MathQuest: Differential Equations

Equilibria and Stability

1. The differential equation $\frac{dy}{dt} = (t-3)(y-2)$ has equilibrium values of

   (a) $y = 2$ only

   (b) $t = 3$ only

   (c) $y = 2$ and $t = 3$

   (d) No equilibrium values

2. Suppose that 3 is an equilibrium value of a differential equation. This means that

   (a) the values will approach 3.

   (b) if the initial value is below 3, the values will decrease.

   (c) if the initial value is 3, then all of the values will be 3.

   (d) all of the above.

3. We know that a given differential equation is in the form $y' = f(y)$, where f is a differentiable function of y. Suppose that $f(5) = 2$ and $f(-1) = -6$.

   (a) y must have an equilibrium value between $y = 5$ and $y = -1$.

   (b) y must have an equilibrium value between $y = 2$ and $y = -6$.

   (c) This does not necessarily indicate that any equilibrium value exists.

4. We know that a given differential equation is in the form $y' = f(y)$, where f is a differentiable function of y. Suppose that $f(10) = 0$, $f(9) = 3$, and $f(11) = -3$.

   (a) This means that $y = 10$ is a stable equilibrium.

   (b) $y = 10$ is an equilibrium, but it might not be stable.

   (c) This does not tell us for certain that $y = 10$ is an equilibrium.

5. We know that a given differential equation is in the form $y' = f(y)$, where f is a differentiable function of y. Suppose that $f(6) = 0$, $f(14) = 0$, and $y(10) = 10$.

   (a) This means that $y(0)$ must have been between 6 and 14.

(b) This means that $y(20) = 0$ is impossible.

(c) This means that $y(20) = 20$ is impossible.

(d) All of the above.

(e) None of the above.

6. We know that a given differential equation is in the form $y' = f(y)$, where f is a differentiable function of y. Suppose that $f(2) = 3$ and that $y(0) = 0$. Which of the following is impossible?

   (a) $y(10) = 6$

   (b) $y(10) = -6$

   (c) $y(-10) = 6$

   (d) $y(-10) = -6$

   (e) All of these are possible

7. We know that a given differential equation is in the form $y' = f(y)$, where f is a differentiable function of y. Suppose that $f(5) = -2$, $f(10) = 4$, and that $y(10) = 3$.

   (a) $y(0)$ must be below 5.

   (b) $y(20)$ must be below 5.

   (c) $y(5)$ could be above 10.

   (d) $y(15)$ must be less than 3.

8. A differential equation has a stable equilibrium value of $T = 6$. Which of the following functions is definitely not a solution?

   (a) $T(t) = 5e^{-3t} + 6$

   (b) $T(t) = -4e^{-2t} + 6$

   (c) $T(t) = 4e^{2t} + 10$

   (d) They could all be solutions

9. Consider the differential equation $\frac{df}{dx} = \sin(f)$

   (a) $f = 0$ is a stable equilibrium.

   (b) $f = 0$ is an unstable equilibrium.

   (c) $f = 0$ is not an equilibrium.

10. Consider the differential equation $\frac{df}{dx} = af + b$, where a and b are positive parameters. If we increase b, what will happen to the equilibrium value?

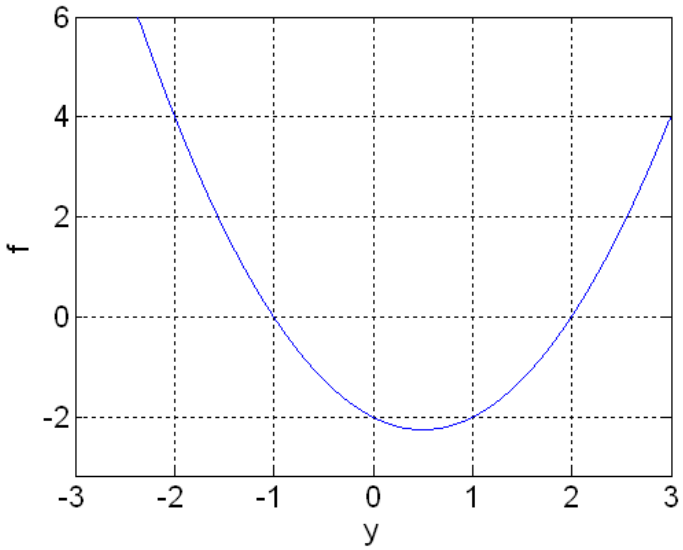
    (a) it increases

    (b) is decreases

    (c) it stays the same

    (d) not enough information is given

11. Suppose that $\frac{dy}{dt} = f(y)$, which is plotted below. What are the equilibrium values of the system?

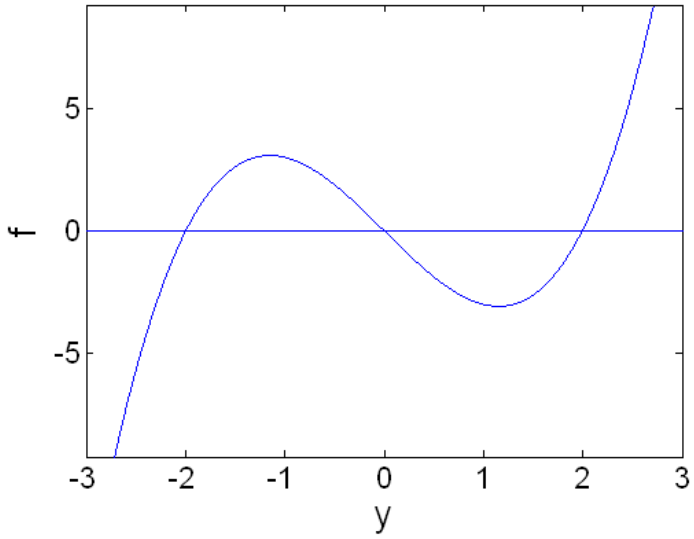


    (a) $y = \frac{1}{2}$ is the only equilibrium.

    (b) $y = -1$ and $y = 2$ are both equilibria.

    (c) Not enough information is given.

12. Suppose that $\frac{dy}{dt} = f(y)$, which is plotted below. What can we say about the equilibria of this system?



(a) $y = 0$ is stable, $y = \pm 2$ are unstable.

(b) $y = 0$ is unstable, $y = \pm 2$ are stable.

(c) $y = -2, 0$ are stable, $y = 2$ is unstable.

(d) $y = -2$ is unstable, $y = 0, 2$ are unstable

(e) None of the above

13. **True or False** A differential equation could have infinitely many equilibria.

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

14. **True or False** A differential equation could have infinitely many equilibria over a finite range.

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

15. Consider the differential equation $\frac{df}{dx} = af + b$, where a and b are non-negative parameters. This equation would have no equilibrium if

(a) $a = 0$

(b) $b = 0$

(c) $a = 1$

(d) More than one of the above

16. What is the equilibrium value of $\frac{dg}{dz} = -\frac{1}{2}g + 3e^z$?

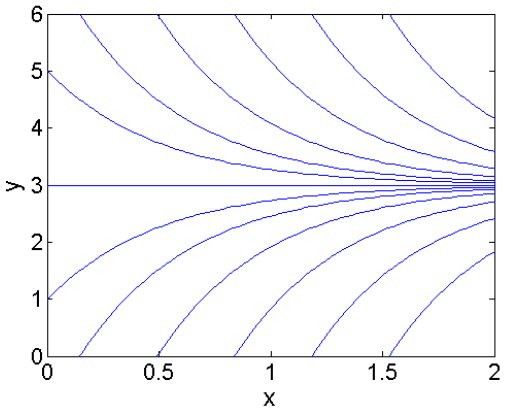
(a) This system is at equilibrium when $g = 6e^z$.

(b) This system is at equlibrium when $z = \ln\left(\frac{g}{6}\right)$.

(c) Both a and b are true.

(d) This equation has no equlibrium.

17. The figure below plots several functions which all solve the differential equation $y' = ay + b$. What could be the values of a and b?



(a) $a = 1$, $b = 3$

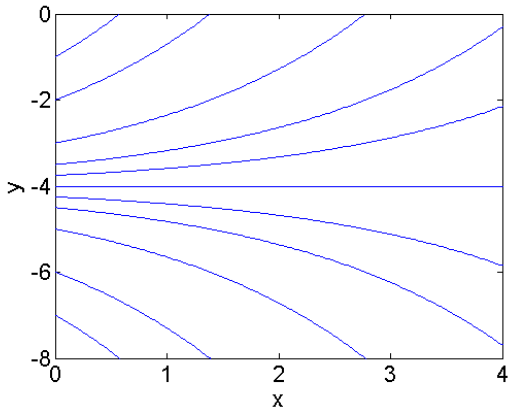
(b) $a = 2$, $b = -6$

(c) $a = -1$, $b = -3$

(d) $a = -2$, $b = 6$

(e) $b = 3$ but a is not easy to tell

18. The figure below plots several functions which all solve the differential equation $\frac{dy}{dx} = ay + b$. What could be the values of a and b?



(a) $a = 0.5$, $b = 2$

(b) $a = 0.5$, $b = -2$

(c) $a = -0.5$, $b = 2$

(d) $a = -0.5$, $b = -2$

(e) None of the above are possible.