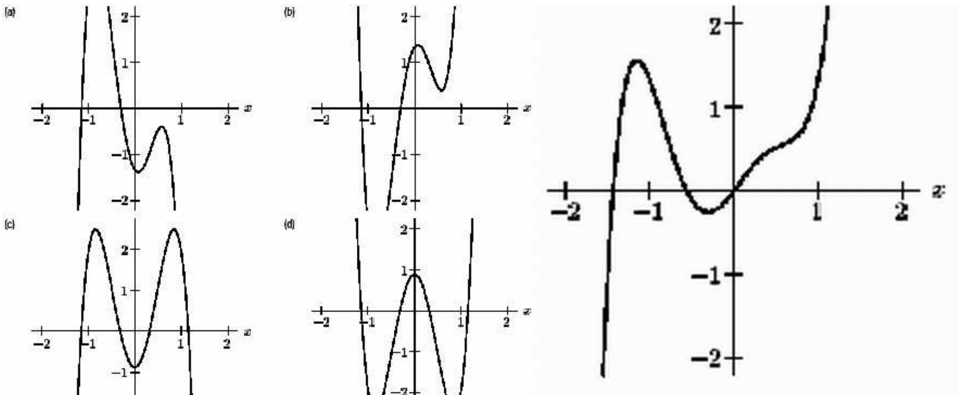


Figure 2.11

18. The graph in Figure 2.12 is the derivative of which of the following functions?

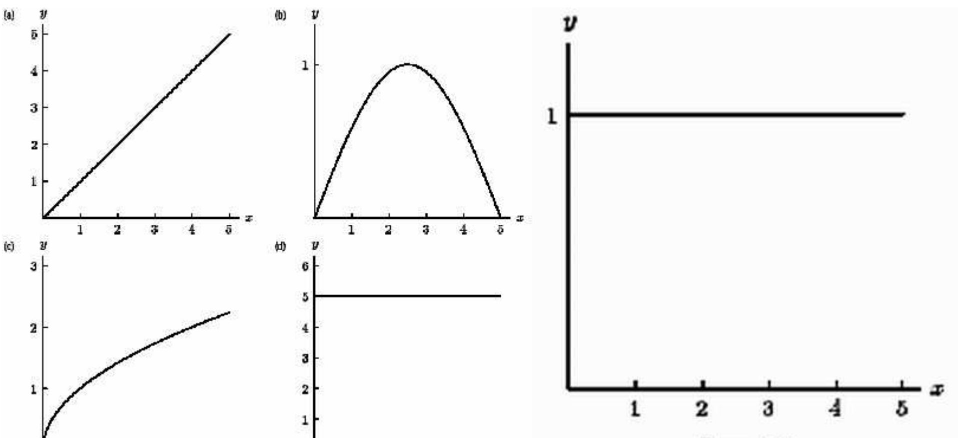


Figure 2.12

19. The graph in Figure 2.13 is the derivative of which of the following functions?

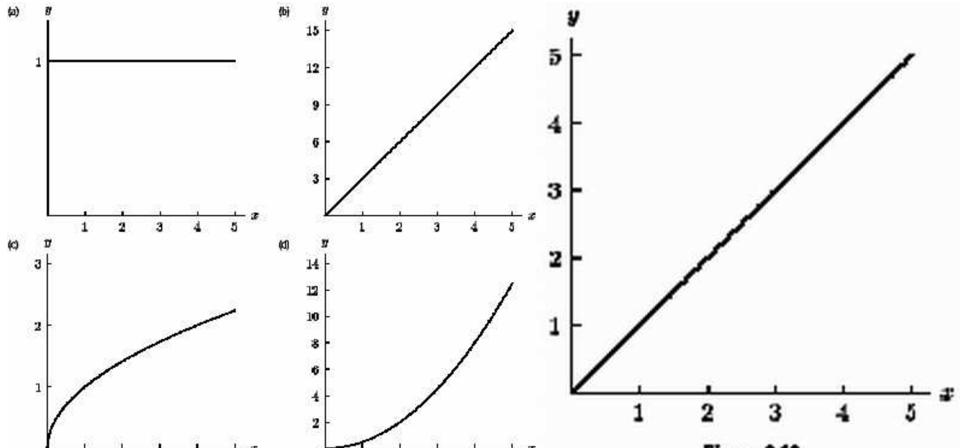


Figure 2.13

20. The graph in Figure 2.14 is the derivative of which of the following functions?

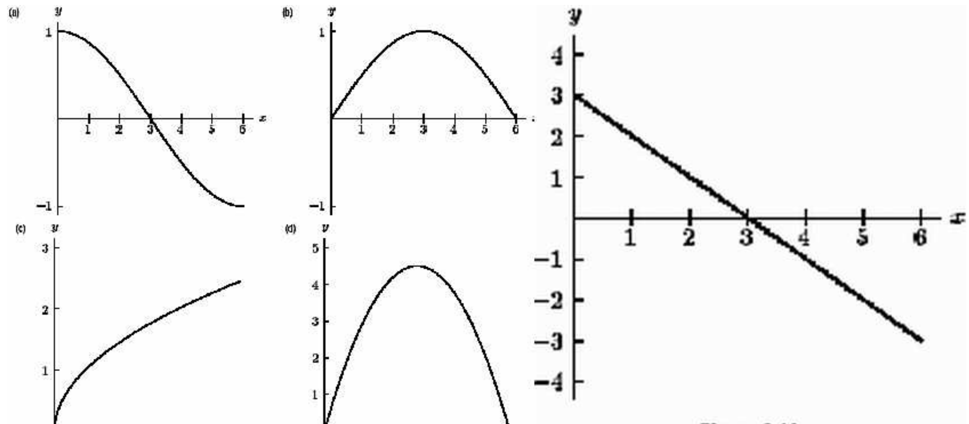


Figure 2.14

21. The graph in Figure 2.15 is the derivative of which of the following functions?

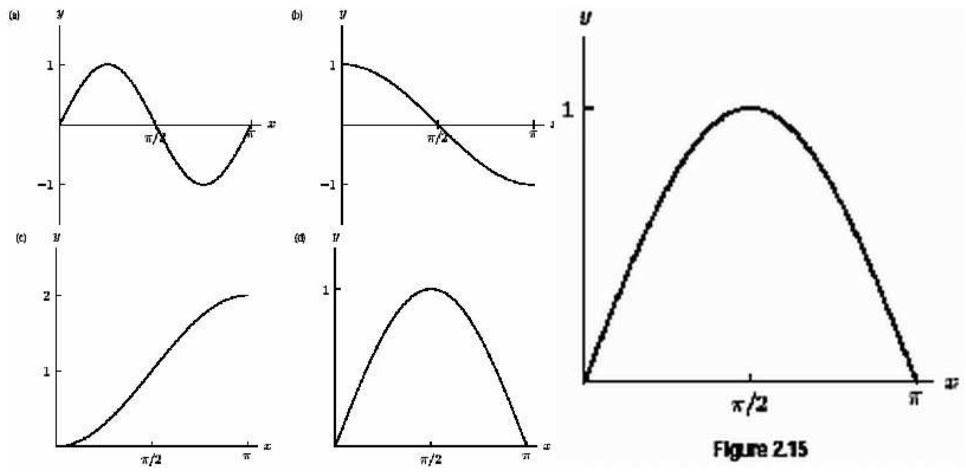


Figure 2.15

22. **True or False:** If $f'(x) = g'(x)$ then $f(x) = g(x)$.

   (a) True, and I am very confident
   (b) True, but I am not very confident
   (c) False, but I am not very confident
   (d) False, and I am very confident

23. Let $f(x) = 2x^3 + 3x^2 + 1$. True or false: On the interval $(-\infty, -1)$, the function f is increasing.

   (a) True, and I am very confident.
   (b) True, but I am not very confident.
   (c) False, but I am not very confident.
   (d) False, and am very confident.

2.4 Interpretations of the Derivative

24. The radius of a snowball changes as the snow melts. The instantaneous rate of change in radius with respect to volume is

(a) $\frac{dV}{dr}$
(b) $\frac{dr}{dV}$
(c) $\frac{dV}{dr} + \frac{dr}{dV}$
(d) None of the above

25. Gravel is poured into a conical pile. The rate at which gravel is added to the pile is

(a) $\frac{dV}{dt}$
(b) $\frac{dr}{dt}$
(c) $\frac{dV}{dr}$
(d) None of the above

26. A slow freight train chugs along a straight track. The distance it has traveled after x hours is given by a function $f(x)$. An engineer is walking along the top of the box cars at the rate of 3 mi/hr in the same direction as the train is moving. The speed of the man relative to the ground is

(a) $f(x) + 3$
(b) $f'(x) + 3$
(c) $f(x) - 3$
(d) $f'(x) - 3$

27. $C(r)$ gives the total cost of paying off a car loan that has an annual interest rate of r %. What are the units of $C'(r)$?

(a) Year / \$
(b) \$ / Year
(c) \$ / %
(d) % / \$

28. $C(r)$ gives the total cost of paying off a car loan that has an annual interest rate of r %. What is the practical meaning of $C'(5)$?

(a) The rate of change of the total cost of the car loan is $C'(5)$.
(b) If the interest rate increases by 1%, then the total cost of the loan increases by $C'(5)$.
(c) If the interest rate increases by 1%, then the total cost of the loan increases by $C'(5)$ when the interest rate is 5%.

(d) If the interest rate increases by 5%, then the total cost of the loan increases by $C'(5)$.

29. $C(r)$ gives the total cost of paying off a car loan that has an annual interest rate of r %. What is the sign of $C'(5)$?

    (a) Positive
    (b) Negative
    (c) Not enough information is given

30. $g(v)$ gives the fuel efficiency, in miles per gallon, of a car going a speed of v miles per hour. What are the units of $g'(v) = \frac{dg}{dv}$?

    (a) (miles)^2/[(gal)(hour)]
    (b) hour/gal
    (c) gal/hour
    (d) (gal)(hour)/(miles)^2

31. $g(v)$ gives the fuel efficiency, in miles per gallon, of a car going a speed of v miles per hour. What is the practical meaning of $g'(55) = -0.54$?

    (a) When the car is going 55 mph, the rate of change of the fuel efficiency decreases to 0.54 miles/gal.

(b) When the car is going 55 mph, the rate of change of the fuel efficiency decreases by 0.54 miles/gal.

(c) If the car speeds up from 55 to 56 mph, then the fuel efficiency is 0.54 miles per gallon.

(d) If the car speeds up from 55 to 56 mph, then the car becomes less fuel efficient by 0.54 miles per gallon.

2.5 The Second Derivative

32. The graph of $y = f(x)$ is shown in figure 2.18. Which of the following is true for f on the interval shown?

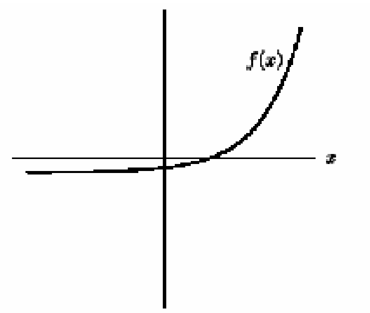


Figure 2.18

i. $f(x)$ is positive
ii. $f(x)$ is increasing
iii. $f'(x)$ is positive
iv. $f'(x)$ is increasing
v. $f''(x)$ is positive

(a) i, ii, and iii only

(b) ii, iii, and v only

(c) ii, iii, iv, and v only

(d) all are true

(e) the correct combination of true statements is not listed here

33. **True or False:** If $f''(x)$ is positive, then $f(x)$ is concave up.

    (a) True, and I am very confident

    (b) True, but I am not very confident

    (c) False, but I am not very confident

    (d) False, and I am very confident

34. **True or False:** If $f''(x)$ is positive, then $f'(x)$ is increasing.

    (a) True, and I am very confident

    (b) True, but I am not very confident

    (c) False, but I am not very confident

    (d) False, and I am very confident

35. **True or False:** If $f'(x)$ is increasing, then $f(x)$ is concave up.

    (a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

36. **True or False:** If the velocity of an object is constant, then its acceleration is zero.

    (a) True, and I am very confident

    (b) True, but I am not very confident

    (c) False, but I am not very confident

    (d) False, and I am very confident

37. In Figure 2.21, the second derivative at points a, b, and c, respectively, is

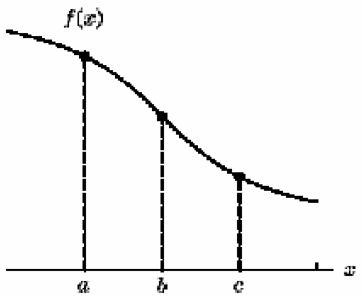


Figure 2.21

(a) +, 0, -

(b) -, 0, +

(c) -, 0, -

(d) +,0,+

(e) +,+,-

(f) -,-,+

38. In figure 2.22, at $x = 0$ the signs of the function and the first and second derivatives, in order, are

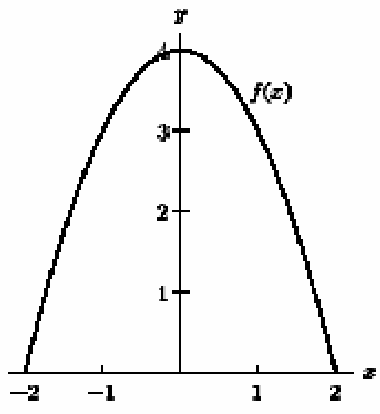


Figure 2.22

(a) +, 0, +

(b) +,0,-

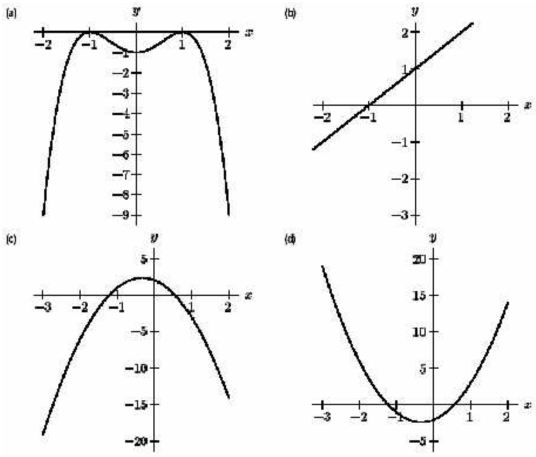
(c) -, +, -

(d) -,+,+

(e) +,-,+

(f) +,+,+

39. Which of the following graphs could represent the second derivative of the function shown in Figure 2.25?



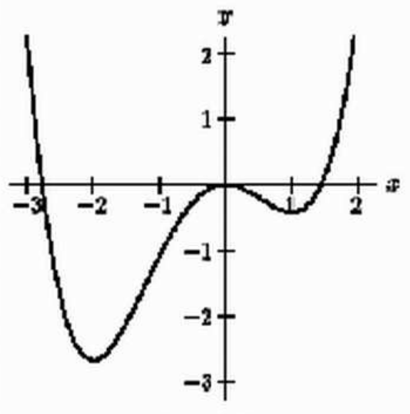


Figure 2.25

40. If an object's acceleration is negative, at that particular instant the object can be

    (a) slowing down only.

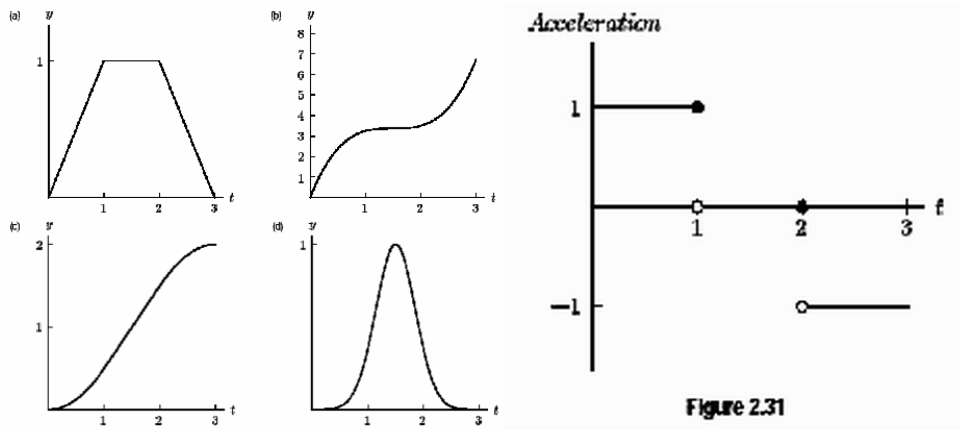
    (b) speeding up only.

    (c) slowing down or momentarily stopped.

    (d) slowing down, momentarily stopped, or speeding up.

41. In *Star Trek: First Contact*, Worf almost gets knocked into space by the Borg. Assume he was knocked into space and his space suit was equipped with thrusters. Worf fires his thrusters for 1 second, which produces a constant acceleration in the positive direction. In the next second he turns off his thrusters. In the third second he fires his thruster producing a constant negative acceleration. The acceleration as a function of

time is given in Figure 2.31. Which of the following graphs represent his position as a function of time?



42. The position of a moving car is given by the function $s(t) = 3t^2 + 3$, where t is in seconds, and s is in feet. What function gives the car's acceleration?

    (a) $a(t) = 3$
    (b) $a(t) = 6t$
    (c) $a(t) = 6$
    (d) $a(t) = 6t + 3$
    (e) $a(t) = 9$

2.6 Differentiability

43. Your mother says "If you eat your dinner, you can have dessert." You know this means, "If you don't

eat your dinner, you cannot have dessert." Your calculus teacher says, "If f is differentiable at x, f is continuous at x." You know this means

(a) if f is not continuous at x, f is not differentiable at x.

(b) if f is not differentiable at x, f is not continuous at x.

(c) knowing f is not continuous at x, does not give us enough information to deduce anything about whether the derivative of f exists at x.

44. If $f'(a)$ exists, $\lim_{x \to a} f(x)$

(a) must exist, but there is not enough information to determine it exactly.

(b) equals $f(a)$.

(c) equals $f'(a)$.

(d) may not exist.

45. **True or False:** The function $f(x) = x^{1/3}$ is continuous at $x = 0$.

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

46. **True or False:** If $f(x) = x^{1/3}$ then there is a tangent line at (0,0).

 (a) True, and I am very confident
 (b) True, but I am not very confident
 (c) False, but I am not very confident
 (d) False, and I am very confident

47. **True or False:** If $f(x) = x^{1/3}$ then $f'(0)$ exists.

 (a) True, and I am very confident
 (b) True, but I am not very confident
 (c) False, but I am not very confident
 (d) False, and I am very confident

3.1 Powers and Polynomials

48. If $f(x) = 2x^2$, then what is $f'(x)$?

 (a) $2x$
 (b) $2x^2$
 (c) 4
 (d) $4x$

(e) $4x^2$

(f) Cannot be determined from what we know

49. If $f(x) = 7$, then what is $f'(x)$?

(a) 7

(b) $7x$

(c) 0

(d) 1

(e) Cannot be determined from what we know

50. If $f(x) = 2x^{2.5}$, then what is $f'(x)$?

(a) $2.5x^{2.5}$

(b) $5x^{2.5}$

(c) $2.5x^{1.5}$

(d) $5x^{1.5}$

(e) Cannot be determined from what we know

51. If $f(x) = \pi^2$, then what is $f'(x)$?

(a) 2π

(b) π^2

(c) 0

(d) 2

(e) Cannot be determined from what we know

52. If $f(x) = 3^x$, then what is $f'(x)$?

(a) $x \cdot 3^{x-1}$
(b) 3^x
(c) $3x^2$
(d) 0
(e) None of the above

53. If $f(x) = 4\sqrt{x}$, then what is $f'(x)$?

(a) $4\sqrt{x}$
(b) $2\sqrt{x}$
(c) $2x^{1/2}$
(d) $4x^{-1/2}$
(e) $2x^{-1/2}$
(f) Cannot be determined from what we know

54. If $f(t) = 3t^2 + 2t$, then what is $f'(t)$?

(a) $3t^2 + 2$
(b) $6t + 2$
(c) $9t^2 + 2t$
(d) $9t + 2$

(e) Cannot be determined from what we know

55. Let $f(x) = -16x^2 + 96x$. Find $f'(2)$.

(a) 0
(b) 32
(c) 128
(d) $f'(2)$ does not exist.

56. If $a + b^2 = 3$, find $\frac{da}{db}$.

(a) $\frac{da}{db} = 0$
(b) $\frac{da}{db} = 2b$
(c) $\frac{da}{db} = -2b$
(d) Cannot be determined from this expression

57. If $r(q) = 4q^{-5}$, then what is $r'(q)$?

(a) $5q^{-5}$
(b) $-20q^{-4}$
(c) $-20q^{-5}$
(d) $-20q^{-6}$
(e) Cannot be determined from what we know

58. If $f(x) = x(x + 5)$, then what is $f'(x)$?

(a) $x + 5$

(b) 1

(c) $2x + 5$

(d) $2x$

(e) Cannot be determined from what we know

59. If $f(x) = \frac{2}{x^3}$, then what is $f'(x)$?

(a) $\frac{2}{3x^2}$

(b) $\frac{-6}{x^4}$

(c) $6x^{-2}$

(d) $-3x^{-4}$

(e) Cannot be determined from what we know

60. If $f(x) = x^2 + \frac{3}{x}$, then what is $f'(x)$?

(a) $2x - 3x^{-2}$

(b) $2x + 3x^{-1}$

(c) $2x - 3x^2$

(d) $x^2 - 3x^{-1}$

(e) Cannot be determined from what we know

61. If $f(x) = 4\sqrt{x} + \frac{5}{x^2}$, then what is $f'(x)$?

(a) $2x^{-1/2} - 10x^{-3}$

(b) $4x^{1/2} + 5x^{-2}$

(c) $2x^{1/2} - 10x^{-3}$

(d) $2x^{-1/2} + 10x^{-3}$

(e) Cannot be determined from what we know

62. If $f(x) = \frac{x^2+5x}{x}$, then what is $f'(x)$?

(a) $2x + 5$

(b) $x + 5$

(c) 1

(d) 0

(e) Cannot be determined from what we know

63. If $f(x) = \frac{x}{x^2+5x}$, then what is $f'(x)$?

(a) $\frac{1}{2x+5}$

(b) $-x^{-2}$

(c) $\frac{1}{x} + \frac{1}{5}$

(d) 1

(e) Cannot be determined from what we know

64. If $f(m) = am^2 + bm$, then what is $f'(m)$?

(a) $m^2 + m$

(b) $2am + b$

(c) am

(d) 0

(e) Cannot be determined from what we know

65. If $p(q) = \frac{2q-8}{q^2}$, then what is $p'(2)$?

(a) $\frac{2}{2q}$

(b) $-2q^{-2} + 16q^{-3}$

(c) $\frac{1}{2}$

(d) $\frac{3}{2}$

(e) 0

(f) Cannot be determined from what we know

66. If $f(d) = ad^2 + bd + d + c$, then what is $f'(d)$?

(a) $2ad + b + d$

(b) $2ad + b + 1$

(c) $2ad + b + c$

(d) $2ad + b$

(e) $2ad + b + 1 + c$

(f) $2ad + b + 2$

67. If $g(d) = ab^2 + 3c^3d + 5b^2c^2d^2$, then what is $g''(d)$?

(a) $3c^3 + 10b^2c^2d$

(b) $10b^2c^2$
(c) $42 + 18cd$
(d) $2ab + 9c^2d + 40bcd$
(e) Cannot be determined from what we know

68. Find the equation of the line that is tangent to the function $f(x) = 3x^2$ when $x = 2$. Recall that this line not only has the same slope as $f(x)$ at $x = 2$, but also has the same value of y when $x = 2$.

(a) $y = 12x - 12$
(b) $y = 6x$
(c) $y = 3x + 6$
(d) $y = 12x$
(e) $y = 6x + 6$

69. Which is the equation of the line tangent to $y = x^2$ at $x = 4$?

(a) $y = (2x)x + 4$
(b) $y = 8x + 4$
(c) $y = 8x - 16$
(d) $y = 16x - 48$

70. A ball is thrown into the air and its height h (in meters) after t seconds is given by the function $h(t) = 10 + 20t - 5t^2$. When the ball reaches its maximum height, its velocity will be zero. At what time will the ball reach its maximum height?

(a) $t = 0$ seconds

(b) $t = 1$ second

(c) $t = 2$ seconds

(d) $t = 3$ seconds

(e) $t = 4$ seconds

71. A ball is thrown into the air and its height h (in meters) after t seconds is given by the function $h(t) = 10 + 20t - 5t^2$. When the ball reaches its maximum height, its velocity will be zero. What will be the ball's maximum height?

(a) $h = 10$ meters

(b) $h = 20$ meters

(c) $h = 30$ meters

(d) $h = 40$ meters

(e) $h = 50$ meters

72. Suppose a stone is thrown vertically upward with an initial velocity of 64 ft/s from a bridge 96 ft above a river. By Newton's laws of motion, the position of the stone (measured as the height above the ground) after t seconds is $s(t) = -16t^2 + 64t + 96$. How many seconds after it is thrown will the stone reach its maximum height?

(a) $(2 - \sqrt{10})$ s

(b) 2 s

(c) $(2 + \sqrt{10})$ s

(d) 4 s

3.2 The Exponential Function

73. $\frac{d}{dx}(e^x)$ is

(a) xe^{x-1}

(b) e^x

(c) $e^x \ln x$

(d) 0

(e) Cannot be determined from what we know

74. $\frac{d}{dx}(5^x)$ is

(a) $x5^{x-1}$

(b) 5^x

(c) $5^x \ln x$

(d) $5^x \ln 5$

(e) Cannot be determined from what we know

75. $\frac{d}{dx}(x^e)$ is

(a) ex^{e-1}

(b) x^e

(c) $x^e \ln x$

(d) ex

(e) Cannot be determined from what we know

76. $\frac{d}{dx}\left(e^7\right)$ is

(a) $7e^6$

(b) e^7

(c) $e^7 \ln 7$

(d) 0

(e) Cannot be determined from what we know

77. $\frac{d}{dx}(3e^x)$ is

(a) $3xe^{x-1}$

(b) $3e^x$

(c) $e^x \ln 3$

(d) 3

(e) Cannot be determined from what we know

78. $\frac{d}{dx}\left(2 \cdot 5^x\right)$ is

(a) 10^x

(b) $2 \cdot 5^x$

(c) $10^x \ln 10$

(d) $2 \cdot 5^x \ln 5$

(e) $10^x \ln 5$

(f) Cannot be determined from what we know

79. $\frac{d}{dx}\left(xe^x\right)$ is

(a) $x^2 e^{x-1}$

(b) xe^x

(c) $e^x \ln x$

(d) Cannot be determined from what we know

80. If $\ln x - y = 0$, find $\frac{dx}{dy}$.

(a) $\frac{dx}{dy} = e^x$

(b) $\frac{dx}{dy} = e^{-x}$

(c) $\frac{dx}{dy} = e^{y}$

(d) $\frac{dx}{dy} = e^{-y}$

(e) Cannot be determined from this expression

81. $\frac{d}{dx}\left(e^{x+2}\right)$ is

(a) $(x+2)e^{x+1}$

(b) $e^2 e^x$

(c) e^2

(d) 0

(e) Cannot be determined from what we know

82. $\frac{d}{dx}\left(e^{2x}\right)$ is

(a) e^{2x}

(b) $e^2 e^x$

(c) 0

(d) Cannot be determined from what we know

83. If $u = 5^v$, find $\frac{d^2u}{dv^2}$.

(a) $\frac{d^2u}{dv^2} = 0$

(b) $\frac{d^2u}{dv^2} = 5^v$

(c) $\frac{d^2u}{dv^2} = 5^v \ln 5$

(d) $\frac{d^2u}{dv^2} = 5^v (\ln 5)^2$

(e) $\frac{d^2u}{dv^2} = v(v-1)5^{v-2}$

(f) Cannot be determined from what we know

84. If $u = ve^w + xy^v$, find $\frac{du}{dv}$.

    (a) $\frac{du}{dv} = e^w + xy^v \ln y$

    (b) $\frac{du}{dv} = ve^w + xy^v \ln y$

    (c) $\frac{du}{dv} = e^w + xy^v \ln v$

    (d) $\frac{du}{dv} = ve^w + xy^v \ln v$

    (e) Cannot be determined from what we know

85. Find the equation of the line that is tangent to the function $g(x) = 2e^x$ at $x = 1$.

    (a) $y = 2e^x x$

    (b) $y = 2ex$

    (c) $y = 2e^x x + 2e$

    (d) $y = 2ex + 2e$

    (e) None of the above

3.3 The Product and Quotient Rules

86. $\frac{d}{dx}\left(x^2e^x\right) =$

    (a) $2xe^x$

    (b) x^2e^x

    (c) $2xe^x + x^2e^{x-1}$

    (d) $2xe^x + x^2e^x$

87. $\frac{d}{dx}\left(xe^x\right) =$

    (a) $xe^x + x^2e^x$

    (b) $e^x + xe^x$

    (c) $2xe^x + xe^x$

    (d) e^x

88. $\frac{d}{dt}\left(\left(t^2+3\right)e^t\right) =$

    (a) $2te^t + \left(t^2+3\right)e^t$

    (b) $(2t+3)\,e^t + \left(t^2+3\right)e^t$

    (c) $2te^t$

    (d) $2te^t + t^2e^t$

    (e) $\left(t^2+3\right)e^t$

89. $\frac{d}{dx}\left(x^3 4^x\right) =$

(a) $3x^2 4^x \ln 4$

(b) $x^3 4^x + x^3 4^x \ln 4$

(c) $3x^2 4^x + x^3 4^x$

(d) $3x^2 4^x + x^3 4^x \ln 4$

90. When differentiating a constant multiple of a function (like $3x^2$) the Constant Multiple Rule tells us $\frac{d}{dx} cf(x) = c \frac{d}{dx} f(x)$ and the Product Rule says $\frac{d}{dx} cf(x) = c \frac{d}{dx} f(x) + f(x) \frac{d}{dx} c$. Do these two rules agree?

(a) Yes, they agree, and I am very confident.

(b) Yes, they agree, but I am not very confident.

(c) No, they do not agree, but I am not very confident.

(d) No, they do not agree, and I am very confident.

91. $\frac{d}{dx} \frac{x}{e^x} =$

(a) $e^x + xe^x$

(b) $\frac{e^x - xe^x}{e^{2x}}$

(c) $\frac{xe^x - e^x}{x^2}$

(d) $\frac{xe^x - e^x}{e^{2x}}$

92. $\frac{d}{dx} \frac{x^{1.5}}{3^x} =$

(a) $\frac{1.5x^{0.5} - 3^x \ln 3}{3^{2x}}$

(b) $\frac{1.5x^{0.5}3^x - x^{1.5}3^x \ln 3}{3^{2x}}$

(c) $\frac{1.5x^{0.5}3^x - x^{1.5}3^x \ln 3}{1.5x^{0.5}}$

(d) $1.5x^{0.5}3^x + x^{1.5}3^x \ln 3$

93. If $e^a - \frac{b}{a^2} = 5$, find $\frac{db}{da}$.

(a) $\frac{db}{da} = e^a$

(b) $\frac{db}{da} = a^2e^a$

(c) $\frac{db}{da} = a^2e^a - 5a^2$

(d) $\frac{db}{da} = 2ae^a + a^2e^a - 10a$

(e) $\frac{db}{da} = 2ae^a + a^2e^a - 10ae^a - 5a^2e^a$

(f) Cannot be determined from this expression

94. $\frac{d}{dx}\left(25x^2e^x\right) =$

(a) $50x^2e^x + 25x^2e^x$

(b) $25xe^x + 25x^2e^x$

(c) $50xe^x + 25x^2e^x$

(d) $50xe^x + 25xe^x$

95. $\frac{d}{dt}\frac{3t+1}{5t+2} =$

(a) $\frac{3(5t+2)-(3t+1)5}{(5t+2)^2}$

(b) $\frac{3(5t+2)-(3t+1)5}{(3t+1)^2}$

(c) $\frac{(3t+1)(5t+2)-(3t+1)5}{(5t+2)^2}$

(d) $\frac{3(5t+2)-(3t+1)(5t+2)}{(5t+2)^2}$

96. $\frac{d}{dt}\frac{\sqrt{t}}{t^2+1} =$

(a) $\frac{\frac{1}{2}t^{-1/2}-2t}{(t^2+1)^2}$

(b) $\frac{\frac{1}{2}t^{-1/2}t^2-2t\sqrt{t}}{(t^2+1)^2}$

(c) $\frac{\frac{1}{2}t^{-1/2}(t^2+1)-2t\sqrt{t}}{(t^2+1)^2}$

(d) $\frac{t^{-1/2}(t^2+1)-2t\sqrt{t}}{(t^2+1)^2}$

97. If $f(3) = 2$, $f'(3) = 4$, $g(3) = 1$, $g'(3) = 3$, and $h(x) = f(x)g(x)$, then what is $h'(3)$?

(a) 2

(b) 10

(c) 11

(d) 12

(e) 14

98. If $f(3) = 2$, $f'(3) = 4$, $g(3) = 1$, $g'(3) = 3$, and $h(x) = \frac{f(x)}{g(x)}$, then what is $h'(3)$?

(a) -2

(b) 2

(c) $\frac{-2}{9}$

(d) $\frac{2}{9}$

(e) 5

99. If $h = \frac{ab^2e^b}{c^3}$ then what is $\frac{dh}{db}$?

(a) $\frac{2abe^b}{c^3}$

(b) $\frac{2abe^b}{3c^2}$

(c) $\frac{2abe^b+ab^2e^b}{c^3}$

(d) $\frac{2abe^bc^3-3c^2ab^2e^b}{c^6}$

100. My old uncle Stanley has a collection of rare and valuable books: He has a total of 4,000 books, that are worth an average of \$60 each. His books are rising in value over time, so that each year, the average price per book goes up by \$0.50. However he also has to sell 30 books per year in order to pay for his snowboarding activities. The value of the collection is

(a) increasing by approximately \$240,000 per year.

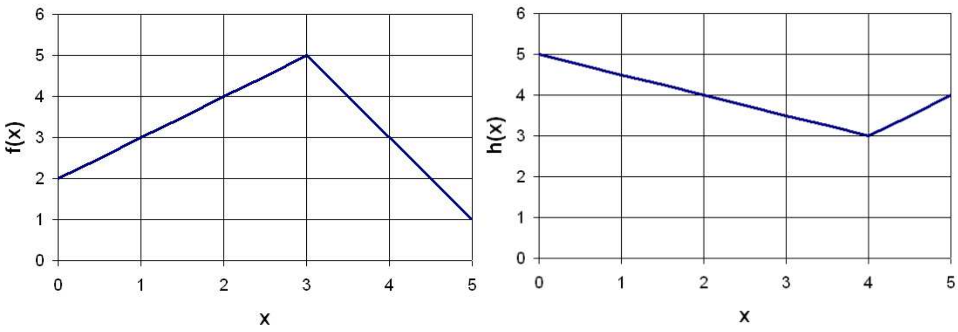
(b) increasing by approximately \$2000 per year.

(c) increasing by approximately \$200 per year.

(d) decreasing by approximately \$1,800 per year.

(e) decreasing by approximately \$119,970 per year.

101. The functions $f(x)$ and $h(x)$ are plotted below. The function $g = 2fh$. What is $g'(2)$?



(a) $g'(2) = -1$

(b) $g'(2) = 2$

(c) $g'(2) = 4$

(d) $g'(2) = 32$

(e) None of the above

3.4 The Chain Rule

102. $\frac{d}{dx}\left(x^2+5\right)^{100} =$

(a) $100(x^2+5)^{99}$

(b) $100x(x^2+5)^{99}$

(c) $200x(x^2+5)^{99}$

(d) $200x(2x+5)^{99}$

103. $\frac{d}{dx}e^{3x} =$

(a) $3e^{3x}$

(b) e^{3x}

(c) $3xe^{3x}$

(d) $3e^3$

104. $\frac{d}{dx}\sqrt{1-x} =$

(a) $\frac{1}{2}(1-x)^{-1/2}$

(b) $-\frac{1}{2}(1-x)^{-1/2}$

(c) $-(1-x)^{-1/2}$

(d) $-\frac{1}{2}(1-x)^{1/2}$

105. $\frac{d}{dx}e^{x^2} =$

(a) x^2e^{2x}

(b) $x^2e^{x^2}$

(c) $2xe^{x^2}$

(d) xe^{x^2}

106. $\frac{d}{dx}3^{4x+1} =$

(a) $4 \cdot 3^{4x+1}\ln 4$

(b) $4 \cdot 3^{4x+1}\ln 3$

(c) $(4x+1)\cdot 3^{4x+1}\ln 3$

(d) $(4x+1)\cdot 3^{4x+1}\ln 4$

107. $\frac{d}{dx}\left(e^x+x^2\right)^2 =$

(a) $2\left(e^x+x^2\right)$

(b) $2\left(e^x+2x\right)\left(e^x+x^2\right)^2$

(c) $2\left(e^x+x^2\right)^2$

(d) $2\left(e^x+2x\right)\left(e^x+x^2\right)$

108. $\frac{d}{dx}x^2e^{-2x} =$

(a) $x^2e^{-2x}-2x^2e^{-2x}$

(b) $2xe^{-2x}-x^2e^{-2x}$

(c) $2xe^{-2x}-2x^2e^{-2x}$

(d) $-2x^2e^{-2x}$

109. $\frac{d}{dx}3^{e^{2x}} =$

(a) $2e^{2x}3^{e^{2x}}\ln 3$

(b) $2e^{2x}3^{e^{2x}}$

(c) $2\cdot 3^{e^{2x}}\ln 3$

(d) $e^{2x}3^{e^{2x}}\ln 3$

(e) $2\cdot 3^{e^{2x}}$

110. If $\frac{dy}{dx} = 5$ and $\frac{dx}{dt} = -2$ then $\frac{dy}{dt} =$

(a) 5

(b) -2

(c) -10

(d) cannot be determined from the information given

111. If $\frac{dz}{dx} = 12$ and $\frac{dy}{dx} = 2$ then $\frac{dz}{dy} =$

(a) 24

(b) 6

(c) $1/6$

(d) cannot be determined from the information given

112. If $y = 5x^2$ and $\frac{dx}{dt} = 3$, then when $x = 4$, $\frac{dy}{dt} =$

(a) 12

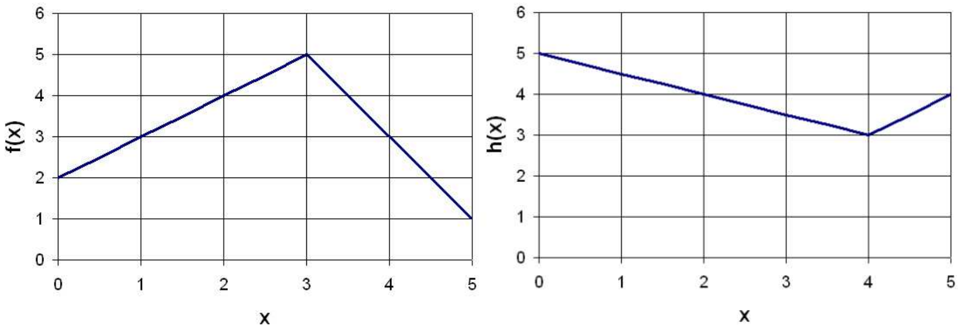
(b) 80

(c) 120

(d) $15x^2$

(e) cannot be determined from the information given

113. The functions $f(x)$ and $h(x)$ are plotted below. The function $g(x) = f(h(x))$. What is $g'(2)$?



(a) $g'(2) = -\frac{1}{2}$

(b) $g'(2) = 1$

(c) $g'(2) = 3$

(d) $g'(2) = 4$

(e) $g'(2)$ is undefined

3.5 The Trigonometric Functions

114. Find a sinusoidal function to model the water level resulting from the tides if we have high tide at 2am with a water level of $W = 32$ feet, and we have a low tide at 7am with a water level of $W = 8$ feet. Let us set time $t = 0$ at midnight.

(a) $W(t) = 8 + 32\sin(t)$.

(b) $W(t) = 8 + 32\cos(t-2)$.

(c) $W(t) = 12 + 20\cos(10(t-2))$

(d) $W(t) = 12 + 20\cos\left(\frac{\pi}{5}(t-2)\right)$

(e) $W(t) = 20 + 12\cos\left(\frac{\pi}{5}(t-2)\right)$

(f) $W(t) = 8 + 32\cos\left(\frac{\pi}{5}(t-2)\right)$

(g) $W(t) = 20 + 12\cos\left(20(t-2)\right)$

(h) None of the above

115. $\frac{d}{dx}\left(-3\sin x\right)$ is

(a) $\cos x$

(b) $-3\sin x$

(c) $3\cos x$

(d) $-3\cos x$

116. $\frac{d}{dx}\frac{\cos x}{25}$ is

(a) $(\sin x)/25$

(b) $-\sin x$

(c) $(-\sin x)/25$

(d) $(-\cos x)/25$

117. $\frac{d}{dx}\left(10\sin\left(12x\right)\right)$ is

(a) $120\cos(12x)$

(b) $10\cos(12x)$

(c) $120\sin(12x)$

(d) $-120\cos(12x)$

118. The 4th derivative of $\sin x$ is

(a) $\sin x$

(b) $\cos x$

(c) $-\sin x$

(d) $-\cos x$

119. The 10th derivative of $\sin x$ is

(a) $\sin x$

(b) $\cos x$

(c) $-\sin x$

(d) $-\cos x$

120. The 100th derivative of $\sin x$ is

(a) $\sin x$

(b) $\cos x$

(c) $-\sin x$

(d) $-\cos x$

121. The 41st derivative of $\sin x$ is

(a) $\sin x$

(b) $\cos x$

(c) $-\sin x$

(d) $-\cos x$

122. The 4th derivative of $\cos x$ is

(a) $\sin x$

(b) $\cos x$

(c) $-\sin x$

(d) $-\cos x$

123. The 30th derivative of $\cos x$ is

(a) $\sin x$

(b) $\cos x$

(c) $-\sin x$

(d) $-\cos x$

124. If $f(x) = \frac{x}{\sin x}$, then $f'(x) =$

(a) $\frac{\sin x - x\cos x}{\sin^2 x}$

(b) $\frac{\sin x - x\cos x}{\cos^2 x}$

(c) $\frac{x\cos x - x\sin x}{\sin^2 x}$

(d) $\frac{\cos x - x\cos x}{\sin^2 x}$

125. $\frac{d}{dx}\sin(\cos x)$ is

(a) $-\cos x\cos(\cos x)$

(b) $-\sin x \cos(\sin x)$
(c) $-\sin x \sin(\cos x)$
(d) $-\sin x \cos(\cos x)$

126. If $f(x) = \sin x \cos x$, then $f'(x) =$

(a) $1 - 2\sin^2 x$
(b) $2\cos^2 x - 1$
(c) $\cos 2x$
(d) All of the above
(e) None of the above

127. If $f(x) = \tan x$, then $f'(x) =$

(a) $\sec^2 x$
(b) $\cot x$
(c) $-\cot x$
(d) All of the above
(e) None of the above

128. We know that $\frac{d}{dx} \sin x = \cos x$. **True or False:** $\frac{d}{dx} \sin(2x) = \cos(2x)$.
(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident

(d) False, and I am very confident

129. $\frac{d}{dx}\left(e^x \sin x\right)$ is

(a) $e^x \cos x$

(b) $e^x \sin x$

(c) $e^x \cos x + e^x \sin x$

(d) $e^x \sin x - e^x \cos x$

130. $\frac{d}{dx}\left(\sin\left(x^2+5\right)\right)$ is

(a) $\cos(x^2+5)$

(b) $\sin(2x+5)$

(c) $2x\sin(x^2+5)$

(d) $2x\cos(x^2+5)$

131. $\frac{d}{dx}\left(\sin^2(ax)\right)$ is

(a) $2\sin(ax)$

(b) $2\cos(ax)$

(c) $2a\sin(ax)$

(d) $2a\sin(ax)\cos(ax)$

132. $\frac{d}{dx}\left(\sin x + e^{\sin x}\right)$ is

(a) $\cos x + e^{\cos x}$

(b) $\cos x + e^{\sin x}$

(c) $\cos x + e^{\sin x} \cos x$

(d) None of the above

133. The equation of the line tangent to the graph of $\cos x$ at $x = 0$ is

(a) $y = 1$

(b) $y = 0$

(c) $y = \cos x$

(d) $y = x$

134. The equation of the line tangent to the graph of $2 \sin 3x$ at $x = \frac{\pi}{3}$ is

(a) $y = 6x - 2\pi$

(b) $y = 6x \cos 3x - 2\pi$

(c) $y = -6x + 2\pi$

(d) $y = -6x + 2\pi - 1$

135. Use linear approximation to estimate $\sin(6)$.

(a) $\sin(6) \approx 1$

(b) $\sin(6) \approx +0.28$

(c) $\sin(6) \approx 0$

(d) $\sin(6) \approx -0.28$

(e) $\sin(6) \approx 2\pi$

(f) $\sin(6) \approx \sin(6) - 0.28\cos(6)$

(g) None of the above

3.6 The Chain Rule and Inverse Functions

136. $\ln(e^{3t})$ is

(a) $\ln(e^3) + \ln(e^t)$

(b) $3\ln(e^t)$

(c) $3\ln(e^3 t)$

(d) $te^{\ln 3}$

(e) $3t$

(f) None of the above

137. $\frac{d}{dt}\ln(t^2+1)$ is

(a) $2t\ln(t^2+1)$

(b) $\frac{2t}{t^2+1}$

(c) $\frac{dt}{\ln(t^2+1)}$

(d) $\frac{1}{t^2+1}$

138. $\frac{d}{dx}\ln(1-x)$ is

(a) $-\ln(1-x)$
(b) $-2x(1-x^2)^{-1}$
(c) $-(1-x)$
(d) $-(1-x)^{-1}$

139. $\frac{d}{dx}\ln(\pi)$ is

(a) $\frac{1}{\pi}$
(b) $\frac{\ln(\pi)}{\pi}$
(c) e^{π}
(d) 0

140. $\frac{d}{d\theta}\ln(\cos\theta)$ is

(a) $\frac{\sin\theta}{\cos\theta}$
(b) $-\sin\theta\ln(\cos\theta)$
(c) $-\frac{\sin\theta}{\cos\theta}$
(d) $-\frac{\sin\theta}{\ln(\cos\theta)}$

141. Find $f'(x)$ if $f(x)=\log_5(2x+1)$.

(a) $f'(x)=\dfrac{2}{\ln 5}\cdot\dfrac{1}{2x+1}$

(b) $f'(x) = \dfrac{2\ln 5}{2x+1}$

(c) $f'(x) = \dfrac{2}{\log_5(2x+1)}$

(d) $f'(x) = \dfrac{2}{2x+1}$

142. If $g(x) = \sin^{-1} x$, then $g'(x)$ is

   (a) $\frac{1}{\sqrt{1-x^2}}$

   (b) $\frac{1}{\cos x}$

   (c) $-\frac{\cos x}{\sin^2 x}$

   (d) $\csc x \cot x$

143. If $g(x) = (\sin x)^{-1}$, then $g'(x)$ is

   (a) $\frac{1}{\sqrt{1-x^2}}$

   (b) $\frac{1}{\cos x}$

   (c) $-\frac{\cos x}{\sin^2 x}$

   (d) $\csc x \cot x$

144. If $p(x) = 3\ln(2x+7)$, then $p'(2)$ is

   (a) $\frac{6}{11}$

   (b) $\frac{6}{2x+7}$

(c) $\frac{3}{2}$

(d) $\frac{3}{x}$

(e) $\frac{3}{11}$

145. If $q = a^2 \ln(a^3 c \sin b + b^2 c)$, then $\frac{dq}{db}$ is

(a) $\frac{a^2}{a^3 c \sin b + b^2 c}$

(b) $\frac{a^5 c \cos b + 2a^2 bc}{a^3 c \sin b + b^2 c}$

(c) $\frac{a^3 c \cos b + 2bc}{a^3 c \sin b + b^2 c}$

(d) $\frac{6a^3 \cos b + 4ab}{a^3 c \sin b + b^2 c}$

3.7 Implicit Differentiation

146. Find $\dfrac{dy}{dx}$ implicitly if $y^3 = x^2 + 1$.

(a) $\dfrac{dy}{dx} = \dfrac{2}{3}x$

(b) $\dfrac{dy}{dx} = 0$

(c) $\dfrac{dy}{dx} = \dfrac{x^2 + 1}{3y^2}$

(d) $\dfrac{dy}{dx} = \dfrac{2x}{3y^2}$

147. Find $\dfrac{dy}{dx}$ implicitly if $x^2 + y^2 = 4$.

(a) $\dfrac{dy}{dx} = -\dfrac{x}{y}$

(b) $\dfrac{dy}{dx} = \dfrac{2}{y} - \dfrac{x}{y}$

(c) $\dfrac{dy}{dx} = -2x$

(d) $\dfrac{dy}{dx} = 0$

(e) $\dfrac{dy}{dx} = -xy$

(f) None of the above

148. Find $\dfrac{dy}{dx}$ implicitly if $x = y^4 + 3$

(a) $\dfrac{dy}{dx} = -\dfrac{1}{2y^3}$

(b) $\dfrac{dy}{dx} = \dfrac{1}{4y^3}$

(c) $\dfrac{dy}{dx} = \dfrac{x}{4y^3}$

(d) $\dfrac{dy}{dx} = \left(\frac{1}{4}\right)^{1/3}$

(e) $\dfrac{dy}{dx} = 0$

(f) None of the above

3.8 L'Hospital's Rule

149. Evaluate $\lim_{x \to \pi/2^-} \dfrac{1 + \tan x}{\sec x}$.

(a) 0
(b) 1
(c) ∞
(d) $-\infty$

3.9 Linear Approximation and the Derivative

150. If $e^{0.5}$ is approximated by using the tangent line to the graph of $f(x) = e^x$ at (0,1), and we know $f'(0) = 1$, the approximation is

(a) 0.5
(b) $1 + e^{0.5}$
(c) $1 + 0.5$

151. The line tangent to the graph of $f(x) = \sin x$ at (0,0) is $y = x$. This implies that

(a) $\sin(0.0005) \approx 0.0005$

(b) The line $y = x$ touches the graph of $f(x) = \sin x$ at exactly one point, (0,0).

(c) $y = x$ is the best straight line approximation to the graph of f for all x.

152. The line $y = 1$ is tangent to the graph of $f(x) = \cos x$ at (0,1). This means that

(a) the only x-values for which $y = 1$ is a good estimate for $y = \cos x$ are those that are close enough to 0.

(b) tangent lines can intersect the graph of f infinitely many times.

(c) the farther x is from 0, the worse the linear approximation is.

(d) All of the above

153. Suppose that $f''(x) < 0$ for x near a point a. Then the linearization of f at a is

(a) an over approximation

(b) an under approximation

(c) unknown without more information.

154. Peeling an orange changes its volume V. What does ΔV represent?

(a) the volume of the rind

(b) the surface area of the orange

(c) the volume of the "edible part" of the orange

(d) $-1\times$ (the volume of the rind)

155. You wish to approximate $\sqrt{9.3}$. You know the equation of the line tangent to the graph of $f(x) = \sqrt{x}$ where $x = 9$. What value do you put into the tangent line equation to approximate $\sqrt{9.3}$?

(a) $\sqrt{9.3}$

(b) 9

(c) 9.3

(d) 0.3

156. We can use a tangent line approximation to $\sqrt{x}$ to approximate square roots of numbers. If we do that for each of the square roots below, for which one would we get the smallest error?

(a) $\sqrt{4.2}$

(b) $\sqrt{4.5}$

(c) $\sqrt{9.2}$

(d) $\sqrt{9.5}$

(e) $\sqrt{16.2}$

(f) $\sqrt{16.5}$

3.10 Theorems about Differentiable Functions

157. A function intersects the x-axis at points a and b, where $a < b$. The slope of the function at a is positive and the slope at b is negative. Which of the following is true for any such function? There exists some point on the interval (a, b) where

(a) the slope is zero and the function has a local maximum.

(b) the slope is zero but there is not a local maximum.

(c) there is a local maximum, but there doesn't have to be a point at which the slope is zero.

(d) none of the above have to be true.

158. A function intersects the x-axis at points a and b, where $a < b$. The slope of the function at a is positive and the slope at b is negative. Which of the following is true for any such function for which the limit of the function exists and is finite at every point? There exists some point on the interval (a, b) where

(a) the slope is zero and the function has a local maximum.
(b) the slope is zero but there is not a local maximum.
(c) there is a local maximum, but there doesnt have to be a point at which the slope is zero.
(d) none of the above have to be true.

159. A continuous function intersects the x-axis at points a and b, where $a < b$. The slope of the function at a is positive and the slope at b is negative. Which of the following is true for any such function? There exists some point on the interval (a, b) where

(a) the slope is zero and the function has a local maximum.
(b) the slope is zero but there is not a local maximum.
(c) there is a local maximum, but there doesnt have to be a point at which the slope is zero.
(d) none of the above have to be true.

160. A continuous and differentiable function intersects the x-axis at points a and b, where $a < b$. The slope of the function at a is positive and the slope at b is negative. Which of the following is true for any such function? There exists some point on the interval (a, b) where

(a) the slope is zero and the function has a local maximum.
(b) the slope is zero but there is not a local maximum.
(c) there is a local maximum, but there doesnt have to be a point at which the slope is zero.
(d) none of the above have to be true.

161. On a toll road a driver takes a time stamped toll-card from the starting booth and drives directly to the end of the toll section. After paying the required toll, the driver is surprised to receive a speeding ticket along with the toll receipt. Which of the following best describes the situation?

(a) The booth attendant does not have enough information to prove that the driver was speeding.
(b) The booth attendant can prove that the driver was speeding during his trip.
(c) The driver will get a ticket for a lower speed than his actual maximum speed.
(d) Both (b) and (c).

162. **True or False:** For $f(x) = |x|$ on the interval $\left[-\frac{1}{2}, 2\right]$, you can find a point c in $\left(-\frac{1}{2}, 2\right)$, such that $f'(c) = \frac{f(2)-f(-\frac{1}{2})}{2-(-\frac{1}{2})}$.

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

163. A racer is running back and forth along a straight path. He finishes the race at the place where he began.

     True or False: There had to be at least one moment, other than the beginning and the end of the race, when he "stopped" (i.e., his speed was 0).

     (a) True, and I am very confident

     (b) True, but I am not very confident

     (c) False, but I am not very confident

     (d) False, and I am very confident

164. Two racers start a race at the same moment and finish in a tie. Which of the following must be true?

     (a) At some point during the race the two racers were not tied.

     (b) The racers' speeds at the end of the race must have been exactly the same.

     (c) The racers must have had the same speed at exactly the same time at some point in the race.

(d) The racers had to have the same speed at some moment, but not necessarily at exactly the same time.

165. Two horses start a race at the same time and one runs slower than the other throughout the race. **True or False:** The Racetrack Principle can be used to justify the fact that the slower horse loses the race.

   (a) True, and I am very confident
   (b) True, but I am not very confident
   (c) False, but I am not very confident
   (d) False, and I am very confident

166. **True or False:** The Racetrack Principle can be used to justify the statement that if two horses start a race at the same time, the horse that wins must have been moving faster than the other throughout the race.

   (a) True, and I am very confident
   (b) True, but I am not very confident
   (c) False, but I am not very confident
   (d) False, and I am very confident

167. Which of the following statements illustrates a correct use of the Racetrack Principle?

(a) Since $\sin 0 = 0$ and $\cos x \leq 1$ for all x, the Racetrack Principle tells us that $\sin x \leq x$ for all $x \geq 0$.

(b) For $a < b$, if $f'(x)$ is positive on $[a, b]$ then the Ractrack Principle tells us that $f(a) < f(b)$.

(c) Let $f(x) = x$ and $g(x) = x^2 - 2$. Since $f(-1) = g(-1) = -1$ and $f(1) > g(1)$, the Racetrack Principle tells us that $f'(x) > g'(x)$ for $-1 < x < 1$.

(d) All are correct uses of the Racetrack Principle.

(e) Exactly 2 of a, b, and c are correct usese of the Racetrack Principle.

4.1 Using First and Second Derivatives

168. **True or False:** If $f''(a) = 0$, then f has an inflection point at a.

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

169. **True or False:** A local maximum of f only occurs at a point where $f'(x) = 0$.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

170. **True or False:** If $x = p$ is not a local minimum or maximum of f, then $x = p$ is not a critical point of f.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

171. **True or False:** If $f'(x)$ is continuous and $f(x)$ has no critical points, then f is everywhere increasing or everywhere decreasing.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

172. **True or False:** If $f'(x) \geq 0$ for all x, then $f(a) \leq f(b)$ whenever $a \leq b$.

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

173. Imagine that you are skydiving. The graph of your speed as a function of time from the time you jumped out of the plane to the time you achieved terminal velocity is

(a) increasing and concave up

(b) decreasing and concave up

(c) increasing and concave down

(d) decreasing and concave down

174. Water is being poured into a "Dixie cup" (a standard cup that is smaller at the bottom than at the top). The height of the water in the cup is a function of the volume of water in the cup. The graph of this function is

(a) increasing and concave up

(b) increasing and concave down

(c) a straight line with positive slope.

4.3 Families of Curves

175. The functions in Figure 4.4 have the form $y = A \sin x$. Which of the functions has the largest A? Assume the scale on the vertical axes is the same for each graph.

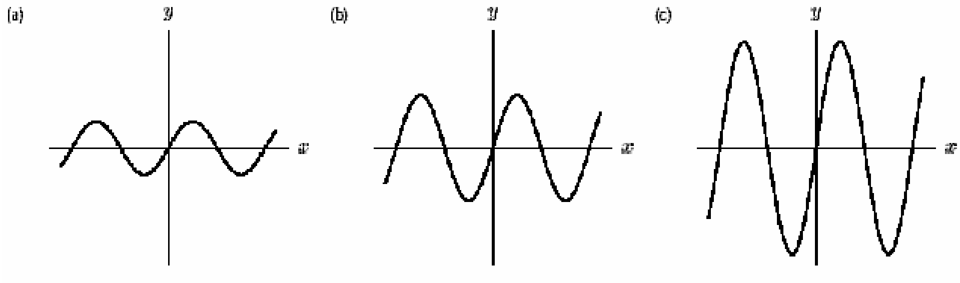


Figure 4.4

176. The functions in Figure 4.5 have the form $y = \sin(Bx)$. Which of the functions has the largest B? Assume the scale on the horizontal axes is the same for each graph.

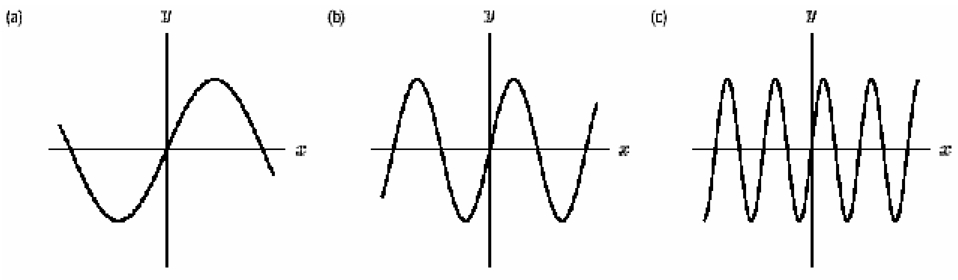


Figure 4.5

177. Let $f(x) = ax + b/x$. What are the critical points of $f(x)$?

(a) $-b/a$

(b) 0

(c) $\pm\sqrt{b/a}$

(d) $\pm\sqrt{-b/a}$

(e) No critial points

178. Let $f(x) = ax + b/x$. Suppose a and b are positive. What happens to $f(x)$ as b increases?

  (a) The critical *points* move further apart.

  (b) The critical *points* move closer together.

179. Let $f(x) = ax + b/x$. Suppose a and b are positive. What happens to $f(x)$ as b increases?

  (a) The critical *values* move further apart.

  (b) The critical *values* move closer together.

180. Let $f(x) = ax + b/x$. Suppose a and b are positive. What happens to $f(x)$ as a increases?

  (a) The critical *points* move further apart.

  (b) The critical *points* move closer together.

181. Let $f(x) = ax + b/x$. Suppose a and b are positive. What happens to $f(x)$ as a increases?

  (a) The critical *values* move further apart.

  (b) The critical *values* move closer together.

182. Find a formula for a parabola with its vertex at (3,2) and with a second derivative of -4.

(a) $y = -4x^2 + 48x - 106$.
(b) $y = -4x^2 + 24x - 34$.
(c) $y = -2x^2 + 12x - 16$.
(d) $y = -2x^2 + 4x + 8$.

4.2 Optimization

183. **True or False:** If $f(x)$ is continuous on a closed interval, then it is enough to look at the points where $f'(x) = 0$ in order to find its global maxima and minima.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

184. **True or False:** A function defined on all points of a closed interval always has a global maximum and a global minimum.

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

185. Let f be a continuous function on the closed interval $0 \leq x \leq 1$. There exists a positive number A so that the graph of f can be drawn inside the rectangle $0 \leq x \leq 1$, $-A \leq y \leq A$.

   The above statement is:

   (a) Always true.

   (b) Sometimes true.

   (c) Not enough information.

186. Let $f(x) = x^2$. **True or False:** f has an upper bound on the interval $(0, 2)$.

   (a) True, and I am very confident

   (b) True, but I am not very confident

   (c) False, but I am not very confident

   (d) False, and I am very confident

187. Let $f(x) = x^2$. **True or False:** f has a global maximum on the interval $(0, 2)$.

   (a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

188. Let $f(x) = x^2$. **True or False:** f has a global minimum on the interval $(0, 2)$.

   (a) True, and I am very confident

   (b) True, but I am not very confident

   (c) False, but I am not very confident

   (d) False, and I am very confident

189. Let $f(x) = x^2$. **True or False:** f has a global minimum on any interval $[a, b]$.

   (a) True, and I am very confident

   (b) True, but I am not very confident

   (c) False, but I am not very confident

   (d) False, and I am very confident

190. Consider $f(x) = -3x^2 + 12x + 7$ on the interval $-2 \leq x \leq 4$. Where does this function have its global maximum value?

   (a) $x = -2$

   (b) $x = 0$

(c) $x = 2$

(d) $x = 4$

191. Consider $f(x) = -3x^2 + 12x + 7$ on the interval $-2 \leq x \leq 4$. Where does this function have its global minimum value?

(a) $x = -2$

(b) $x = 0$

(c) $x = 2$

(d) $x = 4$

Introduction to Sensitivity Analysis

192. What are the critical points of the function $f(x) = x^3 - ax$?

(a) $x = 0, \pm\sqrt{a}$

(b) $x = \pm\sqrt{a/3}$

(c) $x = \pm\sqrt{3a}$

(d) $x = \pm\sqrt{-a/3}$

(e) $x = \pm\sqrt{-3a}$

(f) No critical points exist.

193. Consider the critical points of the function $f(x) = x^3 - ax$. Are these critical points more sensitive to large values of a or to small values of a?

   (a) Slight changes to large values of a affect the critical points more.
   (b) Slight changes to small values of a affect the critical points more.
   (c) Slight changes to a affect the critical points equally, whether a is large or small.

194. We have a system where the critical points are as follows: $x = \pm 5A^2/B$. Will the critical points be more sensitive to changes in A when A is small, or when A is large?

   (a) The critical points will be more sensitive to changes in A when A is small.
   (b) The critical points will be more sensitive to changes in A when A is large.
   (c) The critical points will be equally affected by changes in A, no matter whether A is small or large.

195. What are the critical points of the function $f(z) = ze^{-z^2/\sigma}$?

   (a) $z = 0$

(b) $z = \pm\sqrt{\frac{\sigma}{2}}$

(c) $z = \pm\sqrt{1/2}$

(d) $z = \pm\sqrt{\sigma}$

(e) $z = \frac{\sigma}{2}$

(f) No critical points exist.

196. Consider the critical points of the function $f(z) = ze^{-z^2/\sigma}$. Are these critical points more sensitive to large or small values of σ?

(a) The critical points are more sensitive to small values of σ.

(b) The critical points are more sensitive to large values of σ.

(c) The critical points are equally sensitive to large and small values of σ.

197. What are the critical points of the function $f(N) = \frac{N}{1+kN^2}$?

(a) $N = \sqrt{\frac{1}{k}}$

(b) $N = \sqrt{k}$

(c) $N = \pm\sqrt{\frac{1}{k}}$

(d) $N = \pm\sqrt{\frac{1}{2k}}$

(e) No critical points exist.

198. Consider the critical points of the function $f(N) = \frac{N}{1+kN^2}$. Will the critical points be more sensitive to changes in k when k is large or when k is small?

  (a) Critical points will be more sensitive to changes in k when k is large.

  (b) Critical points will be more sensitive to changes in k when k is small.

  (c) Critical points will be equally affected by changes in k no matter whether k is small or large.

199. What are the critical points of the function $f(t) = a(1 - e^{-bt})$?

  (a) $t = 0$

  (b) $t = \pm\sqrt{\frac{b}{a}}$

  (c) $t = -\frac{1}{b}$

  (d) $t = -\frac{1}{b}\ln\frac{1}{ab}$

  (e) No critical points exist.

4.6 Rates and Related Rates

200. If $\frac{dy}{dx} = 5$ and $\frac{dx}{dt} = -2$ then $\frac{dy}{dt} =$

(a) 5
(b) -2
(c) -10
(d) cannot be determined from the information given

201. If $\frac{dz}{dx} = 12$ and $\frac{dy}{dx} = 2$ then $\frac{dz}{dy} =$

(a) 24
(b) 6
(c) $1/6$
(d) cannot be determined from the information given

202. If $y = 5x^2$ and $\frac{dx}{dt} = 3$, then when $x = 4$, $\frac{dy}{dt} =$

(a) 30
(b) 80
(c) 120
(d) $15x^2$
(e) cannot be determined from the information given

203. The radius of a snowball changes as the snow melts. The instantaneous rate of change in radius with respect to volume is

(a) $\frac{dV}{dr}$
(b) $\frac{dr}{dV}$
(c) $\frac{dV}{dr} + \frac{dr}{dV}$
(d) None of the above

204. Gravel is poured into a conical pile. The rate at which gravel is added to the pile is

(a) $\frac{dV}{dt}$
(b) $\frac{dr}{dt}$
(c) $\frac{dV}{dr}$
(d) None of the above

205. Suppose a deli clerk can slice a stick of pepperoni so that its length L changes at a rate of 2 inches per minute and the total weight W of pepperoni that has been cut increases at a rate of 0.2 pounds per minute. The pepperoni weighs:

(a) 0.4 pounds per inch
(b) 0.1 pounds per inch
(c) 10 pounds per inch
(d) 2.2 pounds per inch
(e) None of the above

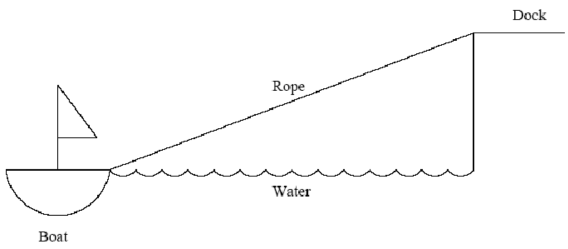
206. The area of a circle, $A = \pi r^2$, changes as its radius changes. If the radius changes with respect to time, the change in area with respect to time is

(a) $\frac{dA}{dt} = 2\pi r$
(b) $\frac{dA}{dt} = 2\pi r + \frac{dr}{dt}$
(c) $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$
(d) Not enough information

207. As gravel is being poured into a conical pile, its volume V changes with time. As a result, the height h and radius r also change with time. Knowing that at any moment $V = \frac{1}{3}\pi r^2 h$, the relationship between the changes in the volume, radius and height, with respect to time, is

(a) $\frac{dV}{dt} = \frac{1}{3}\pi \left(2r\frac{dr}{dt}h + r^2\frac{dh}{dt}\right)$
(b) $\frac{dV}{dt} = \frac{1}{3}\pi \left(2r\frac{dr}{dt} \cdot \frac{dh}{dt}\right)$
(c) $\frac{dV}{dt} = \frac{1}{3}\pi \left(2rh + r^2\frac{dh}{dt}\right)$
(d) $\frac{dV}{dt} = \frac{1}{3}\pi \left((r^2)(1) + 2r\frac{dr}{dh}h\right)$

208. A boat is drawn close to a dock by pulling in a rope as shown. How is the rate at which the rope is pulled in related to the rate at which the boat approaches the dock?



(a) One is a constant multiple of the other.

(b) They are equal.

(c) It depends on how close the boat is to the dock.

209. A boat is drawn close to a dock by pulling in the rope at a constant rate.

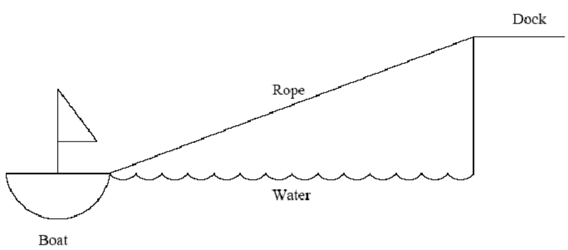
True or False: The closer the boat gets to the dock, the faster it is moving.

(a) True, and I am very confident

(b) True, but I am not very confident

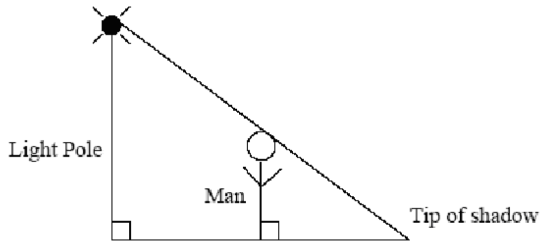
(c) False, but I am not very confident

(d) False, and I am very confident



210. A streetlight is mounted at the top of a pole. A man walks away from the pole. How are the rate at which

he walks away from the pole and the rate at which his shadow grows related?

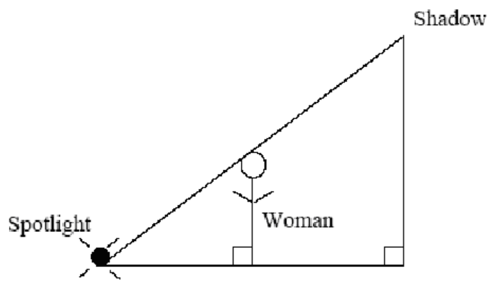


(a) One is a constant multiple of the other.

(b) They are equal.

(c) It depends also on how close the man is to the pole.

211. A spotlight installed in the ground shines on a wall. A woman stands between the light and the wall casting a shadow on the wall. How are the rate at which she walks away from the light and rate at which her shadow grows related?



(a) One is a constant multiple of the other.

(b) They are equal.

(c) It depends also on how close the woman is to the pole.