MathQuest: Differential Equations

Slope Fields and Euler's Method

1. What does the differential equation $\frac{dy}{dx} = 2y$ tell us about the slope of the solution curves at at any point?

    (a) The slope is always 2.

    (b) The slope is equal to the x-coordinate.

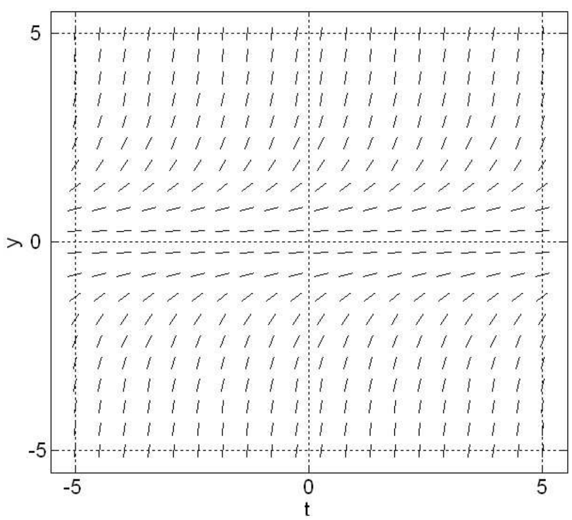
    (c) The slope is equal to the y-coordinate.

    (d) The slope is equal to two times the x-coordinate.

    (e) The slope is equal to two times the y-coordinate.

    (f) None of the above.

2. The slopefield below indicates that the differential equation has which form?

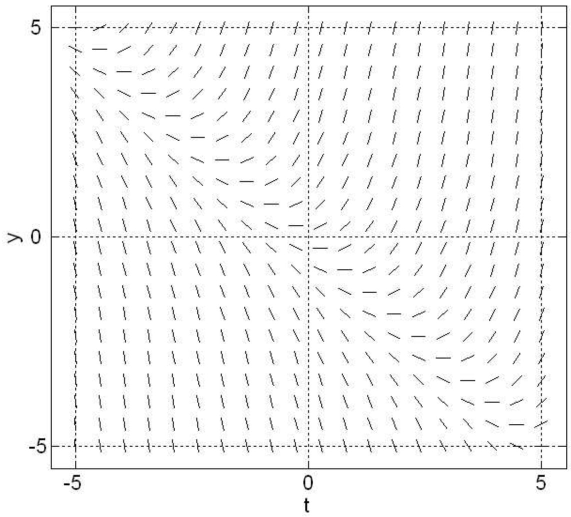


    (a) $\frac{dy}{dt} = f(y)$

    (b) $\frac{dy}{dt} = f(t)$

    (c) $\frac{dy}{dt} = f(y, t)$

3. The slopefield below indicates that the differential equation has which form?

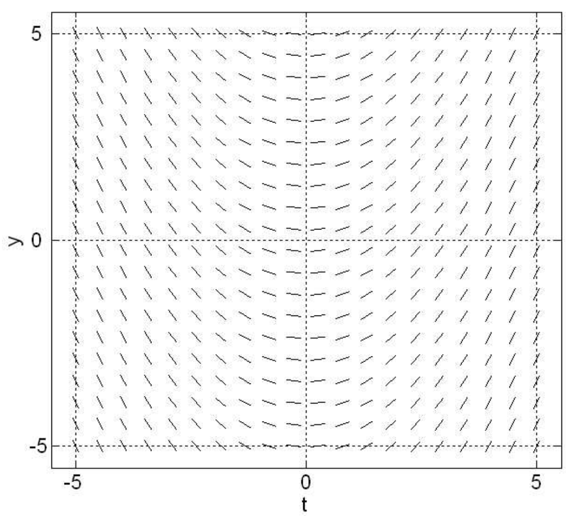


(a) $\frac{dy}{dt} = f(y)$

(b) $\frac{dy}{dt} = f(t)$

(c) $\frac{dy}{dt} = f(y, t)$

4. The slopefield below indicates that the differential equation has which form?

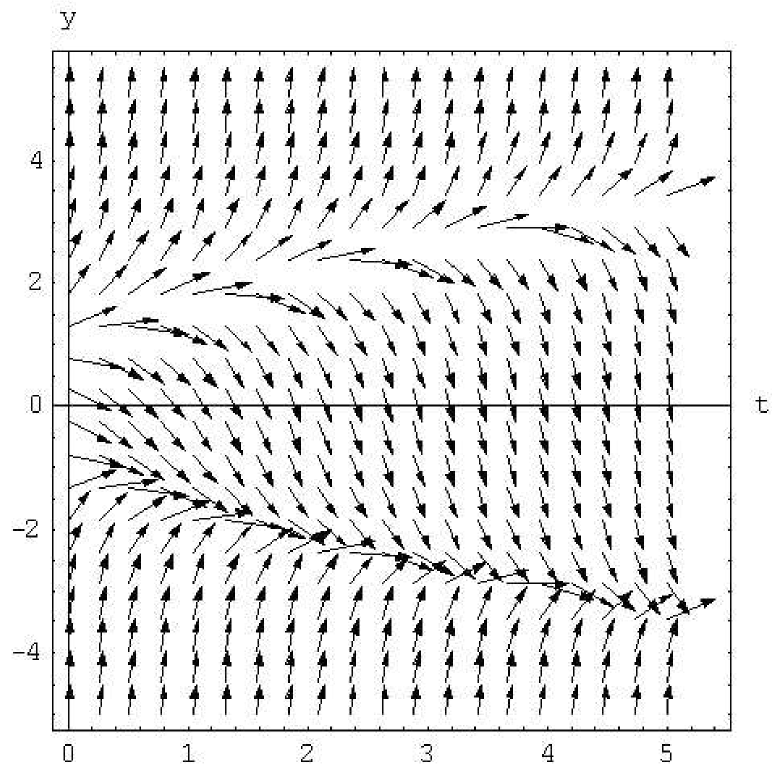


(a) $\frac{dy}{dt} = f(y)$

(b) $\frac{dy}{dt} = f(t)$

(c) $\frac{dy}{dt} = f(y, t)$

5. The arrows in the slope field below have slopes that match the derivative y' for a range of values of the function y and the independent variable t. Suppose that $y(0) = 0$. What would you predict for $y(5)$?



(a) $y(5) \approx -3$

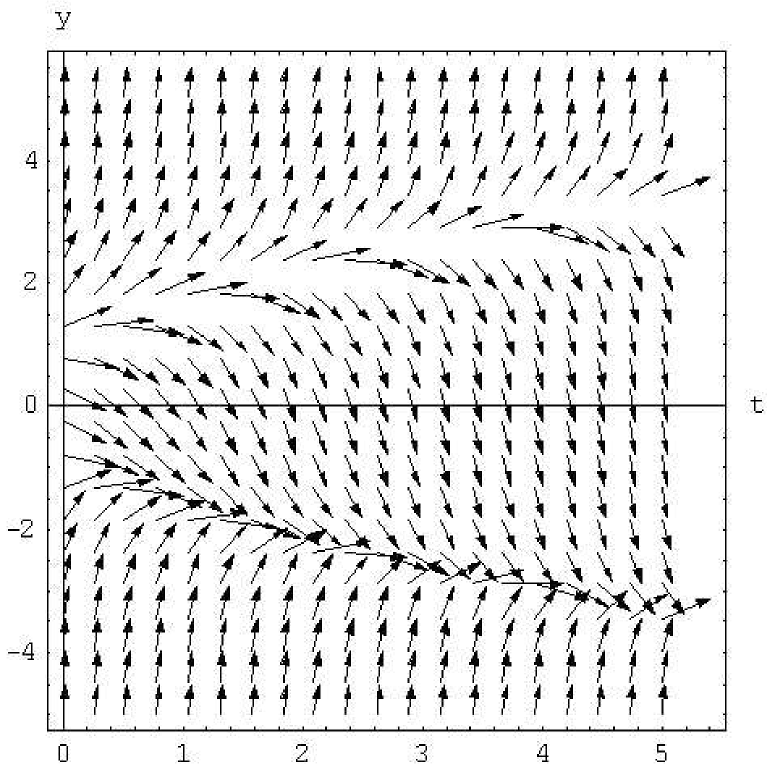
(b) $y(5) \approx +3$

(c) $y(5) \approx 0$

(d) $y(5) < -5$

(e) None of the above

6. The arrows in the slope field below give the derivative y' for a range of values of the function y and the independent variable t. Suppose that $y(0) = -4$. What would you predict for $y(5)$?



(a) $y(5) \approx -3$

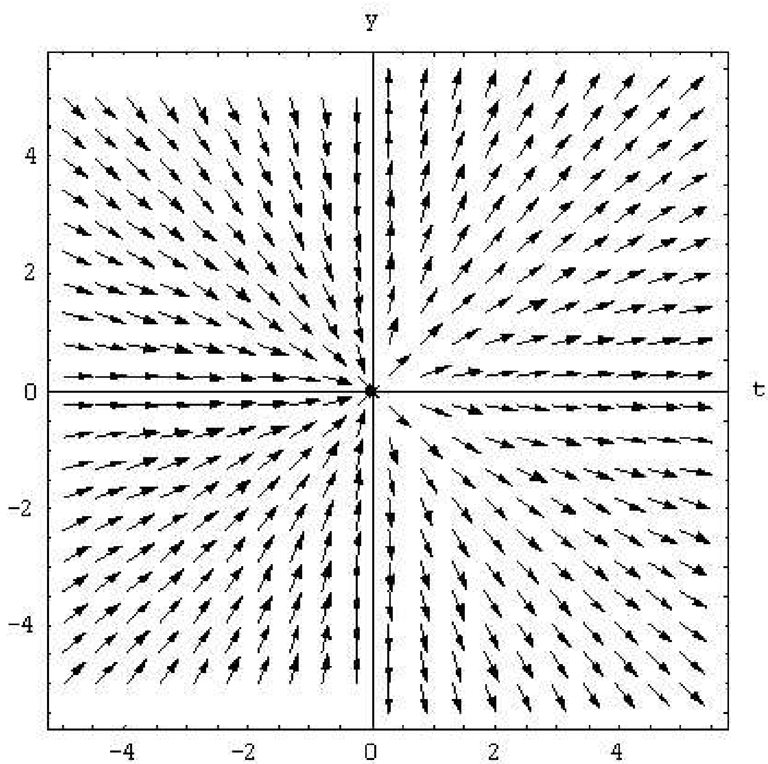
(b) $y(5) \approx +3$

(c) $y(5) \approx 0$

(d) $y(5) < -5$

(e) None of the above

7. The slope field below represents which of the following differential equations?



(a) $y' = yt$

(b) $y' = \frac{y}{t}$

(c) $y' = -yt$

(d) $y' = -\frac{y}{t}$

8. Consider the differential equation $y' = ay + b$ with parameters a and b. To approximate this function using Euler's method, what difference equation would we use?

   (a) $y_{n+1} = ay_n + b$

   (b) $y_{n+1} = y_n + ay_n\Delta t + b\Delta t$

   (c) $y_{n+1} = ay_n\Delta t + b\Delta t$

   (d) $y_{n+1} = y_n\Delta t + ay_n\Delta t + b\Delta t$

   (e) None of the above

9. Using Euler's method, we set up the difference equation $y_{n+1} = y_n + c\Delta t$ to approximate a differential equation. What is the differential equation?

   (a) $y' = cy$

   (b) $y' = c$

(c) $y' = y + c$
(d) $y' = y + c\Delta t$
(e) None of the above

10. We know that $f(2) = -3$ and we use Euler's method to estimate that $f(2.5) \approx -3.6$, when in reality $f(2.5) = -3.3$. This means that between $x = 2$ and $x = 2.5$,

    (a) $f(x) > 0$.
    (b) $f'(x) > 0$.
    (c) $f''(x) > 0$.
    (d) $f'''(x) > 0$.
    (e) None of the above

11. We have used Euler's method and $\Delta t = 0.5$ to approximate the solution to a differential equation with the difference equation $y_{n+1} = y_n + 0.2$. We know that the function $y = 7$ when $t = 2$. What is our approximate value of y when $t = 3$?

    (a) $y(3) \approx 7.2$
    (b) $y(3) \approx 7.4$
    (c) $y(3) \approx 7.6$
    (d) $y(3) \approx 7.8$
    (e) None of the above

12. We have used Euler's method to approximate the solution to a differential equation with the difference equation $z_{n+1} = 1.2z_n$. We know that the function $z(0) = 3$. What is the approximate value of $z(2)$?

    (a) $z(2) \approx 3.6$
    (b) $z(2) \approx 4.32$
    (c) $z(2) \approx 5.184$
    (d) Not enough information is given.

13. We have used Euler's method and $\Delta t = 0.5$ to approximate the solution to a differential equation with the difference equation $y_{n+1} = y_n + t + 0.2$. We know that the function $y = 7$ when $t = 2$. What is our approximate value of y when $t = 3$?

    (a) $y(3) \approx 7.4$

(b) $y(3) \approx 11.4$

(c) $y(3) \approx 11.9$

(d) $y(3) \approx 12.9$

(e) None of the above

14. We have a differential equation for $\frac{dx}{dt}$, we know that $x(0) = 5$, and we want to know $x(10)$. Using Euler's method and $\Delta t = 1$ we get the result that $x(10) \approx 25.2$. Next, we use Euler's method again with $\Delta t = 0.5$ and find that $x(10) \approx 14.7$. Finally we use Euler's method and $\Delta t = 0.25$, finding that $x(10) \approx 65.7$. What does this mean?

    (a) These may all be correct. We need to be told which stepsize to use, otherwise we have no way to know which is the right approximation in this context.

    (b) Fewer steps means fewer opportunities for error, so $x(10) \approx 25.2$.

    (c) Smaller stepsize means smaller errors, so $x(10) \approx 65.7$.

    (d) We have no way of knowing whether any of these estimates is anywhere close to the true value of $x(10)$.

    (e) Results like this are impossible: We must have made an error in our calculations.

15. We have a differential equation for $f'(x)$, we know that $f(12) = 0.833$, and we want to know $f(15)$. Using Euler's method and $\Delta t = 0.1$ we get the result that $f(15) \approx 0.275$. Next, we use $\Delta t = 0.2$ and find that $f(15) \approx 0.468$. When we use $\Delta t = 0.3$, we get $f(15) \approx 0.464$. Finally, we use $\Delta t = 0.4$ and we get $f(15) \approx 0.462$. What does this mean?

    (a) These results appear to be converging to $f(15) \approx 0.46$.

    (b) Our best estimate is $f(15) \approx 0.275$.

    (c) This data does not allow us to make a good estimate of $f(15)$.