MathQuest: Differential Equations

First Order Linear Models

1. Water from a thunderstorm flows into a reservoir at a rate given by the function $g(t) = 250e^{-0.1t}$, where g is in gallons per day, and t is in days. The water in the reservoir evaporates at a rate of 2.25% per day. What equation could describe this scenario?

   (a) $f'(t) = -0.0225f + 250e^{-0.1t}$
   (b) $f'(t) = -0.0225(250e^{-0.1t})$
   (c) $f'(t) = 0.9775f + 250e^{-0.1t}$
   (d) None of the above

2. The state of ripeness of a banana is described by the differential equation $R'(t) = 0.05(2 - R)$ with $R = 0$ corresponding to a completely green banana and $R = 1$ a perfectly ripe banana. If all bananas start completely green, what value of R describes the state of a completely black, overripe banana?

   (a) $R = 0.05$
   (b) $R = \frac{1}{2}$
   (c) $R = 1$
   (d) $R = 2$
   (e) $R = 4$
   (f) None of the above.

3. The evolution of the temperature T of a hot cup of coffee cooling off in a room is described by $\frac{dT}{dt} = -0.01T + 0.6$, where T is in °F and t is in hours. What is the temperature of the room?

   (a) 0.6
   (b) -0.01
   (c) 60
   (d) 0.006
   (e) 30
   (f) none of the above

4. The evolution of the temperature of a hot cup of coffee cooling off in a room is described by $\frac{dT}{dt} = -0.01(T-60)$, where T is in °F and t is in hours. Next, we add a small heater to the coffee which adds heat at a rate of 0.1 °F per hour. What happens?

   (a) There is no equilibrium, so the coffee gets hotter and hotter.

   (b) The coffee reaches an equilibrium temperature of 60°F.

   (c) The coffee reaches an equilibrium temperature of 70°F.

   (d) The equilibrium temperature becomes unstable.

   (e) None of the above

5. A drug is being administered intravenously into a patient at a certain rate d and is breaking down at a certain fractional rate $k > 0$. If $c(t)$ represents the concentration of the drug in the bloodstream, which differential equation represents this scenario?

   (a) $\frac{dc}{dt} = -k + d$

   (b) $\frac{dc}{dt} = -kc + d$

   (c) $\frac{dc}{dt} = kc + d$

   (d) $\frac{dc}{dt} = c(d - k)$

   (e) None of the above

6. A drug is being administered intravenously into a patient. The drug is flowing into the bloodstream at a rate of 50 mg/hr. The rate at which the drug breaks down is proportional to the total amount of the drug, and when there is a total of 1000 mg of the drug in the patient, the drug breaks down at a rate of 300 mg/hr. If y is the number of milligrams of drug in the bloodstream at time t, what differential equation would describe the evolution of the amount of the drug in the patient?

   (a) $y' = -0.3y + 50$

   (b) $y' = -0.3t + 50$

   (c) $y' = 0.7y + 50$

   (d) None of the above

7. The amount of a drug in the bloodstream follows the differential equation $c' = -kc+d$, where d is the rate it is being added intravenously and k is the fractional rate at which it breaks down. If the initial concentration is given by a value $c(0) > d/k$, then what will happen?

   (a) This equation predicts that the concentration of the drug will be negative, which is impossible.

(b) The concentration of the drug will decrease until there is none left.

(c) This means that the concentration of the drug will get smaller, until it reaches the level $c = d/k$, where it will stay.

(d) This concentration of the drug will approach but never reach the level d/k.

(e) Because $c(0) > d/k$ this means that the concentration of the drug will increase, so the dose d should be reduced.

8. The amount of a drug in the bloodstream follows the differential equation $c' = -kc+d$, where d is the rate it is being added intravenously and k is the fractional rate at which is breaks down. If we double the rate at which the drug flows in, how will this change the equilibrium value?

   (a) It will be double the old value.

   (b) It will be greater than the old, but not quite doubled.

   (c) It will be more than doubled.

   (d) It will be the same.

   (e) Not enough information is given.

9. If we construct an electric circuit with a battery, a resistor, and a capacitor all in series, then the voltage is described by the equation $V_{bat} = \frac{Q}{C} + IR$. Here V_{bat} is the voltage produced by the battery, and the constants C and R give the capacitance and resistance respectively. $Q(t)$ is the charge on the capacitor and $I(t) = \frac{dQ}{dt}$ is the current flowing through the circuit. What is the equilibrium charge on the capacitor?

   (a) $Q_e = V_{bat}C$

   (b) $Q_e = V_{bat}/R$

   (c) $Q_e = 0$

   (d) Not enough information is given.

10. If we construct an electric circuit with a battery, a resistor, and a capacitor all in series, then the voltage is described by the equation $V_{bat} = \frac{Q}{C} + IR$. Here V_{bat} is the voltage produced by the battery, and the constants C and R give the capacitance and resistance respectively. $Q(t)$ is the charge on the capacitor and $I(t) = \frac{dQ}{dt}$ is the current flowing through the circuit. Which of the following functions could describe the charge on the capacitor $Q(t)$?

    (a) $Q(t) = 5e^{-t/RC}$

    (b) $Q(t) = 4e^{-RCt} + V_{bat}C$

(c) $Q(t) = 3e^{-t/RC} - V_{bat}C$

(d) $Q(t) = -6e^{-t/RC} + V_{bat}C$

(e) None of the above

11. If we construct an electric circuit with a battery, a resistor, and a capacitor all in series, then the voltage is described by the equation $V_{bat} = \frac{Q}{C} + IR$. Here V_{bat} is the voltage produced by the battery, and the constants C and R give the capacitance and resistance respectively. $Q(t)$ is the charge on the capacitor and $I(t) = \frac{dQ}{dt}$ is the current flowing through the circuit. Which of the following functions could describe the current flowing through the circuit $I(t)$?

    (a) $I(t) = 5e^{-t/RC}$

    (b) $I(t) = 4e^{-RCt} + V_{bat}C$

    (c) $I(t) = 3e^{-t/RC} - V_{bat}C$

    (d) $I(t) = -6e^{-t/RC} + V_{bat}C$

    (e) None of the above