

MathQuest: Differential Equations

First Order Linear Models

1. Water from a thunderstorm flows into a reservoir at a rate given by the function $g(t) = 250e^{-0.1t}$, where g is in gallons per day, and t is in days. The water in the reservoir evaporates at a rate of 2.25% per day. What equation could describe this scenario?

- (a) $f'(t) = -0.0225f + 250e^{-0.1t}$
- (b) $f'(t) = -0.0225(250e^{-0.1t})$
- (c) $f'(t) = 0.9775f + 250e^{-0.1t}$
- (d) None of the above

2. The state of ripeness of a banana is described by the differential equation $R'(t) = 0.05(2 - R)$ with $R = 0$ corresponding to a completely green banana and $R = 1$ a perfectly ripe banana. If all bananas start completely green, what value of R describes the state of a completely black, overripe banana?

- (a) $R = 0.05$
- (b) $R = \frac{1}{2}$
- (c) $R = 1$
- (d) $R = 2$
- (e) $R = 4$
- (f) None of the above.

3. The evolution of the temperature T of a hot cup of coffee cooling off in a room is described by $\frac{dT}{dt} = -0.01T + 0.6$, where T is in $^{\circ}\text{F}$ and t is in hours. What is the temperature of the room?

- (a) 0.6
- (b) -0.01
- (c) 60
- (d) 0.006
- (e) 30
- (f) none of the above

4. The evolution of the temperature of a hot cup of coffee cooling off in a room is described by $\frac{dT}{dt} = -0.01(T - 60)$, where T is in $^{\circ}\text{F}$ and t is in hours. Next, we add a small heater to the coffee which adds heat at a rate of 0.1°F per hour. What happens?
- There is no equilibrium, so the coffee gets hotter and hotter.
 - The coffee reaches an equilibrium temperature of 60°F .
 - The coffee reaches an equilibrium temperature of 70°F .
 - The equilibrium temperature becomes unstable.
 - None of the above
5. A drug is being administered intravenously into a patient at a certain rate d and is breaking down at a certain fractional rate $k > 0$. If $c(t)$ represents the concentration of the drug in the bloodstream, which differential equation represents this scenario?
- $\frac{dc}{dt} = -k + d$
 - $\frac{dc}{dt} = -kc + d$
 - $\frac{dc}{dt} = kc + d$
 - $\frac{dc}{dt} = c(d - k)$
 - None of the above
6. A drug is being administered intravenously into a patient. The drug is flowing into the bloodstream at a rate of 50 mg/hr. The rate at which the drug breaks down is proportional to the total amount of the drug, and when there is a total of 1000 mg of the drug in the patient, the drug breaks down at a rate of 300 mg/hr. If y is the number of milligrams of drug in the bloodstream at time t , what differential equation would describe the evolution of the amount of the drug in the patient?
- $y' = -0.3y + 50$
 - $y' = -0.3t + 50$
 - $y' = 0.7y + 50$
 - None of the above
7. The amount of a drug in the bloodstream follows the differential equation $c' = -kc + d$, where d is the rate it is being added intravenously and k is the fractional rate at which it breaks down. If the initial concentration is given by a value $c(0) > d/k$, then what will happen?
- This equation predicts that the concentration of the drug will be negative, which is impossible.

- (b) The concentration of the drug will decrease until there is none left.
- (c) This means that the concentration of the drug will get smaller, until it reaches the level $c = d/k$, where it will stay.
- (d) This concentration of the drug will approach but never reach the level d/k .
- (e) Because $c(0) > d/k$ this means that the concentration of the drug will increase, so the dose d should be reduced.
8. The amount of a drug in the bloodstream follows the differential equation $c' = -kc + d$, where d is the rate it is being added intravenously and k is the fractional rate at which it breaks down. If we double the rate at which the drug flows in, how will this change the equilibrium value?
- (a) It will be double the old value.
- (b) It will be greater than the old, but not quite doubled.
- (c) It will be more than doubled.
- (d) It will be the same.
- (e) Not enough information is given.
9. If we construct an electric circuit with a battery, a resistor, and a capacitor all in series, then the voltage is described by the equation $V_{bat} = \frac{Q}{C} + IR$. Here V_{bat} is the voltage produced by the battery, and the constants C and R give the capacitance and resistance respectively. $Q(t)$ is the charge on the capacitor and $I(t) = \frac{dQ}{dt}$ is the current flowing through the circuit. What is the equilibrium charge on the capacitor?
- (a) $Q_e = V_{bat}C$
- (b) $Q_e = V_{bat}/R$
- (c) $Q_e = 0$
- (d) Not enough information is given.
10. If we construct an electric circuit with a battery, a resistor, and a capacitor all in series, then the voltage is described by the equation $V_{bat} = \frac{Q}{C} + IR$. Here V_{bat} is the voltage produced by the battery, and the constants C and R give the capacitance and resistance respectively. $Q(t)$ is the charge on the capacitor and $I(t) = \frac{dQ}{dt}$ is the current flowing through the circuit. Which of the following functions could describe the charge on the capacitor $Q(t)$?
- (a) $Q(t) = 5e^{-t/RC}$
- (b) $Q(t) = 4e^{-RCt} + V_{bat}C$

- (c) $Q(t) = 3e^{-t/RC} - V_{bat}C$
- (d) $Q(t) = -6e^{-t/RC} + V_{bat}C$
- (e) None of the above

11. If we construct an electric circuit with a battery, a resistor, and a capacitor all in series, then the voltage is described by the equation $V_{bat} = \frac{Q}{C} + IR$. Here V_{bat} is the voltage produced by the battery, and the constants C and R give the capacitance and resistance respectively. $Q(t)$ is the charge on the capacitor and $I(t) = \frac{dQ}{dt}$ is the current flowing through the circuit. Which of the following functions could describe the current flowing through the circuit $I(t)$?

- (a) $I(t) = 5e^{-t/RC}$
- (b) $I(t) = 4e^{-RCt} + V_{bat}C$
- (c) $I(t) = 3e^{-t/RC} - V_{bat}C$
- (d) $I(t) = -6e^{-t/RC} + V_{bat}C$
- (e) None of the above