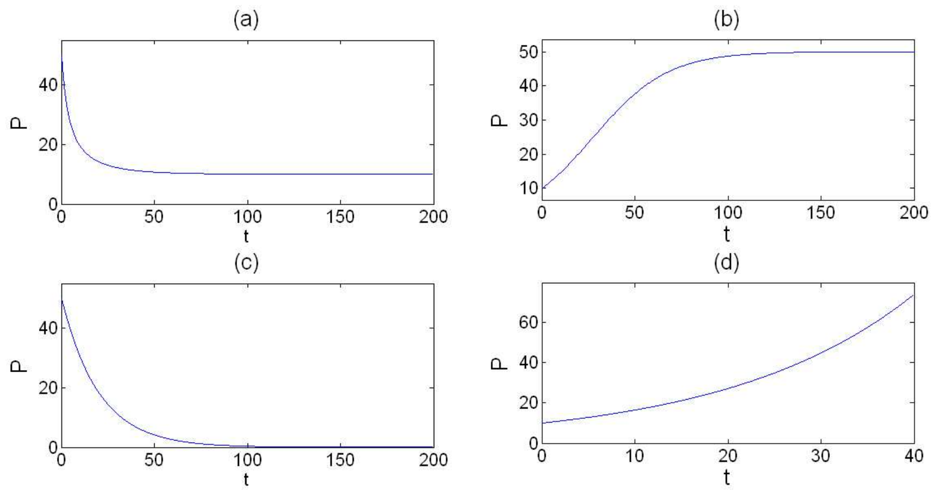
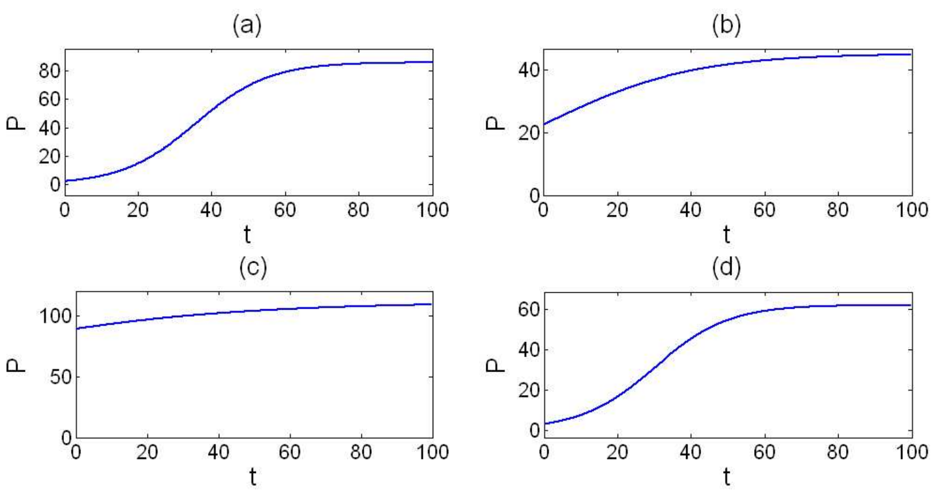
MathQuest: Differential Equations

Logistic Models

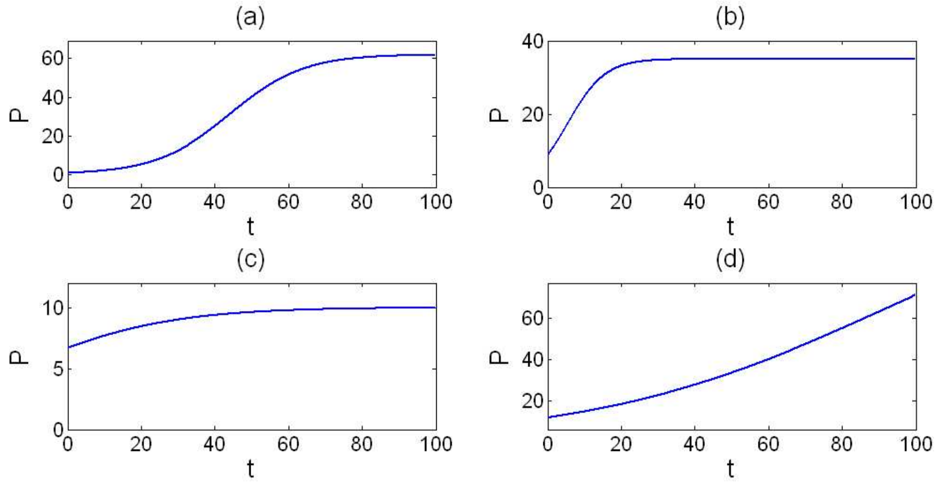
1. Consider the function $P = \frac{L}{1+Ae^{-kt}}$, where $A = (L - P_0)/P_0$. Suppose that $P_0 = 10$, $L = 50$, and $k = 0.05$, which of the following could be a graph of this function?



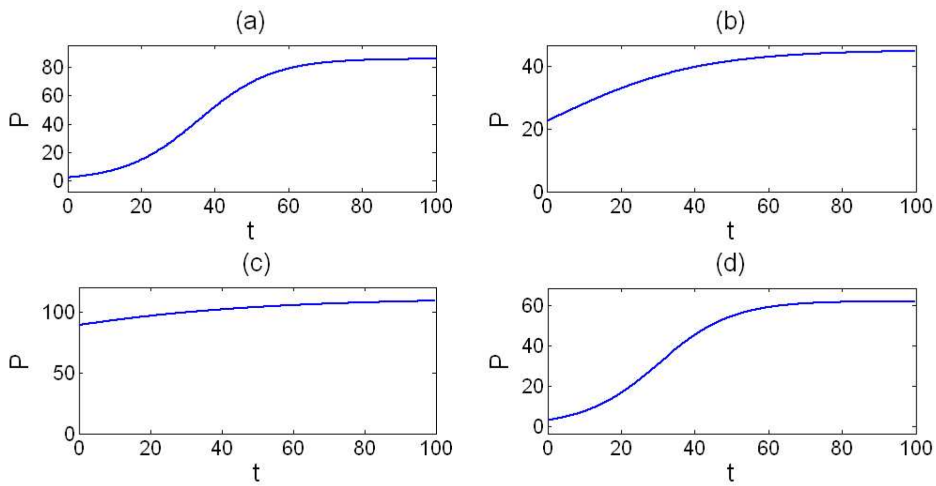
2. The following graphs all plot the function $P = \frac{L}{1+Ae^{-kt}}$. The function plotted in which graph has the largest value of L?



3. The following graphs all plot the function $P = \frac{L}{1+Ae^{-kt}}$. The function plotted in which graph has the largest value of k?



4. The following graphs all plot the function $P = \frac{L}{1+Ae^{-kt}}$. The function plotted in which graph has the largest value of A?



5. Consider the differential equation $\frac{dP}{dt} = kP\left(1 - \frac{P}{L}\right)$, called the logistic equation. What are the equilibria of this system?

   i. $k = 0$ is a stable equilibrium.
   ii. $L = 0$ is an unstable equilibrium.
   iii. $P = L$ is a stable equilibrium.
   iv. $P = 0$ is an unstable equilibrium.
   v. $P = L$ is an unstable equilibrium.
   vi $P = 0$ is a stable equilibrium.

   (a) i
   (b) ii
   (c) Both iii and v

(d) Both v and vi
(e) Both iv and v
(f) Both iii and iv

6. The population of rainbow trout in a river system is modeled by the differential equation $P' = 0.2P - 4 \times 10^{-5}P^2$. What is the maximum number of trout that the river system could support?

(a) 4×10^5 trout
(b) 4,000 trout
(c) 5,000 trout
(d) 25,000 trout
(e) Not enough information is given

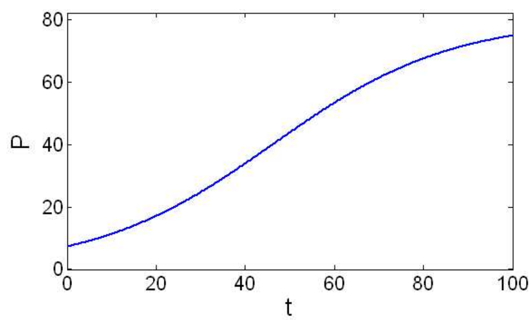
7. The solution to the logistic equation $\frac{dP}{dt} = kP\left(1 - \frac{P}{L}\right)$ is $P = \frac{L}{1+Ae^{-kt}}$, where $A = (L - P_0)/P_0$. If we are modeling a herd of elk, with an initial population of 50, in a region with a carrying capacity of 300, and knowing that the exponential growth rate of an elk population is 0.07, which function would describe our elk population as a function of time?

(a) $P(t) = \frac{300}{1+5e^{-0.07t}}$
(b) $P(t) = \frac{50}{1+\frac{5}{6}e^{0.07t}}$
(c) $P(t) = \frac{300}{1+\frac{6}{5}e^{0.07t}}$
(d) $P(t) = \frac{300}{1+\frac{1}{6}e^{-0.07t}}$

8. The population of mice on a farm is modeled by the differential equation $\frac{3000}{P}\frac{dP}{dt} = 200 - P$. If we know that today there are 60 mice on the farm, what function will describe how the mouse population will develop in the future?

(a) $P = \frac{200}{1+\frac{7}{3}e^{-t/15}}$
(b) $P = \frac{200}{1+\frac{7}{3}e^{-200t}}$
(c) $P = \frac{3000}{1+49e^{-t/15}}$
(d) $P = \frac{3000}{1+49e^{-t/20}}$
(e) None of the above

9. The function plotted below could be a solution to which of the following differential equations?



(a) $\frac{dP}{dt} = -0.05P\left(1 - \frac{P}{80}\right)$

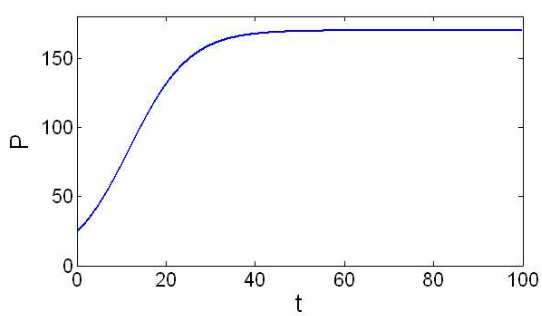
(b) $P = \frac{P'}{\frac{1}{20} - 6.25 \times 10^{-4} P}$

(c) $40\frac{P'}{P^2} + \frac{1}{40} = \frac{2}{P}$

(d) $-20\frac{dP}{dt} + P = \frac{P^2}{80}$

(e) All of the above

10. The function plotted below could be a solution of which of the following?



(a) $\frac{dP}{dt} = 0.15P\left(1 - \frac{P}{170}\right)$

(b) $\frac{dP}{dt} = 0.15P\left(1 - \frac{P}{240}\right)$

(c) $\frac{dP}{dt} = 0.05P\left(1 - \frac{P}{170}\right)$

(d) $\frac{dP}{dt} = 0.05P\left(1 - \frac{P}{240}\right)$

(e) None of the above