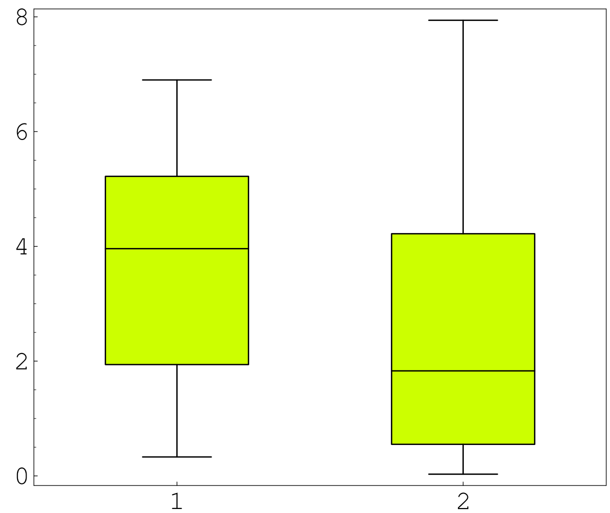
Math 216 Spring 2008
Answers to Clicker Questions

§1.2 Summary Statistics

1. **Clicker Question 1:** Suppose you sample 10 steel bars produced by Machine A and find the sample has an average length of 1 meter with a standard deviation of 5 millimeters. Suppose you sample 10 steel bars produced by Machine B and find the sample has an average weight of 100 kilograms with a standard deviation of 1 kilogram. All the steel bars produced by either machine should be identical. Which of the following statements is best supported by these data?
   (a) Machine A is more reliable than Machine B.
   (b) Machine B is more reliable than Machine A.
   (c) The reliability of Machines A and B cannot be compared using these data.
2. **Answer:** C. Although one could argue that Machine A is more reliable since its standard deviation is smaller in proportion to its average than Machine B's, we can't compare these data directly. It could be that when comparing length to length or weight to weight, we get a different result.
3. **Clicker Question 2:** Suppose you sample 10 steel bars produced by Machine A and find the sample has an average length of 1 meter with a standard deviation of 5 millimeters. Suppose you sample 10 steel bars produced by Machine B and find the sample has an average length of 1 meter with a standard deviation of 10 millimeters. All the steel bars produced by either machine should be identical. Which of the following statements is best supported by these data?
   (a) Machine A is more reliable than Machine B.
   (b) Machine B is more reliable than Machine A.
   (c) The reliability of Machines A and B cannot be compared using these data.
4. **Answer:** A. This time we're comparing apples to apples, so we can say that Machine A is more reliable. However, given the relatively small sample sizes (10 bars from each machine), we can't be too sure. Later in the course, we'll learn how to quantify this uncertainly precisely.

§1.3 Graphical Summaries

1. **Clicker Question 1:** Suppose you take a random sample of 10 Vanderbilt engineering juniors who received the same model laptop when they were first-years. You test these 10 laptops to determine how long their batteries last before needing to be recharged, and you obtain the following data (in hours): 1.2, 1.3, 3.8, 3.9, 3.9, 4.0, 4.1, 4.1, 4.2, 4.3. What should be done with the values 1.2 and 1.3? Which of the following is the best course of action?
   (a) Delete them from the data set since they are outliers.
   (b) Keep them in the data set even though they are outliers.
   (c) Determine why these values were so much lower than the rest, then delete them.
   (d) Determine why these values were so much lower than the rest, then keep them in the data set, provided they weren't due to data entry errors.
2. **Answer:** D. That's the best course of action listed. However, what you'll probably want to do is to determine why these values were so low, then decide whether or not to keep them. It may be that they are the result of some process that you wish to exclude from your analysis–perhaps they are a result of students who greatly modified their laptops. It may be, however, that some number of these laptop batteries just don't hold up well over time. If these values were produced by the same process that produced the other values, then they should be kept.
3. **Clicker Question 2:** Below are boxplots for two data sets.



TRUE or FALSE: There is a greater proportion of values outside the box for the set on the right than for the set on the left.

(a) True
(b) False

4. **Answer:** False. These are boxplots, so the box represents the middle 50% of data in both cases, meaning that what's outside of the box is also 50% in both cases. (The only exception is if the data set has a lot of repeated values right at the first or third quartile. These values would be "in" the box and could increase the proportion of data in the box beyond the standard 50%).

§2.1 Basic Probability

1. **Clicker Question 1:** In a certain semester, 500 students enrolled in both Calculus I and Physics I. Of these students, 82 got an A in calculus, 73 got an A in physics, and 42 got an A in both courses. Which of the following probabilities is the smallest? The probability that a randomly chosen student...

   (a) Got an A in at least one of the two courses
   (b) Got less than an A in at least one of the two courses
   (c) Got an A in both of the two courses
   (d) Got an A in calculus but not in physics
   (e) Got an A in physics but not calculus

2. **Answer:** E. Use a Venn diagram to see why.

§2.3 Conditional Probability

1. **Clicker Question 1:** Three cards are placed in a hat–one card is blue on both sides, one card is red on both sides, and one card has one side blue and one side red. A card is drawn at random from the hat and you see that one side is blue. What is the probability that the other side is also blue?

   (a) 1/3
   (b) 1/2
   (c) 2/3

2. **Answer:** C. Draw a tree diagram to see why.

3. **Clicker Question 2:** Consider tossing a fair coin, that is, one that comes up heads half of the time and tails half of the time. Let A be the event "the first toss is a head," B be the event "the second toss is a tails," C be the event "the two outcomes are the same," D be the event "two heads turn up." Which of the following pairs of events is not independent?

   (a) A and B

   (b) A and C

   (c) A and D

4. **Answer:** C. Use the definition of independence ($P(A|B) = P(A)$ must be true) to see why.

5. **Clicker Question 3:** Suppose A is the event that it rains today and B is the event that I brought my umbrella into work today. What is wrong with the following argument? "These events are independent because bringing an umbrella to work doesn't effect whether or not it rains today."

   (a) These events are not independent, because one's decision of bringing an umbrella is dependent on the likelihood of rain. (However, rain is definitely not dependent on one carrying an umbrella although Murphy's Law might prove the opposite.)

   (b) Although bringing an umbrella to work doesn't cause it to rain, given that you've brought your umbrella to work, the probability that it's a rainy day is higher than the chance of rain on any random day. s

   (c) These events are independent because the probability of bringing an umbrella to work doesn't effect the probability of the event its rains today and vice versa.

   (d) It is false because the fact that it is raining today means that it was probably predicted to rain. If you checked that prediction then you would be more likely to bring in an umbrella making the events linked.

6. **Answer:** B. Here's another example: Suppose you're running an ice cream stand. Let A be the event that it is a hot day and let B be the event that you sell more ice cream than usual. It seems clear that these events aren't independent since it's more likely that B will occur if A occurs–you'll sell more ice cream than usual on hot days. Mathematically, we know that if $P(B) \neq P(B|A)$, then $P(A) \neq P(A|B)$, but what does this mean in terms of the events? It doesn't mean that selling more ice cream than usual *causes* it to be a hot day. It does, however, mean that when you just look at days on which you sell more ice cream, the proportion of hot days is higher than the proportion of hot days to all days. That is, given that you're selling more ice cream than usual, the probability that it's a hot day is greater than the probability of any given day being hot. This is a great example, since causation creates the dependence in one direction, but not in the other direction.

7. **Clicker Question 4:** Through accounting procedures, it is known that about 10% of the employees in a store are stealing. The managers would like to fire the thieves, but their only tool in distinguishing them from the honest employees is a lie detector test that is only 90% accurate. That is, if an employee is a thief, he or she will fail the test with probability 0.9, and if an employee is not a thief, he or she will pass the test with probability 0.9. If an employee fails the test, what is the probability that he or she is a thief?

   (a) 90%

   (b) 75%

   (c) 66 2/3%

   (d) 50%

8. **Answer:** D. Construct a tree diagram to see why.

§2.4 Random Variables

1. **Clicker Question 1:** Suppose your instructor asks you a multiple-choice question with three answer choices in class. You are to submit your answer and also rate the confidence (low, medium, or high) with which you believe in that answer. You will be scored based on the following chart.

Confidence	Correct Answer	Incorrect Answer
Low	3	2
Medium	4	1
High	5	0

   If have no idea what the answer to the question is and you have to guess randomly among the three available answer choices, what confidence level should you choose in order to maximize your points?[1]

   (a) Low
   (b) Medium
   (c) High
   (d) It doesn't matter.

2. **Answer:** A. Note that since your chance of guessing incorrectly is twice as much as your chance of guessing correctly, the expected value for "low" is greater than the expected values for "medium" or "high."

3. **Clicker Question 2:** Suppose your instructor asks you a multiple-choice question with *two* answer choices in class. You are to submit your answer and also rate the confidence (low, medium, or high) with which you believe in that answer. You will be scored based on the following chart.

Confidence	Correct Answer	Incorrect Answer
Low	3	2
Medium	4	1
High	5	0

   If have no idea what the answer to the question is and you have to guess randomly among the two available answer choices, what confidence level should you choose in order to maximize your points?[2]

   (a) Low
   (b) Medium
   (c) High
   (d) It doesn't matter.

4. **Answer:** It depends. In this case, the expected values for "low," "medium," and "high" are the same. However, the variance for each choice differs. Since the variance for "low" is the smallest, "low" is your least risky choice. Why should this effect your confidence level? Over a large number of questions, all three confidence levels will give you the same average score. However, over a small number of questions, the "medium" and "high" confidence levels will have more volatility. It all depends on if you feel lucky or not.

5. Draw the following dart board: A dart board is constructed from three concentric circles with radii 1 inch, 2 inches, and 3 inches, respectively. If a dart lands in the innermost circle, the player receives 4 points. If the dart lands between the innermost circle and the middle circle, the player receives 2 points. If the dart lands between the middle circle and the outermost circle, the player receives 1 point. Assume that the probability of a dart landing in any particular region is proportional to the area of that region.

[1] Adapted from Dennis Jacobs' CRS question point scheme.
[2] Adapted from Dennis Jacobs' CRS question point scheme.

6. **Clicker Question 3:** Define the random variable X to be the sum of the player's score on two successive throws. Then X is what type of random variable?
   (a) discrete
   (b) continuous
7. **Answer:** A. The possible values for X are 2, 3, 4, 5, 6, and 8–a countable number of values.
8. **Clicker Question 4:** Same dart board, but now suppose that a player's score on a single dart throw is defined to be the distance between the dart and the center of the board. Define the random variable X to be the sum of the player's score on two successive throws. Then X is what type of random variable?
   (a) discrete
   (b) continuous
9. **Answer:** B. The possible values for X are any number between 0 and 6–an uncountable number of values.
10. **Clicker Question 5:** A radioactive mass emits particles at an average rate of 15 particles per minute. Define the random variable X to be the number of particles emitted in a 10-minute time frame. Then X is what type of random variable?
    (a) discrete
    (b) continuous
11. **Answer:** A. The possibles values for X are all integers between 0 and the number of particles in the mass. Even if there were an infinite number of particles in the mass, this would still be a discrete random variable, since the possible values are countable (1, 2, 3, ...).
12. **Clicker Question 6:** A radioactive mass emits particles at an average rate of 15 particles per minute. A particle is emitted at noon today. Define the random variable X to be the time elapsed between noon and the next emission. Then X is what type of random variable?
    (a) discrete
    (b) continuous
13. **Answer:** B. X can take on any positive value, which is an uncountable set of values.
14. **Clicker Question 7:** Consider the continuous random variable X = the weight in pounds of a randomly selected newborn baby born in the United States during 2006. Let f be the probability density function for X. It is probably safe to say that $P(X < 0) = 0$ and $P(X < 20) = 1$. Which of the following is *not* a justifiable conclusion about f given this information?
    (a) No portion of the graph of f can lie below the x-axis.
    (b) The area under the entire graph of f equals 1.
    (c) The area under the graph of f between $x = 0$ and $x = 20$ is 1.
    (d) The nonzero portion of the graph of f lies entirely between $x = 0$ and $x = 19$.
15. **Answer:** D. Since X is a continuous random variable, it can take on values between 19 and 20. There may be some nonzero portion of the graph of f that lies between $x = 19$ and $x = 20$. (In fact, the Guinness Book of World Records lists as the heaviest baby born to a healthy mother a boy weighing 22 pounds, 8 ounces, born in Aversa, Italy, in September 1955.) Draw a graph for f and illustrate these three properties.

16. **Clicker Question 8:** Consider the continuous random variable X = the weight in pounds of a randomly selected newborn baby born in the United States during 2006. Let F be the cumulative distribution function for X. It is probably safe to say that $P(X < 0) = 0$ and $P(X < 20) = 1$. Which of the following is *not* a justifiable conclusion about F given this information?

    (a) $F(x) = 0$ for all $x \leq 0$.
    (b) $F(x) = 1$ for all $x \geq 20$.
    (c) The area under the graph of F between $x = 0$ and $x = 20$ is 1.
    (d) F is an non-decreasing function between $x = 0$ and $x = 20$.

17. **Answer:** C. That statement is true of f, but not of F. In fact, the area under the curve between $x = 19$ and $x = 20$ alone is probably pretty close to 1.

§4.2 The Binomial Distribution

1. **Clicker Question 1:** Consider the following experiment. On a Friday night, a highway patrol officer sets up a roadblock and stops 100 drivers. A given driver is considered a success if he or she is wearing a seat belt; the driver is considered a failure otherwise. Can we consider this experiment a binomial experiment?

    (a) Yes
    (b) No

2. **Answer:** B is probably the best choice here. If there's a line at the checkpoint, then the drivers waiting in line are likely to buckle their seat belts, which means that the probability that a driver is wearing a seat belt is not likely to be the same from car to car.

3. **Clicker Question 2:** Consider the following experiment. A particular car club has 100 members, 70 of which regularly wear their seat belts and 30 of which do not. Ten of these members are selected at random without replacement as they leave a car show. A given driver is considered a success if he or she is wearing a seat belt. The driver is considered a failure otherwise. Can we consider this experiment a binomial experiment?

    (a) Yes
    (b) No

4. **Answer:** B is the technically correct choice. Since the selection is done without replacement, the probability of selecting a driver wearing a seat belt changes as each selection is made, which means that the "trials" are not independent. However, in this case, the sample size (10) is fairly small compared to the population size (100), so those probabilities won't change much, so this is pretty close to a binomial experiment.

§4.3 The Poisson Distribution

1. **Clicker Question 1:** Suppose that trucks arrive at a receiving dock with an average arrival rate of 3 per hour. What is the probability exactly 5 trucks will arrive in a two-hour period?

    (a) $\dfrac{e^{-3}3^5}{5!}$
    (b) $\dfrac{e^{-3}3^{2.5}}{2.5!}$
    (c) $\dfrac{e^{-6}6^5}{5!}$
    (d) $\dfrac{e^{-5}5^6}{6!}$

2. **Answer:** C. In this case, λ, the expected outcome, is equal to 6 since we expect 6 trucks to arrive in a two-hour period.

§4.5 The Normal Distribution

1. **Clicker Question 1:** Consider the continuous random variable X = the weight in pounds of a randomly selected newborn baby born in the United States last year. Suppose that X can be modeled with a normal distribution with mean $\mu = 7.57$ and standard deviation $\sigma = 1.06$. If the standard deviation were $\sigma = 1.26$ instead, how would that change the graph of the pdf of X?

   (a) The graph would be narrower and have a greater maximum value.
   (b) The graph would be narrower and have a lesser maximum value.
   (c) The graph would be narrower and have the same maximum value.
   (d) The graph would be wider and have a greater maximum value.
   (e) The graph would be wider and have a lesser maximum value.
   (f) The graph would be wider and have the same maximum value.

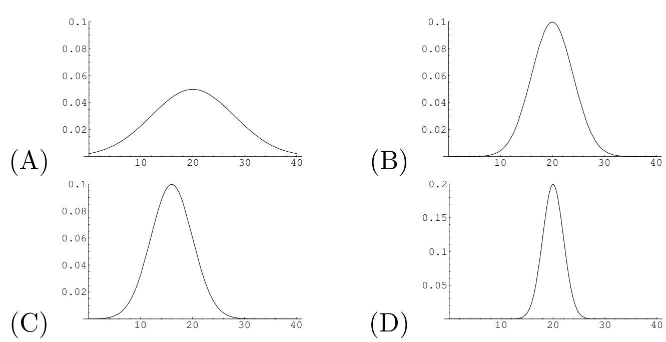
2. **Answer:** E. Increasing the standard deviation increases the spread of the random variable, which means the graph will be wider. Since the area under the curve must be 1, making the graph wider means making it shorter as well so that the area remains constant.

3. **Clicker Question 2:** Consider the continuous random variable X = the weight in pounds of a randomly newborn baby born in the United States during 2006. Suppose that X can be modeled with a normal distribution with mean $\mu = 7.57$ and standard deviation $\sigma = 1.06$. If the mean were $\mu = 7.27$ instead, how would that change the graph of the pdf of X?

   (a) The graph would be shifted to the left.
   (b) The graph would be shifted to the right.
   (c) The graph would become more negatively skewed.
   (d) The graph would become more positively skewed.
   (e) The graph would have a greater maximum value.
   (f) The graph would have a lesser maximum value.

4. **Answer:** A. Decreasing the mean without changing the standard deviation moves the entire graph to the left, 0.3 units in this case.

5. **Clicker Question 3:** If X is a normal random variable with mean $\mu = 20$ and standard deviation $\sigma = 4$, which of the following could be the graph of the pdf of X?

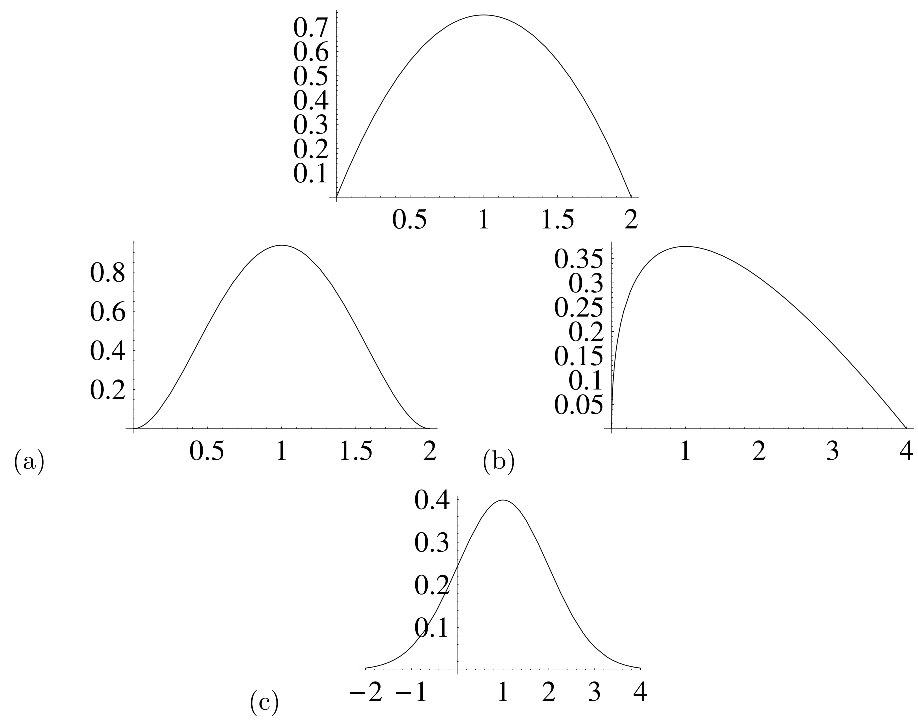


6. **Answer:** B. Note that the vertical axis for the fourth graph is different. The third choice has the wrong mean, the first choice has too great a standard deviation, and the fourth choice has too small a standard deviation. Note that 68% of the area under the curve should fall between $\mu - \sigma$ and $\mu + \sigma$. Also, one can show that the graph changes concavity at $\mu \pm \sigma$.

7. **Clicker Question 4:** Let Z be a standard normal random variable. Which of the following probabilities is the smallest?
   (a) $P(-2 < Z < -1)$
   (b) $P(0 < Z < 2)$
   (c) $P(Z < 1)$
   (d) $P(Z > 2)$
8. **Answer:** D. Looking at the graph of the pdf for Z quickly narrows the choices down. Adding in the rules of thumb about standard deviations (e.g. 68% of the population lies within one standard deviation of the mean) helps us finalize our answer.
9. **Clicker Question 5:** Let Z be a standard normal random variable. Which of the following probabilities is the smallest?
   (a) $P(0 \leq Z \leq 2.07)$
   (b) $P(-0.64 \leq Z \leq -0.11)$
   (c) $P(Z > -1.06)$
   (d) $P(Z < -0.88)$
10. **Answer:** D. Looking at the graph of the pdf for Z narrows the choices down to 2 or 4. Using Table A.2, we can compute these and determine which is smaller.

§4.6 The Lognormal Distribution

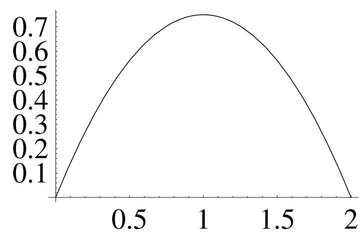
1. **Clicker Question 1:** Suppose that the random variable X has the distribution given in the first graph below. Which of the latter graphs gives the distribution of the random variable X^2?



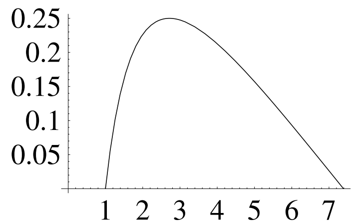
2. **Answer:** B. Note that the random variable X can take on values between 0 and 2. Thus, the random variable X^2 (which can be seen as a new way of "scoring" the same set of outcomes that X "scores") takes on values between 0 and 4. Only choice B reflects this. Furthermore, since there's a 50% chance

that X will take on a value between 0 and 1, there should be a 50% chance that X^2 takes on a value between $0^2 = 0$ and $1^2 = 1$, which is seen in the fact that half the area under the curve in choice B occurs to the left of $x = 1$.

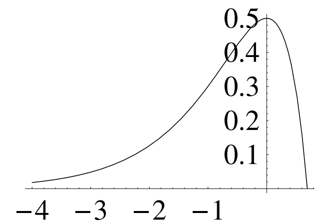
3. **Clicker Question 2:** Suppose that the random variable X has the distribution given in the first graph below. Which of the latter graphs gives the distribution of the random variable e^X?



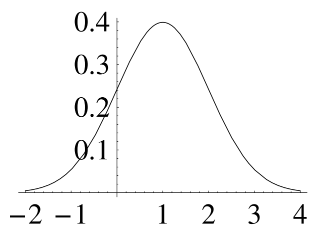
(a)



(b)



(c)



4. **Answer:** A. Since X takes on values between 0 and 2, e^X takes on values between $e^0 = 1$ and $e^2 \approx 7.39$, which corresponds to choice C. Also, since there's a 50% chance that X is less than 1, there's a 50% chance that e^X is less than $e^1 \approx 2.72$, which is seen in the fact that half the area under the curve in choice C occurs to the left of 2.72 or so.

§4.7 The Exponential Distribution

1. **Clicker Question 1:** A certain type of component can be purchased new or used. Fifty percent of all new components last more than five years, but only 30% of used components last more than five years. Is it possible that the lifetimes of new components are exponentially distributed?

   (a) Yes

   (b) No

2. **Answer:** No. If the lifetimes were exponentially distributed, the proportion of used components lasting longer than five years would be the same as the proportion of new components lasting longer than five years because of the lack of memory property. If $T =$ the number of years a randomly chosen component lasts, then we want to show that $P(T > 5) = P(T > 5 + s | T > s)$, where s is the age in years of a used component. (Since s can be anything, this handles the fact that used components can be of any age.) Use the cumulative distribution function for T to show this statement is true.

3. **Clicker Question 2:** Which of the following random variables is best modeled by an exponential distribution?

   (a) The distance between consecutive weak spots in a length of copper wire

   (b) The number of days between drawings of the Powerball lottery that have winners

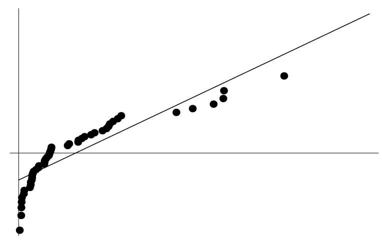
(c) The number of accidents that occur in a certain intersection in a given year

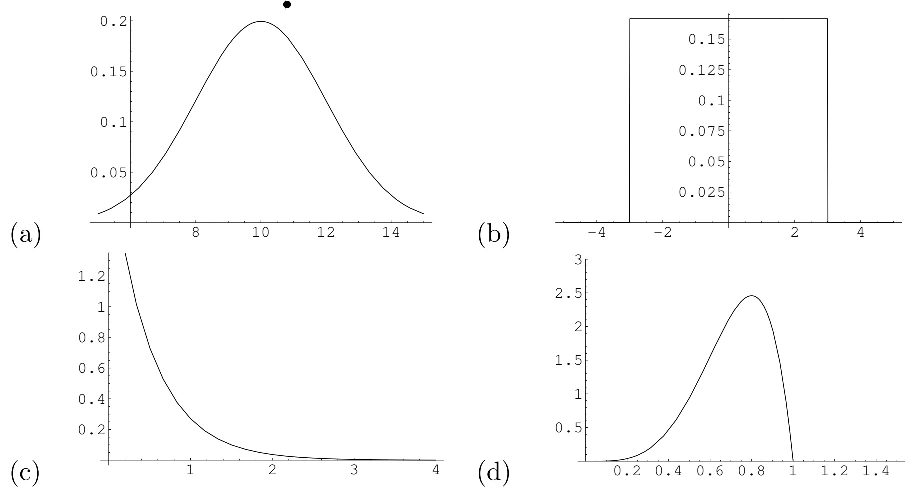
(d) The amount of rainfall in Nashville in a week given an average rainfall of 0.2 inches per day

4. **Answer:** Choice (b) isn't a continuous random variable nor does it involve a Poisson process. Choice (c) involves a Poisson process, but is a Poisson random variable, not an exponetial one. Choice (d) doesn't involve a Poisson process. Choice (a), on the other hand, can be modeled by an exponential random variable with parameter $\lambda =$ average number of weak spots per unit of distance.

§4.10 Probability Plots

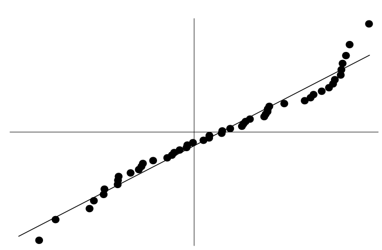
1. **Clicker Question 1:** Which of the following distributions matches the given probability plot?

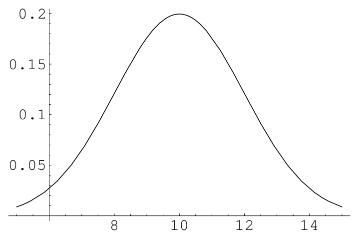


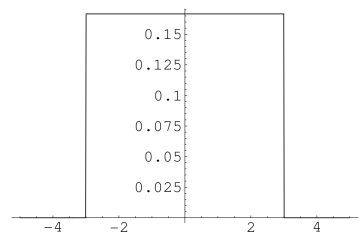


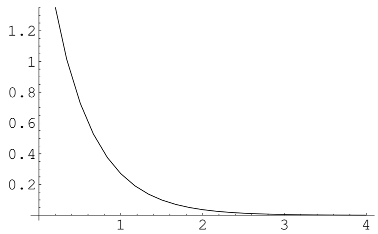
2. **Answer:** (c) In this case, on the right side of the plot, the points on the probability plot to the right of the line of best fit indicate that the data we have (given by the x-coordinates of these points) are greater than we would expect from a normal distribution. This indicates that our distribution has a longer right tail than the normal distribution. On the left side of the plot, the points to the right of the line of best fit indicate that the data we have are greater than we would expect from a normal distribution. This indicates that our distribution's left tail isn't as long as the left tail of a normal distribution. These observations together indicate that our distribution is skewed to the right. Additionally, the clustering we see on the left side of the probability plots indicates that our distribution has a very short left tail. All of this indicates that our distribution is the one in choice 3, which happens to be an exponential distribution.

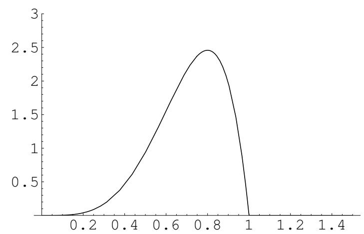
3. **Clicker Question 2:** Which of the following distributions matches the given probability plot?



(a)


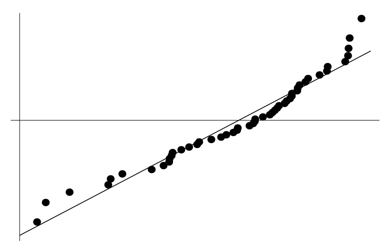
(b)


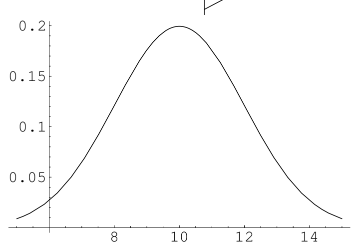
(c)


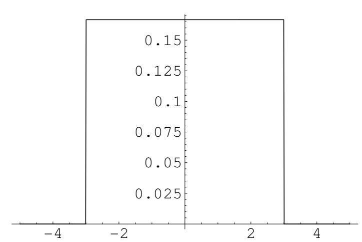
(d)


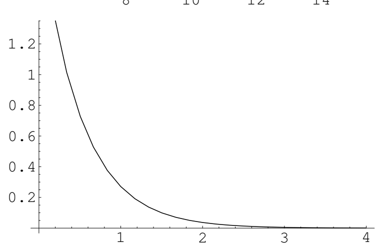
4. **Answer:** (b) In this case, on the right side of the plot, the points to the left of the line of best fit indicate that the data we have are less than what we would expect from a normal distribution. This indicates that our distribution's right tail is shorter than a normal distribution's right tail. On the left side of the plot, the points to the right of the line of best fit indicate that our data is greater than we would expect from a normal distribution, indicating that our distribution's left tail is also shorter than a normal distribution's left tail. Since the uniform distribution has very short tails, it makes sense that our distribution is a uniform one.

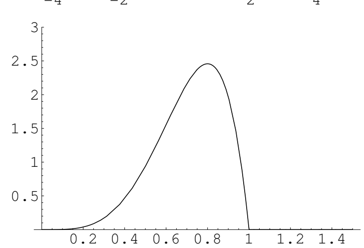
5. **Clicker Question 3:** Which of the following distributions matches the given probability plot?



(a)


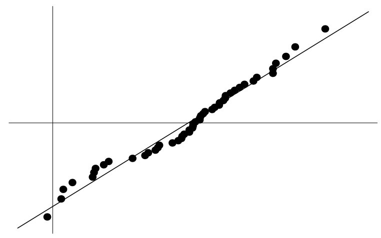
(b)


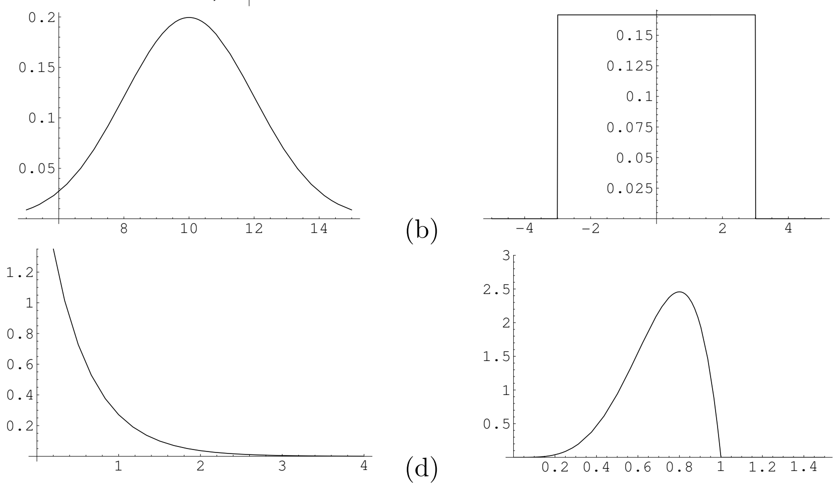
(c)


(d)


6. **Answer:** (d) Similar reasoning holds here.

7. **Clicker Question 4:** Which of the following distributions matches the given probability plot?





(a) (b) (c) (d)

8. **Answer:** (a) Similar reasoning holds here.

§4.11 The Central Limit Theorem

1. **Clicker Question[3]:** In 1938, Duke University researchers Pratt and Woodruff conducted an experiment looking for evidence of ESP (extrasensory perception). In the experiment, students were presented with five standard ESP symbols (square, wavy lines, circle, star, cross). The experimenter shuffled a desk of ESP cards, each of which had one of the five symbols on it. The experimenter drew a card from this deck, looked at it, and concentrated on the symbol on the card. The student would then guess the symbol, perhaps by reading the experimenter's mind. This experiment was repeated with 32 students for a total of 60,000 trials. The students were correct 12,489 times. If the students were selecting one of the five symbols as random, the probability of success would be $p = 0.2$ and we would expect the students to be correct 12,000 times out of 60,000. Should we write off the observed excess of 489 as nothing more than random variation?

   (a) Yes

   (b) No

2. **Answer:** No. The Central Limit Theorem gives us that if $X \sim \text{Bin}(n, p)$, then X is approximately normal with the same mean and standard deviation. This fact can be used to compute $P(X \geq 12489)$, which turns out to be a very small number.

§5.1 Large-Sample Confidence Intervals for Population Means

1. **Clicker Question 1:** Given the confidence interval just constructed, is it correct to say that there is a 95% chance that μ is between 6.85 and 7.61. Is this statement correct?

   (a) Yes

   (b) No

[3]Larsen and Marx, 3rd edition, Case Study 4.3.1

2. **Answer:** Technically, no. We're not saying that there's a 95% chance that the interval $(6.85, 7.61)$ contains μ. What we're saying is that there's a 95% chance that the interval

$$\left(\overline{X} - 1.96\frac{\sigma}{\sqrt{n}}, \overline{X} - 1.96\frac{\sigma}{\sqrt{n}}\right)$$

contains μ. We will get different intervals for different values of the random variable $\overline{X}$. In the long run, we would expect 95% of these intervals to contain the constant μ. One such interval is $(6.85, 7.61)$, but we don't know if it lies in the 95% that contain μ or the 5% that don't contain μ–and we have no way of determining how likely it is that it's in either category.

3. **Clicker Question 2:** Given the confidence interval just constructed, is it correct to say the following? If the process of selecting a sample of size 30 and then computing the corresponding 95% confidence interval is repeated 100 times, 95 of the resulting intervals will include μ.

   (a) Yes

   (b) No

4. **Answer:** No. In the long run, 95% of such intervals can be expected to include μ. As the number of times we repeat the experiment grows, the proportion of intervals containing μ will approach 95%. One hundred experiments is not enough to guarantee that we will hit 95% exactly.

5. **Clicker Question 3:** Given the confidence interval just constructed, it is correct to say that 95% of all birth weights will be between 6.85 and 7.61 pounds?

   (a) Yes

   (b) No

6. **Answer:** No. This confidence interval gives us a sense of where the population mean lies, not which individual observations are likely to occur. Given that the population is normally distributed and that we know the population standard deviation σ, if we knew the population mean μ, then we could say that 95% of all birth weights should be in the interval $(\mu - 2\sigma, \mu + 2\sigma)$. (That's our 95% rule of thumb.) However, we don't know μ–the point of the confidence interval is to estimate μ. μ could be near 6.85 or near 7.61 or somewhere in the middle or somewhere else entirely, so we don't have enough information to apply our 95% rule of thumb.

7. **Clicker Question 4:** Without consulting Table A.2, determine which of the following statements is TRUE.

   (a) As α increases, z_α increases.

   (b) As α increases, z_α decreases.

8. **Answer:** (b) Since z_α has the property that the area under the graph of the standard normal distribution to the *right* of z_α is equal to α, it follows that as α increases, the z-score z_α must move to the left.

9. **Clicker Question 5:** Suppose that for babies born in the United States, birth weight is normally distributed about some unknown mean μ with standard deviation $\sigma = 1.06$ pounds. Suppose also that a random sample of 30 babies born at a particular hospital has an average birth weight of $\bar{x} = 7.23$ pounds. Which of the following confidence intervals for μ calculated from this sample is the smallest in width?

   (a) 90% confidence interval

   (b) 95% confidence interval

   (c) 99% confidence interval

10. **Answer:** Choice (a) is the correct choice. The smallest interval is the 90% confidence interval. Confidence and interval width are always trade-offs. Note that the width of a confidence interval is proportional to $z_{\alpha/2}$, and, as we saw in an earlier clicker question, as $\alpha/2$ increases, $z_{\alpha/2}$ increases. Similarly, as $\alpha/2$ decreases, $z_{\alpha/2}$ decreases and the confidence interval gets narrower.

11. **Clicker Question 6:** Suppose you construct a 95% confidence interval from a random sample of size $n = 20$ with sample mean $\overline{x} = 100$ taken from a population with unknown mean μ and known standard deviation $\sigma = 10$, and the interval is fairly wide. Which of the following conditions would NOT lead to a narrower confidence interval?

    (a) If you decreased your confidence level

    (b) If you increased your sample size

    (c) If the sample mean was smaller

    (d) If the population standard deviation was smaller

12. **Answer:** Choice (c) is the correct answer. As we saw in the last question, decreasing your confidence level results in decreasing $z_{\alpha/2}$ which results in a narrower confidence interval. Since the width of the confidence interval is proportional to $1/\sqrt{n}$, increasing the sample size will also narrow the interval. Likewise, the width of the confidence interval is proportional to σ, so a smaller population standard devation will result in a narrower interval. On the other hand, the interval is centered at the sample mean, but its width doesn't depend on the sample mean, so decreasing the sample mean won't narrow the interval–it will just shift it to the left.

13. **Clicker Question 7:** Suppose that for babies born in the United States, birth weight is normally distributed about some unknown mean μ with standard deviation $\sigma = 1.06$ pounds. What is the minimum sample size necessary to ensure that the resulting 99% confidence interval has a width of at most 0.5?

    (a) 70

    (b) 119

    (c) 120

    (d) 140

14. **Answer:** (c) If we use $z_{.005} = 2.575$, then we find that $n \geq 119.2$, thus $n = 120$ is the minimize sample size to ensure a 99% CI of width at most 0.5. Of course, any $n \geq 120$ will do, so choice (d) would work as well, it's just not the minimum sample size.

15. **Clicker Question 8:** Suppose a random sample of size $n = 50$ of ball bearings produced by a particular machine is taken and the diameter of each ball bearing in the sample is measured. Suppose that for this sample, $\bar{x} = 5.14$ and $s = 0.34$. Which of the following is the corresponding 95% confidence interval for the population mean μ?

    (a) $\left(5.14 - z_{0.025} \cdot \frac{0.34}{\sqrt{50}}, 5.14 + z_{0.025} \cdot \frac{0.34}{\sqrt{50}}\right)$

    (b) $\left(5.14 - t_{49,0.025} \cdot \frac{0.34}{\sqrt{50}}, 5.14 + t_{49,0.025} \cdot \frac{0.34}{\sqrt{50}}\right)$

    (c) There is not enough information given to construct a 95% confidence interval for μ.

16. **Answer:** Choice (a) is the best choice here. In this case, the sample size is large enough that the distribution of $\overline{X}$ is approximately normal. We don't know the population standard deviation σ, but with a sample size this large, the sample standard deviation s is a good approximation of σ, so we can go with choice (a).

17. **Clicker Question 9:** Suppose a random sample of size $n = 10$ of ball bearings produced by a particular machine is taken and the diameter of each ball bearing in the sample is measured. Suppose that for this sample, $\bar{x} = 5.14$ and $s = 0.34$. Which of the following is the corresponding 95% confidence interval for the population mean μ?

    (a) $\left(5.14 - z_{0.025} \cdot \frac{0.34}{\sqrt{10}}, 5.14 + z_{0.025} \cdot \frac{0.34}{\sqrt{10}}\right)$

    (b) $\left(5.14 - t_{9,0.025} \cdot \frac{0.34}{\sqrt{10}}, 5.14 + t_{9,0.025} \cdot \frac{0.34}{\sqrt{10}}\right)$

    (c) There is not enough information given to construct a 95% confidence interval for μ.

18. **Answer:** Choice (c) is the correct one here. We're in the same situation as the previous question, but now the sample size is much smaller. We can use a t-distribution to model the sample mean only when the the underlying distribution is fairly normal. We don't know enough about the underlying distribution to claim that it's fairly normal. **What if we knew σ in this case?** We would still be out of luck since the sample size isn't large enough to apply the Central Limit Theorem and say that the sample mean has an approximately normal distribution.

19. **Clicker Question 10:** Suppose a random sample of size $n = 10$ of ball bearings produced by a particular machine is taken and the diameter of each ball bearing in the sample is measured. Suppose that for this sample, $\bar{x} = 5.14$ and $s = 0.34$. Suppose also that a probability plot for this sample indicates that it comes from a distribution that is fairly normal. Which of the following is the corresponding 95% confidence interval for the population mean μ?

    (a) $\left(5.14 - z_{0.025} \cdot \frac{0.34}{\sqrt{10}}, 5.14 + z_{0.025} \cdot \frac{0.34}{\sqrt{10}}\right)$

    (b) $\left(5.14 - t_{9,0.025} \cdot \frac{0.34}{\sqrt{10}}, 5.14 + t_{9,0.025} \cdot \frac{0.34}{\sqrt{10}}\right)$

    (c) There is not enough information given to construct a 95% confidence interval for μ.

20. **Answer:** Choice (b) is the correct choice again here. The added assumption that the underlying population distribution is fairly normal one means that we can use our t-distribution.

§5.2 Confidence Interval for Proportions

1. **Clicker Question 1:** In the polling example, if the sample proportion were .9 instead of .5, what would happen to the width of the resulting confidence interval?

    (a) The new CI would be narrower.

    (b) The new CI would have the same width.

    (c) The new CI would be wider.

2. **Answer:** (a) is the correct choice. Verify this computationally, computing $1.96\sqrt{\frac{\tilde{p}(1-\tilde{p})}{\tilde{n}}}$ for the actual sample proportion, 0.5, and a hypothetical sample proportion of 0.9. Why? If the sample proportion is fairly high, then the population proportion is likely high, as well. That means that the binomial random variable X, the number of students in the sample that favor Obama, will have a relatively small standard deviation. Verify this using the population standard deviation, $\sqrt{np(1-p)}$. This means that the sample proportion will have less variability, which means the confidence interval based on this sample proportion will have more precision.

3. **Clicker Question 2:** In our polling example, approximately what sample size would you need if you wanted to cut the margin of error we obtained (with a sample size of 50) in half?

    (a) 25

(b) 100
(c) 200
(d) 400

4. **Answer:** Choice (c) is the correct one. This can be seen from the formula for margin of error. Thanks to the square root in the formula, halving the margin of error requires quadrupling the sample size.

5. **Clicker Question 3:** Which of the following does *not* result in a larger margin of error?

   (a) Increasing the confidence level
   (b) Decreasing the sample size
   (c) Having a larger population size

6. **Answer:** Choice (c) is the correct one. Choices (a) and (b) can be explained using the formula for the margin of error. While one might think that the larger the population, the greater our margin of error (and conversely, the smaller the population, the lesser our margin of error), margin of error doesn't depend on the population size. As long as the population size is sufficiently greater than the sample size (so that the binomial assumption we made way back when holds), it doesn't matter what the population size is. This means that we can take a random sample of 1200 people from the much, much larger population of all registered voters, and still have a margin of error of 2.83%, even though our sample size is much smaller than our population size.

§6.1 Large-Sample Tests for a Population Mean
§6.2 Drawing Conclusions from the Results of Hypothesis Test

1. **Example:** Suppose that Bayside High School was chosen to participate in the evaluation of a new algebra and geometry curriculum. In the recent past, Bayside's students would be considered "typical," having earned scores on standardized exams that are consistent with national averages.

   Two years ago, a cohort of 86 Bayside sophomores, all randomly selected, were assigned to a special set of classes that integrated algebra and geometry. According to test results that have just been released, those students averaged 502 on the SAT-I math exam. Nationwide, seniors averaged 494 with a standard deviation of 124 with a normal distribution of scores. Can it be claimed that the new curriculum had an effect? (Let μ be the mean of the hypothetical population of all Bayside High School students taking the new curriculum.)

2. **Clicker Question 1:** In the Bayside High School example, which of the following should the school administrators choose for their alternate hypothesis?

   (a) $H_1 : \mu < 494$
   (b) $H_1 : \mu \neq 494$
   (c) $H_1 : \mu > 494$

3. **Answer:** Choice (c) is probably the best choice here. We're interested in showing that the new curriculum improves learning here. Note that Choice (c) means that our null hypothesis is $H_0 : \mu \leq 494$.

4. **Clicker Question 2:** In the Bayside High School example, which of the following errors would be more serious?

   (a) Asserting that the new curriculum had a positive effect, when, in fact, it didn't.
   (b) Asserting that the new curriculum had no positive effect, when, in fact, it did.

5. **Answer:** Choice (a) is probably the better choice here. Making this error (asserting that the new curriculum had a positive effect, when, in fact, it didn't) likely means wasting a lot of resources rolling out the new curriculum (new books, training for teachers, etc.) on a new curriculum that doesn't actually improve learning (and may actually harm learning). An argument could be made for choice (b), however.

6. **Clicker Question 3:** Water samples are taken from water used for cooling as it is being discharged from a power plant into a river. It has been determined that as long as the mean temperature of the discharged water is at most 150 degrees Fahrenheit, there will be no negative effects on the river's ecosystem. To investigate whether the plan is in compliance with the regulations that prohibit a mean discharge water temperature above 150 degrees, 50 water samples will be taken at randomly selected times, and the temperatures of each sample recorded. Which of the following hypothesis tests should be used?

   (a) $H_0 : \mu \geq 150$ versus $H_1 : \mu < 150$

   (b) $H_0 : \mu \leq 150$ versus $H_1 : \mu > 150$

7. Option (a) is the hypothesis test $H_0 : \mu \geq 150$ versus $H_1 : \mu < 150$. If we were to get a low P-value with this hypothesis test (e.g. $P = 0.02$), then that would be strong evidence that the water temperature is less than 150 degrees. (There would be only a 2% chance that we would get water temperatures as low as, say, 145 degrees if the temperature were really greater than 150. Since this probability is so low, we would conclude that the water temperature must be lower than 150.)

   Option (b) is the hypothesis test $H_0 : \mu \leq 150$ versus $H_1 : \mu > 150$. If we were to get a low P-value with this hypothesis test (e.g. $P = 0.02$), then that would be strong evidence that the water temperature is greater than 150 degrees. (There would only be a 2% chance that we would get water temperatures as big as, say, 155 degrees if the temperature were really less than 150. Since this probability is so low, we would conclude that the water temperature must be greater than 150.)

   Now suppose you're the power plant owner, and you want to avoid an unnecessary fine by the EPS for high discharge water temperatures. You would want to use Option (b) since you'll want to see strong evidence that your water temperature is too high. If you got a low P-value, then that would be strong evidence that your water temperature is too high, so you would go along with the EPA fine. If you got a high P-value, then there's not strong evidence that your water temperature is too high, so the EPA presumably wouldn't fine you. A low P-value minimizes the chance of an unnecessary fine.

   Now suppose you're Green Peace and you really don't want high water temperatures to kill fish. You would want to use Option (a) since you'll want to see strong evidence that the water temperature is low enough. If you got a low P-value, then that would be strong evidence that the water temperature is low enough, so you would be okay with the EPA not fining the power plant. If you got a high P-value, then there would not be strong evidence that the water temperature is low enough, so you would want the EPA to go ahead and fine the power plant. A low P-value minimizes the chance that the power plant will "get away" with too-high water temperatures.

   In practice, you would typically collect your water samples and compute the sample mean. If the mean were bigger than 150, then if you're the power plant, then you would want to conduct an Option (b) hypothesis test to see if there's really sufficient evidence that your water is too hot. If the mean were bigger than 150 and you're Green Peace, then you would just say, "Fine the power plant."

   If the mean were lower than 150, then if you're the power plant, then you would tell the EPA not to fine you since your water is cool enough. If the sample mean were lower than 150 and you're Green Peace, then you would want to conduct an Option (a) hypothesis test to see if there's really sufficient evidence that the water is cool enough.

8. **Clicker Question 4:** In the Bayside High School example, suppose that one of the following P-values was calculated from the sample data instead of $P = .2743$. Which one would most strongly indicate that the new curriculum works?

   (a) $P = 0.02$

   (b) $P = 0.09$

   (c) $P = 0.14$

9. **Answer:** Choice (a) is the correct answer. In general, the *lower* the P-value for a given data set, the *stronger* the evidence is in support of rejecting the null hypothesis.

10. **Clicker Question 5:** In the Bayside example, we found that the P-value for the sample data was .2743. Which of the following is the lowest level at which this result is statistically significant?

    (a) 1%
    (b) 5%
    (c) 10%
    (d) 30%

11. **Answer:** Choice (d) is the correct answer. See the definition of "statistically significant" for an explanation.

12. **Clicker Question 6:** In the Bayside High School example, we found that if a sample of $n = 86$ students scored an average of 502, then the P-value was .2743. If a random sample of 860 students scored an average of 502, would this raise or lower our P-value?

    (a) Raise
    (b) Lower

13. **Answer:** Choice (b) is the correct choice. It's much less likely that a sample 860 students would average 8 points above the national average than it is for a sample of 86 students to do so. Thus, the data would provide stronger evidence for rejecting the null hypothesis, which means the p-value would be lower. Compute the p-value as a check.

14. **Clicker Question 7:** Suppose that in the Bayside example, we had found that the P-value for the sample data was 0.04. What is the probability that the new curriculum has no positive effect?

    (a) 2%
    (b) 4%
    (c) 8%
    (d) We have insufficient information with which to answer this question.

15. **Answer:** Choice (d) is the correct answer. The P-value is the probability that we would get a result like the one at hand *if* the null hypothesis (that the curriculum has no effect) is true. The P-value is *not* the probability that the null hypothesis is true. The null hypothesis is either true or false. The truth or falsehood of the null hypothesis does not vary each time we collect a sample. Since it is not a random variable (that is, it does not vary), it does not make sense to discuss the "probability" that it is true or not.

§6.3 Tests for a Population Proportion
§6.4 Small-Sample Tests for a Population Mean

1. **Clicker Question 1:** Which of the following test statistics is most appropriate for Example 4 on your handout?

    (a) $z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$
    (b) $z = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$
    (c) $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$
    (d) $z = \dfrac{\hat{p} - p_0}{\sqrt{p_0(1 - p_0)/n}}$

2. **Answer:** Choice (d) is the correct choice. This problem involves a hypothesis test for a population *proportion*, not a population *mean*. Note that the problem involves a binomial random variable–the number of "successes" out of a set number of trials, each of which results in a success or failure. That's your cue that you're looking for a population proportion.

3. **Clicker Question 2:** Which of the following test statistics is most appropriate for Example 5 on your handout?

   (a) $z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$

   (b) $z = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$

   (c) $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$

   (d) $z = \dfrac{\hat{p} - p_0}{\sqrt{p_0(1 - p_0)/n}}$

4. **Answer:** Choice (a) is the correct choice here. Initially, it might seem that choice (c) is the correct one, since we have a small sample size. However, since we know the population standard deviation σ, we don't need to estimate σ with the sample standard deviation s. As a result, we don't need to use a t-test, which is designed to handle the extra variability introduced when we approximate σ with s. We can instead use a z-test.

5. **Clicker Question 3:** Which of the following test statistics is most appropriate for Example 6 on your handout?

   (a) $z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$

   (b) $z = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$

   (c) $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$

   (d) $z = \dfrac{\hat{p} - p_0}{\sqrt{p_0(1 - p_0)/n}}$

6. **Answer:** Choice (b) is the correct choice here. We have a large sample, so we can use a z-test, thanks to the Central Limit Theorem. Since we don't know the population standard deviation σ, we have to use the sample standard deviation s.

§6.8 Paired Tests for Difference Between Two Means

1. **Clicker Question 1:** Two methods are used to predict the shear strength for steel plate girders. Each method is applied to nine specific girders and the ratio of predicted load to observed load is calculated for each method and each girder. What kind of t-test should we use to compare these data?

   (a) Independent t-test

   (b) Paired t-test

2. **Answer:** Choice (b) is the correct one. In this case, each pair of data has something in common–they are generated by applying one of the two methods to the *same* girder.

3. **Clicker Question 2:** Two catalysts are being analyzed to determine how they affect the mean yield of a chemical process. Catalyst 1 is used in the process eight times and the yield in percent is measured each time. Then catalyst 2 is used in the process eight times and the yield is measured each time. What kind of t-test should be used to compare these data?

   (a) Independent t-test

   (b) Paired t-test

4. **Answer:** Choice (a) is the correct one. In this case, catalyst 1 is applied to a *different* set of processes than catalyst 2, thus there is no way to match data from the first set with data from the second set.

5. **Clicker Question 3:** Six river locations are selected and the zinc concentration is determined for both surface water and bottom water at each location. What kind of t-test should be used to compare these data?

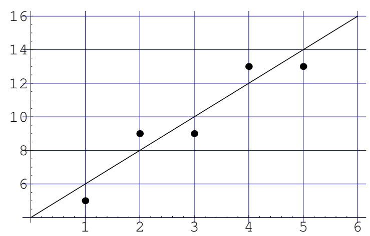
   (a) Independent t-test

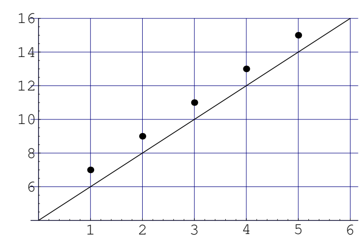
   (b) Paired t-test

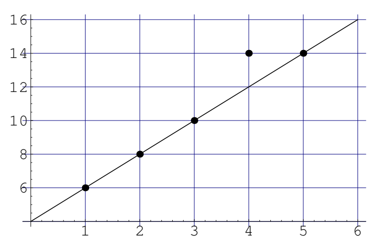
6. **Answer:** Choice (b) is the correct one. In this case, each pair of data has something in common–they are taken from the *same* river.

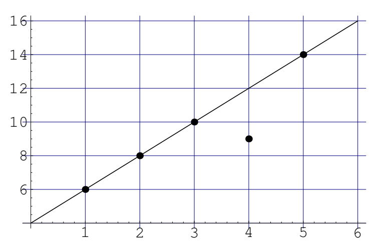
§7.1–7.3 Simple Linear Regression

1. **Clicker Question 1:** The line $y = 4 + 2x$ has been proposed as a line of best for the following four sets of bivariate data. For which data set is this line the *best* fit?

   (a) 

   (b) 

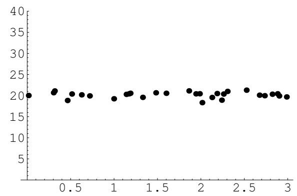
   (c) 

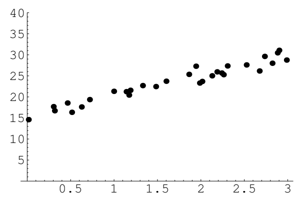
   (d) 

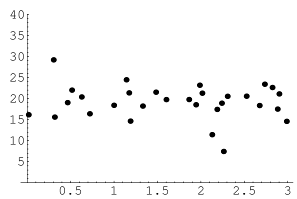
2. **Answer:** Choice (c) is the correct one. As mentioned in the book, the line that best fits a set of data is the one that minimizes the sum of the squares of the *residuals*:

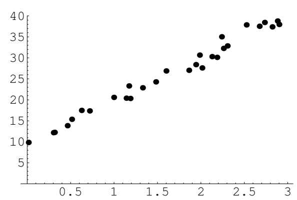
$$S = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 = \sum_{i=1}^{n} \left(y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i\right)^2.$$

3. **Clicker Question 2:** Which of the following scatter plots likely has the greatest value of the total sum of squares?

(a) 

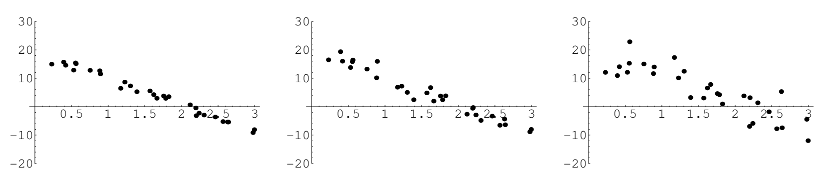
(b) 

(c) 

(d) 

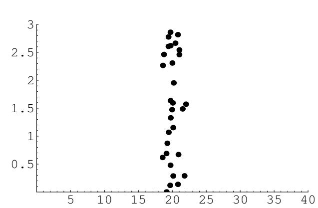
4. **Answer:** Choice (d) is the correct choice. Ignoring the x-values, choice 4 has the greatest sum of squares of differences between the observed y-values and the average of the observed y-values.

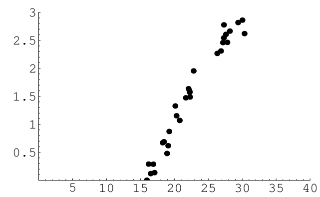
5. **Clicker Question 3:** Which of the following scatter plots likely has the greatest value of the *error sum of squares*?

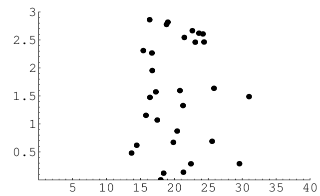


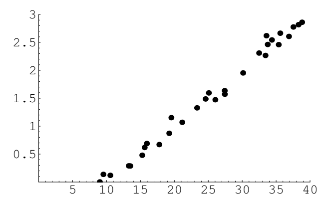
6. **Answer:** Choice (c) is the correct one. Choice 3 has the greatest sum of squares of differences between the observed y-values and the y-values that would be predicted by the line of best fit.

7. **Clicker Question 4:** Which of the following scatter plots likely has the greatest value of $\sum_{i=1}^{n}(x_i - \bar{x})^2$?

(a) 

(b) 

(c) 

(d) 

8. **Answer:** Choice (d) is the correct one. Note that we're adding up the differences between the x-values and the average x-value.

9. **Clicker Question 5:** Which of the following does *not* result in more accurate estimates for β_1?

   (a) An increase in the sample size

   (b) An increase in the coefficient of determination

   (c) An increase in the variance of the observed x-values

   (d) An increase in the variance of the observed y-values

10. **Answer:** Choice (d) is the correct answer.

    (a) Intuitively, a larger sample size (choice (a)) should result in more accurate estimates of any population parameter, including β_1. You can also see from the formula for $s_{\hat{\beta}_1}$, which is our estimate for the standard deviation of $\hat{\beta}_1$, that increasing n results in a decrease in $s_{\hat{\beta}_1}$, which in turn increases our accuracy.

    (b) Intuitively, an increase in the coefficient of determination (choice (b)) means the data fit the linear model better, which should increase the accuracy of our estimate of the slope of that linear model. You can also see from the formula for $s_{\hat{\beta}_1}$ that a larger value for r^2 results in a smaller value for $(1 - r^2)$ and thus a smaller value for $s_{\hat{\beta}_1}$.

    (c) It's a little harder to see how increasing the variance in the observed x-values (choice (c)) increases the accuracy of our estimate, but that variance appears in the denominator in our formula for $s_{\hat{\beta}_1}$, so increasing that variance decreases the standard deviation of $\hat{\beta}_1$, making our estimates more accurate.

    (d) As for choice (d), the variance of the observed y-values appears in the numerator of our formual for $s_{\hat{\beta}_1}$, so increasing that variance increases the standard deviation of $\hat{\beta}_1$, making our estimates less accurate.