MathQuest: Differential Equations

What is a Differential Equation?

1. Which of the following is not a differential equation?

   (a) $y' = 3y$

   (b) $2x^2y + y^2 = 6$

   (c) $tx\frac{dx}{dt} = 2$

   (d) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 7y + 8x = 0$

   (e) All are differential equations.
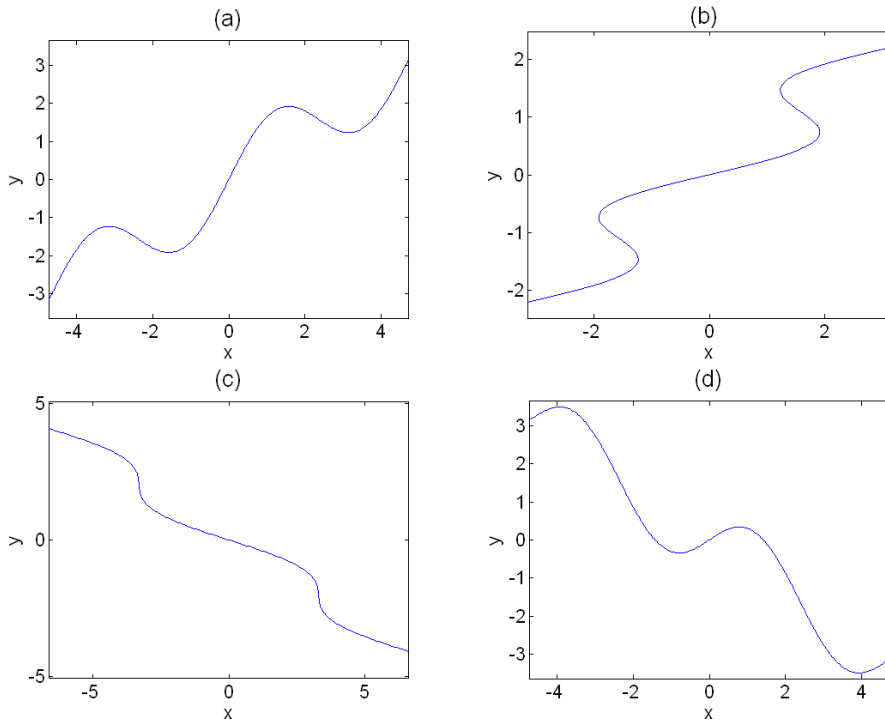
2. Which of the following is not a differential equation?

   (a) $6\frac{dy}{dx} + 3xy$

   (b) $8 = \frac{y'}{y}$

   (c) $2\frac{d^2f}{dt^2} + 7\frac{df}{dt} = f$

   (d) $h(x) + 2h'(x) = g(x)$

   (e) All are differential equations.

3. Which of the following couldn't be the solution of a differential equation?

   (a) $z(t) = 6$

   (b) $y = 3x^2 + 7$

   (c) $x = 0$

   (d) $y = 3x + y'$

   (e) All could be solutions of a differential equation.

4. Which of the following could not be a solution of a differential equation?

5. Which of the following could not be a solution of a differential equation?

   (a) $f = 2y + 7$
   (b) $q(d) = 2d^2 - 6e^d$
   (c) $6y^2 + 2yx = \sqrt{x}$
   (d) $y = 4\sin 8\pi z$
   (e) All could be a solution of a differential equation.

6. True or False? A differential equation is a type of function.

   (a) True, and I am very confident
   (b) True, but I am not very confident
   (c) False, but I am not very confident
   (d) False, and I am very confident

7. Suppose $\dfrac{dx}{dt} = 0.5x$ and $x(0) = 8$. Then the value of $x(2)$ is approximately

   (a) 4
   (b) 8

(c) 9

(d) 12

(e) 16

8. Which of the following is a solution to the differential equation $\dfrac{dy}{dt} = 72 - y$?

   (a) $y(t) = 72t - \frac{1}{2}t^2$

   (b) $y(t) = 72 + e^{-t}$

   (c) $y(t) = e^{-72t}$

   (d) $y(t) = e^{-t}$

9. The amount of a chemical in a lake is decreasing at a rate of 30% per year. If $p(t)$ is the total amount of the chemical in the lake as a function of time t (in years), which differential equation models this situation?

   (a) $p'(t) = -30$

   (b) $p'(t) = -0.30$

   (c) $p'(t) = p - 30$

   (d) $p'(t) = -0.3p$

   (e) $p'(t) = 0.7p$

10. The evolution of the temperature of a hot cup of coffee cooling off in a room is described by $\frac{dT}{dt} = -0.01T + 0.6$, where T is in °F and t is in hours. What are the units of the numbers -0.01 and 0.6?

    (a) -0.01 °F, and 0.6 °F

    (b) -0.01 per hour, and 0.6 °F per hour

    (c) -0.01 °F per hour, and 0.6 °F

    (d) neither number has units

11. We want to test the function $z(x) = 4\sin 3x$ to see if it solves $z'' + 2z' + 4z = 0$, by substituting the function into the differential equation. What is the resulting equation before simplification?

    (a) $-36\sin 3x + 24\cos 3x + 16\sin 3x = 0$

    (b) $4\sin 3x + 8\sin 3x + 16\sin 3x = 0$

    (c) $-36\sin 3x + 12\cos 3x + 4\sin 3x = 0.$

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(d) $4\sin 3x + 8\cos 3x + 4\sin 3x = 0$

(e) none of the above

12. If we test the function $f(x) = ae^{bx}$ to see if it could solve $\frac{df}{dx} = cf^2$, which equation is the result?

(a) $\frac{df}{dx} = ca^2 e^{2bx}$

(b) $abe^{bx} = cf^2$

(c) $ae^{bx} = ca^2 e^{(bx)^2}$

(d) $abe^{bx} = ca^2 e^{2bx}$

(e) $abe^{bx} = cae^{bx}$

(f) None of the above

13. We want to test the function $f(x) = 3e^{2x} + 6x$ to see if it solves the differential equation $\frac{df}{dx} = 2f + 3x$, so we insert the function and its derivative, getting $6e^{2x} + 6 = 2(3e^{2x} + 6x) + 3x$. This means that:

(a) This function is a solution.

(b) This function is a solution if $x = 2/5$.

(c) This function is not a solution.

(d) Not enough information is given.

14. A bookstore is constantly discarding a certain percentage of its unsold inventory and also receiving new books from its supplier so that the rate of change of the number of books in inventory is $B'(t) = -0.02B + 400 + 0.05t$, where B is the number of books and t is in months. If the store begins with 10,000 books in inventory, at what rate is it receiving books from its supplier at $t = 0$?

(a) 200 books per month

(b) 400 books per month

(c) -200 books per month

(d) 900 books per month