

MathQuest: Differential Equations

Equilibria and Stability

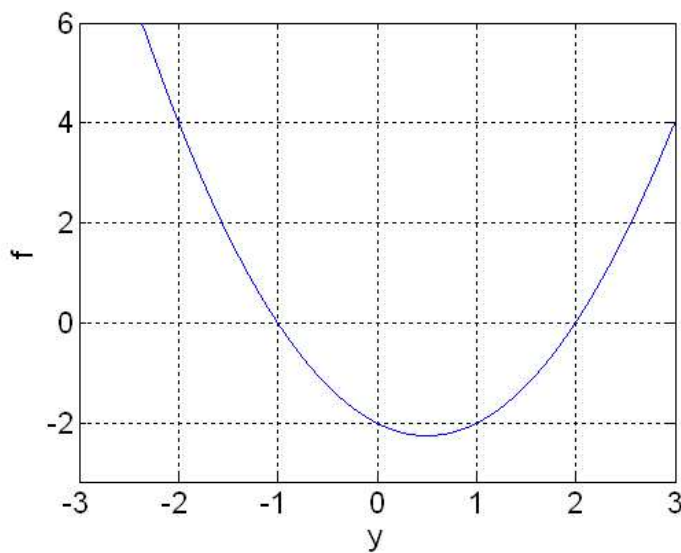
1. The differential equation $\frac{dy}{dt} = (t - 3)(y - 2)$ has equilibrium values of
 - (a) $y = 2$ only
 - (b) $t = 3$ only
 - (c) $y = 2$ and $t = 3$
 - (d) No equilibrium values
2. Suppose that 3 is an equilibrium value of a differential equation. This means that
 - (a) the values will approach 3.
 - (b) if the initial value is below 3, the values will decrease.
 - (c) if the initial value is 3, then all of the values will be 3.
 - (d) all of the above.
3. We know that a given differential equation is in the form $y' = f(y)$, where f is a continuous function of y . Suppose that $f(5) = 2$ and $f(-1) = -6$.
 - (a) y must have an equilibrium value between $y = 5$ and $y = -1$.
 - (b) y must have an equilibrium value between $y = 2$ and $y = -6$.
 - (c) This does not necessarily indicate that any equilibrium value exists.
4. We know that a given differential equation is in the form $y' = f(y)$, where f is a continuous function of y . Suppose that $f(10) = 0$, $f(9) = 3$, and $f(11) = -3$.
 - (a) This means that $y = 10$ is a stable equilibrium.
 - (b) $y = 10$ is an equilibrium, but it might not be stable.
 - (c) This does not tell us for certain that $y = 10$ is an equilibrium.
5. We know that a given differential equation is in the form $y' = f(y)$, where f is a continuous function of y . Suppose that $f(6) = 0$, $f(14) = 0$, and $y(10) = 10$.
 - (a) This means that $y(0)$ must have been between 6 and 14.

- (b) This means that $y(20) = 0$ is impossible.
- (c) This means that $y(20) = 20$ is impossible.
- (d) All of the above.
- (e) None of the above.
6. We know that a given differential equation is in the form $y' = f(y)$, where f is a continuous function of y . Suppose that $f(2) = 3$ and that $y(0) = 0$. Which of the following is impossible?
- (a) $y(10) = 6$
- (b) $y(10) = -6$
- (c) $y(-10) = 6$
- (d) $y(-10) = -6$
- (e) All of these are possible
7. We know that a given differential equation is in the form $y' = f(y)$, where f is a continuous function of y . Suppose that $f(5) = -2$, $f(10) = 4$, and that $y(10) = 3$.
- (a) $y(0)$ must be below 5.
- (b) $y(20)$ must be below 5.
- (c) $y(5)$ could be above 10.
- (d) $y(15)$ must be less than 3.
8. A differential equation has a stable equilibrium value of $T = 6$. Which of the following functions is definitely not a solution?
- (a) $T(t) = 5e^{-3t} + 6$
- (b) $T(t) = -4e^{-2t} + 6$
- (c) $T(t) = 4e^{2t} + 10$
- (d) They could all be solutions
9. Consider the differential equation $\frac{df}{dx} = \sin(f)$
- (a) $f = 0$ is a stable equilibrium.
- (b) $f = 0$ is an unstable equilibrium.
- (c) $f = 0$ is not an equilibrium.

10. Consider the differential equation $\frac{df}{dx} = af + b$, where a and b are positive parameters. If we increase b , what will happen to the equilibrium value?

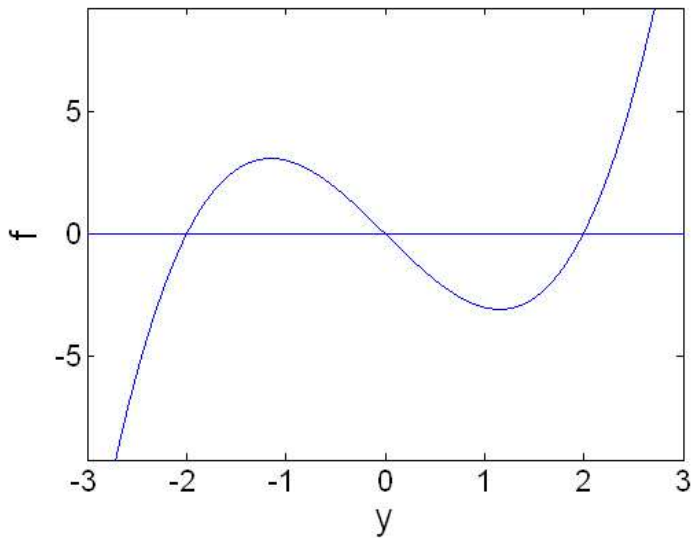
- (a) it increases
- (b) is decreases
- (c) it stays the same
- (d) not enough information is given

11. Suppose that $\frac{dy}{dt} = f(y)$, which is plotted below. What are the equilibrium values of the system?



- (a) $y = \frac{1}{2}$ is the only equilibrium.
- (b) $y = -1$ and $y = 2$ are both equilibria.
- (c) Not enough information is given.

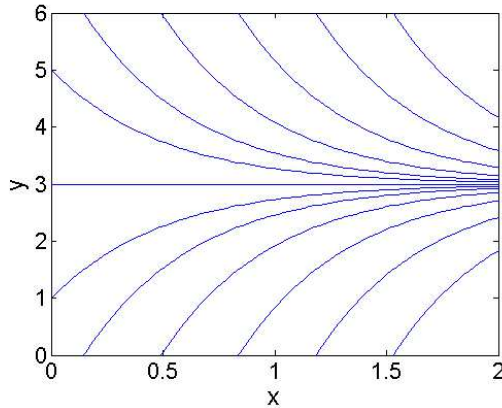
12. Suppose that $\frac{dy}{dt} = f(y)$, which is plotted below. What can we say about the equilibria of this system?



- (a) $y = 0$ is stable, $y = \pm 2$ are unstable.
 (b) $y = 0$ is unstable, $y = \pm 2$ are stable.
 (c) $y = -2, 0$ are stable, $y = 2$ is unstable.
 (d) $y = -2$ is unstable, $y = 0, 2$ are unstable
 (e) None of the above
13. **True or False** Could a differential equation have infinitely many equilibria?
14. **True or False** Could a differential equation have infinitely many equilibria over a finite range?
15. Consider the differential equation $\frac{df}{dx} = af + b$, where a and b are non-negative parameters. This equation would have no equilibrium if
- (a) $a = 0$
 (b) $b = 0$
 (c) $a = 1$
 (d) More than one of the above
16. What is the equilibrium value of $\frac{dg}{dz} = -\frac{1}{2}g + 3e^z$?
- (a) This system is at equilibrium when $g = 6e^z$.
 (b) This system is at equilibrium when $z = \ln\left(\frac{g}{6}\right)$.

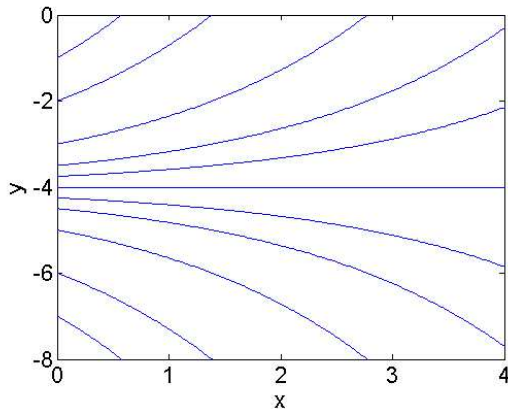
- (c) Both a and b are true.
- (d) This equation has no equilibrium.

17. The figure below plots several functions which all solve the differential equation $y' = ay + b$. What could be the values of a and b ?



- (a) $a = 1, b = 3$
- (b) $a = 2, b = -6$
- (c) $a = -1, b = -3$
- (d) $a = -2, b = 6$
- (e) $b = 3$ but a is not easy to tell

18. The figure below plots several functions which all solve the differential equation $\frac{dy}{dx} = ay + b$. What could be the values of a and b ?



- (a) $a = 0.5, b = 2$
- (b) $a = 0.5, b = -2$

- (c) $a = -0.5, b = 2$
- (d) $a = -0.5, b = -2$
- (e) None of the above are possible.