

MathQuest: Differential Equations

Exponential Solutions, Growth and Decay

1. A star's brightness is decreasing at a rate equal to 10% of its current brightness per million years. If B_0 is a constant with units of brightness and t is in millions of years, what function could describe the brightness of the star?

(a) $B'(t) = -0.1B(t)$

(b) $B(t) = B_0e^t$

(c) $B(t) = B_0e^{-0.1t}$

(d) $B(t) = B_0e^{0.1t}$

(e) $B(t) = B_0e^{0.9t}$

(f) $B(t) = -0.1B_0t$

2. A small company grows at a rate proportional to its size, so that $c'(t) = kc(t)$. We set $t = 0$ in 1990 when there were 50 employees. In 2005 there were 250 employees. What equation must we solve in order to find the growth constant k ?

(a) $50e^{2005k} = 250$

(b) $50e^{15k} = 250$

(c) $250e^{15k} = 50$

(d) $50e^{tk} = 250$

(e) Not enough information is given.

3. What differential equation is solved by the function $f(x) = 0.4e^{2x}$?

(a) $\frac{df}{dx} = 0.4f$

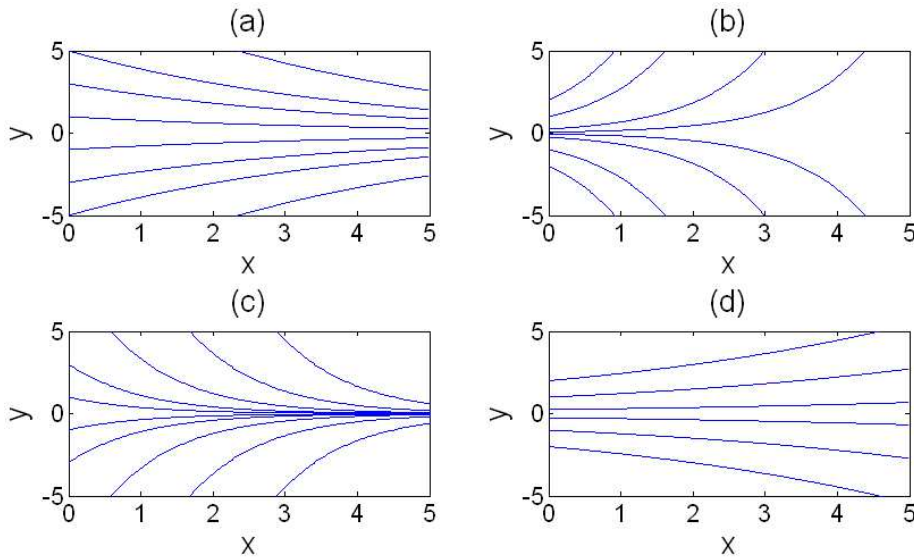
(b) $\frac{df}{dx} = 2f$

(c) $\frac{df}{dx} = 2f + 0.4$

(d) $\frac{df}{dx} = 0.4f + 2$

(e) None of the above.

4. Each of the graphs below show solutions of $y' = k_i y$ for a different k_i . Rank these constants from smallest to largest.



- (a) $k_b < k_d < k_a < k_c$
 (b) $k_d < k_c < k_b < k_a$
 (c) $k_c < k_a < k_d < k_b$
 (d) $k_a < k_b < k_c < k_d$

5. The function $f(y)$ solves the differential equation $f' = -0.1f$ and we know that $f(0) > 0$. This means that:

- (a) When y increases by 1, f decreases by exactly 10%.
 (b) When y increases by 1, f decreases by a little more than 10%.
 (c) When y increases by 1, f decreases by a little less than 10%.
 (d) Not enough information is given.

6. The function $g(z)$ solves the differential equation $\frac{dg}{dz} = 0.03g$. This means that:

- (a) g is an increasing function that changes by 3% every time z increases by 1.
 (b) g is an increasing function that changes by more than 3% every time z increases by 1.
 (c) g is an increasing function that changes by less than 3% every time z increases by 1.
 (d) g is a decreasing function that changes by more than 3% every time z increases by 1.

- (e) g is a decreasing function that changes by less than 3% every time z increases by 1.
- (f) Not enough information is given.
7. 40 grams of a radioactive element with a half-life of 35 days are put into storage. We solve $y' = -ky$ with $k = 0.0198$ to find a function that describes how the amount of this element will decrease over time. Another facility stores 80 grams of the element and we want to derive a similar function. When solving the differential equation, what value of k should we use?
- (a) $k = 0.0099$
- (b) $k = 0.0198$
- (c) $k = 0.0396$
- (d) None of the above
8. A star's brightness is decreasing at a rate equal to 10% of its current brightness per million years, so $B'(t) = -0.1B(t)$, where t is measured in millions of years. If we want t to be measured in years, how would the differential equation change?
- (a) $B'(t) = -0.1B(t)$
- (b) $B'(t) = -10^5 B(t)$
- (c) $B'(t) = -10^{-6} B(t)$
- (d) $B'(t) = -10^{-7} B(t)$
- (e) None of the above
9. The solution to which of the following will approach $+\infty$ as x becomes very large?
- (a) $y' = -2y, y(0) = 2$
- (b) $y' = 0.1y, y(0) = 1$
- (c) $y' = 6y, y(0) = 0$
- (d) $y' = 3y, y(0) = -3$
- (e) None of the above
10. $y' = -\frac{1}{3}y$ with $y(0) = 2$. As x becomes large, the solution will
- (a) diverge to $+\infty$.
- (b) diverge to $-\infty$.
- (c) approach 0 from above.
- (d) approach 0 from below.
- (e) do none of the above.