

# MathQuest: Differential Equations

## First Order Linear Models

1. Water from a thunderstorm flows into a reservoir at a rate given by the function  $g(t) = 250e^{-0.1t}$ , where  $g$  is in gallons per day, and  $t$  is in days. The water in the reservoir evaporates at a rate of 2.25% per day. What equation could describe this scenario?

- (a)  $f'(t) = -0.0225f + 250e^{-0.1t}$
- (b)  $f'(t) = -0.0225(250e^{-0.1t})$
- (c)  $f'(t) = 0.9775f + 250e^{-0.1t}$
- (d) None of the above

2. The state of ripeness of a banana is described by the differential equation  $R'(t) = 0.05(2 - R)$  with  $R = 0$  corresponding to a completely green banana and  $R = 1$  a perfectly ripe banana. If all bananas start completely green, what value of  $R$  describes the state of a completely black, overripe banana?

- (a)  $R = 0.05$
- (b)  $R = \frac{1}{2}$
- (c)  $R = 1$
- (d)  $R = 2$
- (e)  $R = 4$
- (f) None of the above.

3. The evolution of the temperature  $T$  of a hot cup of coffee cooling off in a room is described by  $\frac{dT}{dt} = -0.01T + 0.6$ , where  $T$  is in  $^{\circ}\text{F}$  and  $t$  is in hours. What is the temperature of the room?

- (a) 0.6
- (b) -0.01
- (c) 60
- (d) 0.006
- (e) 30
- (f) none of the above

4. The evolution of the temperature of a hot cup of coffee cooling off in a room is described by  $\frac{dT}{dt} = -0.01(T - 60)$ , where  $T$  is in  $^{\circ}\text{F}$  and  $t$  is in hours. Next, we add a small heater to the coffee which adds heat at a rate of  $0.1$   $^{\circ}\text{F}$  per hour. What happens?
- There is no equilibrium, so the coffee gets hotter and hotter.
  - The coffee reaches an equilibrium temperature of  $60^{\circ}\text{F}$ .
  - The coffee reaches an equilibrium temperature of  $70^{\circ}\text{F}$ .
  - The equilibrium temperature becomes unstable.
  - None of the above
5. A drug is being administered intravenously into a patient at a certain rate  $d$  and is breaking down at a certain fractional rate  $k > 0$ . If  $c(t)$  represents the concentration of the drug in the bloodstream, which differential equation represents this scenario?
- $\frac{dc}{dt} = -k + d$
  - $\frac{dc}{dt} = -kc + d$
  - $\frac{dc}{dt} = kc + d$
  - $\frac{dc}{dt} = c(d - k)$
  - None of the above
6. A drug is being administered intravenously into a patient. The drug is flowing into the bloodstream at a rate of  $50$  mg/hr. The rate at which the drug breaks down is proportional to the total amount of the drug, and when there is a total of  $1000$  mg of the drug in the patient, the drug breaks down at a rate of  $300$  mg/hr. If  $y$  is the number of milligrams of drug in the bloodstream at time  $t$ , what differential equation would describe the evolution of the amount of the drug in the patient?
- $y' = -0.3y + 50$
  - $y' = -0.3t + 50$
  - $y' = 0.7y + 50$
  - None of the above
7. The amount of a drug in the bloodstream follows the differential equation  $c' = -kc + d$ , where  $d$  is the rate it is being added intravenously and  $k$  is the fractional rate at which it breaks down. If the initial concentration is given by a value  $c(0) > d/k$ , then what will happen?
- This equation predicts that the concentration of the drug will be negative, which is impossible.

- (b) The concentration of the drug will decrease until there is none left.
- (c) This means that the concentration of the drug will get smaller, until it reaches the level  $c = d/k$ , where it will stay.
- (d) This concentration of the drug will approach but never reach the level  $d/k$ .
- (e) Because  $c(0) > d/k$  this means that the concentration of the drug will increase, so the dose  $d$  should be reduced.
8. The amount of a drug in the bloodstream follows the differential equation  $c' = -kc + d$ , where  $d$  is the rate it is being added intravenously and  $k$  is the fractional rate at which it breaks down. If we double the rate at which the drug flows in, how will this change the equilibrium value?
- (a) It will be double the old value.
- (b) It will be greater than the old, but not quite doubled.
- (c) It will be more than doubled.
- (d) It will be the same.
- (e) Not enough information is given.
9. If we construct an electric circuit with a battery, a resistor, and a capacitor all in series, then the voltage is described by the equation  $V_{bat} = \frac{Q}{C} + IR$ . Here  $V_{bat}$  is the voltage produced by the battery, and the constants  $C$  and  $R$  give the capacitance and resistance respectively.  $Q(t)$  is the charge on the capacitor and  $I(t) = \frac{dQ}{dt}$  is the current flowing through the circuit. What is the equilibrium charge on the capacitor?
- (a)  $Q_e = V_{bat}C$
- (b)  $Q_e = V_{bat}/R$
- (c)  $Q_e = 0$
- (d) Not enough information is given.
10. If we construct an electric circuit with a battery, a resistor, and a capacitor all in series, then the voltage is described by the equation  $V_{bat} = \frac{Q}{C} + IR$ . Here  $V_{bat}$  is the voltage produced by the battery, and the constants  $C$  and  $R$  give the capacitance and resistance respectively.  $Q(t)$  is the charge on the capacitor and  $I(t) = \frac{dQ}{dt}$  is the current flowing through the circuit. Which of the following functions could describe the charge on the capacitor  $Q(t)$ ?
- (a)  $Q(t) = 5e^{-t/RC}$
- (b)  $Q(t) = 4e^{-RCt} + V_{bat}C$

- (c)  $Q(t) = 3e^{-t/RC} - V_{bat}C$
- (d)  $Q(t) = -6e^{-t/RC} + V_{bat}C$
- (e) None of the above

11. If we construct an electric circuit with a battery, a resistor, and a capacitor all in series, then the voltage is described by the equation  $V_{bat} = \frac{Q}{C} + IR$ . Here  $V_{bat}$  is the voltage produced by the battery, and the constants  $C$  and  $R$  give the capacitance and resistance respectively.  $Q(t)$  is the charge on the capacitor and  $I(t) = \frac{dQ}{dt}$  is the current flowing through the circuit. Which of the following functions could describe the current flowing through the circuit  $I(t)$ ?

- (a)  $I(t) = 5e^{-t/RC}$
- (b)  $I(t) = 4e^{-RCt} + V_{bat}C$
- (c)  $I(t) = 3e^{-t/RC} - V_{bat}C$
- (d)  $I(t) = -6e^{-t/RC} + V_{bat}C$
- (e) None of the above