

Classroom Voting Questions: Calculus I

3.10 Theorems about Differentiable Functions

1. A function intersects the x -axis at points a and b , where $a < b$. The slope of the function at a is positive and the slope at b is negative. Which of the following is true for any such function? There exists some point on the interval (a, b) where
 - (a) the slope is zero and the function has a local maximum.
 - (b) the slope is zero but there is not a local maximum.
 - (c) there is a local maximum, but there doesn't have to be a point at which the slope is zero.
 - (d) none of the above have to be true.

2. A function intersects the x -axis at points a and b , where $a < b$. The slope of the function at a is positive and the slope at b is negative. Which of the following is true for any such function for which the limit of the function exists and is finite at every point? There exists some point on the interval (a, b) where
 - (a) the slope is zero and the function has a local maximum.
 - (b) the slope is zero but there is not a local maximum.
 - (c) there is a local maximum, but there doesn't have to be a point at which the slope is zero.
 - (d) none of the above have to be true.

3. A continuous function intersects the x -axis at points a and b , where $a < b$. The slope of the function at a is positive and the slope at b is negative. Which of the following is true for any such function? There exists some point on the interval (a, b) where
 - (a) the slope is zero and the function has a local maximum.
 - (b) the slope is zero but there is not a local maximum.
 - (c) there is a local maximum, but there doesn't have to be a point at which the slope is zero.
 - (d) none of the above have to be true.

4. A continuous and differentiable function intersects the x -axis at points a and b , where $a < b$. The slope of the function at a is positive and the slope at b is negative. Which of the following is true for any such function? There exists some point on the interval (a, b) where
- the slope is zero and the function has a local maximum.
 - the slope is zero but there is not a local maximum.
 - there is a local maximum, but there doesn't have to be a point at which the slope is zero.
 - none of the above have to be true.
5. On a toll road a driver takes a time stamped toll-card from the starting booth and drives directly to the end of the toll section. After paying the required toll, the driver is surprised to receive a speeding ticket along with the toll receipt. Which of the following best describes the situation?
- The booth attendant does not have enough information to prove that the driver was speeding.
 - The booth attendant can prove that the driver was speeding during his trip.
 - The driver will get a ticket for a lower speed than his actual maximum speed.
 - Both (b) and (c).
6. **True or False:** For $f(x) = |x|$ on the interval $[-\frac{1}{2}, 2]$, you can find a point c in $(-\frac{1}{2}, 2)$, such that $f'(c) = \frac{f(2) - f(-\frac{1}{2})}{2 - (-\frac{1}{2})}$.
7. A racer is running back and forth along a straight path. He finishes the race at the place where he began.
- True or False:** There had to be at least one moment, other than the beginning and the end of the race, when he “stopped” (i.e., his speed was 0).
8. Two racers start a race at the same moment and finish in a tie. Which of the following must be true?
- At some point during the race the two racers were not tied.
 - The racers' speeds at the end of the race must have been exactly the same.
 - The racers must have had the same speed at exactly the same time at some point in the race.
 - The racers had to have the same speed at some moment, but not necessarily at exactly the same time.

9. Two horses start a race at the same time and one runs slower than the other throughout the race. **True or False:** The Racetrack Principle can be used to justify the fact that the slower horse loses the race.
10. **True or False:** The Racetrack Principle can be used to justify the statement that if two horses start a race at the same time, the horse that wins must have been moving faster than the other throughout the race.
11. Which of the following statements illustrates a correct use of the Racetrack Principle?
- (a) Since $\sin 0 = 0$ and $\cos x \leq 1$ for all x , the Racetrack Principle tells us that $\sin x \leq x$ for all $x \geq 0$.
 - (b) For $a < b$, if $f'(x)$ is positive on $[a, b]$ then the Racetrack Principle tells us that $f(a) < f(b)$.
 - (c) Let $f(x) = x$ and $g(x) = x^2 - 2$. Since $f(-1) = g(-1) = -1$ and $f(1) > g(1)$, the Racetrack Principle tells us that $f'(x) > g'(x)$ for $-1 < x < 1$.
 - (d) All are correct uses of the Racetrack Principle.
 - (e) Exactly 2 of a, b, and c are correct uses of the Racetrack Principle.