MathQuest: Differential Equations

Exponential Solutions, Growth and Decay

1. A star's brightness is decreasing at a rate equal to 10% of its current brightness per million years. If B_0 is a constant with units of brightness and t is in millions of years, what function could describe the brightness of the star?

   (a) $B'(t) = -0.1B(t)$
   (b) $B(t) = B_0 e^t$
   (c) $B(t) = B_0 e^{-0.1t}$
   (d) $B(t) = B_0 e^{0.1t}$
   (e) $B(t) = B_0 e^{0.9t}$
   (f) $B(t) = -0.1B_0 t$

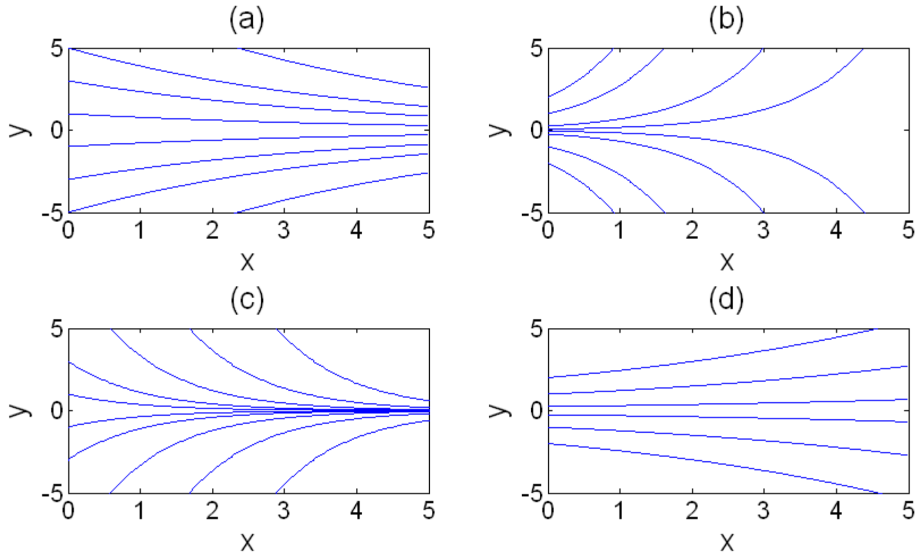
2. A small company grows at a rate proportional to its size, so that $c'(t) = kc(t)$. We set $t = 0$ in 1990 when there were 50 employees. In 2005 there were 250 employees. What equation must we solve in order to find the growth constant k?

   (a) $50e^{2005k} = 250$
   (b) $50e^{15k} = 250$
   (c) $250e^{15k} = 50$
   (d) $50e^{tk} = 250$
   (e) Not enough information is given.

3. What differential equation is solved by the function $f(x) = 0.4e^{2x}$?

   (a) $\frac{df}{dx} = 0.4f$
   (b) $\frac{df}{dx} = 2f$
   (c) $\frac{df}{dx} = 2f + 0.4$
   (d) $\frac{df}{dx} = 0.4f + 2$
   (e) None of the above.

4. Each of the graphs below show solutions of $y' = k_i y$ for a different k_i. Rank these constants from smallest to largest.



(a) $k_b < k_d < k_a < k_c$

(b) $k_d < k_c < k_b < k_a$

(c) $k_c < k_a < k_d < k_b$

(d) $k_a < k_b < k_c < k_d$

5. The function $f(y)$ solves the differential equation $f' = -0.1f$ and we know that $f(0) > 0$. This means that:

(a) When y increases by 1, f decreases by exactly 10%.

(b) When y increases by 1, f decreases by a little more than 10%.

(c) When y increases by 1, f decreases by a little less than 10%.

(d) Not enough information is given.

6. The function $g(z)$ solves the differential equation $\frac{dg}{dz} = 0.03g$. This means that:

(a) g is an increasing function that changes by 3% every time z increases by 1.

(b) g is an increasing function that changes by more than 3% every time z increases by 1.

(c) g is an increasing function that changes by less than 3% every time z increases by 1.

(d) g is a decreasing function that changes by more than 3% every time z increases by 1.

(e) g is a decreasing function that changes by less than 3% every time z increases by 1.

(f) Not enough information is given.

7. 40 grams of a radioactive element with a half-life of 35 days are put into storage. We solve $y' = -ky$ with $k = 0.0198$ to find a function that describes how the amount of this element will decrease over time. Another facility stores 80 grams of the element and we want to derive a similar function. When solving the differential equation, what value of k should we use?

   (a) $k = 0.0099$

   (b) $k = 0.0198$

   (c) $k = 0.0396$

   (d) None of the above

8. A star's brightness is decreasing at a rate equal to 10% of its current brightness per million years, so $B'(t) = -0.1B(t)$, where t is measured in millions of years. If we want t to be measured in years, how would the differential equation change?

   (a) $B'(t) = -0.1B(t)$

   (b) $B'(t) = -10^5 B(t)$

   (c) $B'(t) = -10^{-6} B(t)$

   (d) $B'(t) = -10^{-7} B(t)$

   (e) None of the above

9. The solution to which of the following will approach $+\infty$ as x becomes very large?

   (a) $y' = -2y$, $y(0) = 2$

   (b) $y' = 0.1y$, $y(0) = 1$

   (c) $y' = 6y$, $y(0) = 0$

   (d) $y' = 3y$, $y(0) = -3$

   (e) None of the above

10. $y' = -\frac{1}{3}y$ with $y(0) = 2$. As x becomes large, the solution will

    (a) diverge to $+\infty$.

    (b) diverge to $-\infty$.

(c) approach 0 from above.
(d) approach 0 from below.
(e) do none of the above.

11. Suppose H is the temperature of a hot object placed into a room whose temperature is 70 degrees, and t represents time. Suppose k is a positive number. Which of the following differential equations best corresponds to Newton's Law of Cooling?

    (a) $dH/dt = -kH$
    (b) $dH/dt = k(H - 70)$
    (c) $dH/dt = -k(H - 70)$
    (d) $dH/dt = -k(70 - H)$
    (e) $dH/dt = -kH(H - 70)$

12. Suppose H is the temperature of a hot object placed into a room whose temperature is 70 degrees.The function H giving the object's temperature as a function of time is most likely

    (a) Increasing, concave up
    (b) Increasing, concave down
    (c) Decreasing, concave up
    (d) Decreasing, concave down

13. Suppose H is the temperature of a hot object placed into a room whose temperature is 70 degrees, and t represents time. Then $\lim_{t\to\infty} H$ should equal approximately

    (a) $-\infty$
    (b) 0
    (c) 32
    (d) 70
    (e) Whatever the difference is between the object's initial temperature and 70