MathQuest: Differential Equations

Nonhomogeneous Differential Equations & Undetermined Coefficients

1. Consider the equation $\frac{df}{dx} = 2f + e^{3x}$. When we separate the variables, this equation becomes:

   (a) $\frac{1}{2f}df = e^{3x}dx$

   (b) $df = 2f + e^{3x}dx$

   (c) $\frac{1}{f}df = (2 + e^{3x})dx$

   (d) $-2fdf = e^{3x}dx$

   (e) This equation is not separable

2. $x'(t) + 4x(t) = e^t$ and we want to test the function $x(t) = C_0e^{-4t} + C_1e^t$ to see if it is a solution. What equation is the result?

   (a) $(-4C_0e^{-4t} + C_1e^t) + 4C_0e^{-4t} + 4C_1e^t = e^t$

   (b) $(C_0e^{-4t} + C_1e^t) + 4C_0e^{-4t} + 4C_1e^t = e^t$

   (c) $-4C_0e^{-4t} + 4C_1e^t = e^t$

   (d) None of the above

3. We are testing the function $f(x) = C_0e^{3x}$ as a possible solution to a differential equation. After we substitute the function and its derivative into the differential equation we get: $3C_0e^{3x} = -2C_0e^{3x} + 4e^{3x}$. What was the differential equation?

   (a) $f' = -2f + \frac{4}{C_0}f$

   (b) $f' = -2f + 4e^{3x}$

   (c) $3f = -2f + 4e^{3x}$

   (d) $3C_0e^{3x} = -2f + 4e^{3x}$

   (e) None of the above.

4. We are testing the function $f(x) = C_0e^{2x} + C_1e^{-2x}$ as a possible solution to a differential equation. After we substitute the function and its derivative into the differential equation we get: $2C_0e^{2x} - 2C_1e^{-2x} = -2(C_0e^{2x} + C_1e^{-2x}) + 3e^{2x}$. What value of C_0 will allow this function to work?

(a) $C_0 = \frac{3}{4}$

(b) $C_0 = \frac{3}{2}$

(c) $C_0 = 3$

(d) $C_0 = 2$

(e) Any value of C_0 will work.

(f) No value of C_0 will work.

5. We are testing the function $f(x) = C_0e^{2x} + C_1e^{-2x}$ as a possible solution to a differential equation. After we substitute the function and its derivative into the differential equation we get: $2C_0e^{2x} - 2C_1e^{-2x} = -2(C_0e^{2x} + C_1e^{-2x}) + 3e^{2x}$. What value of C_1 will allow this function to work?

(a) $C_1 = \frac{3}{4}$

(b) $C_1 = \frac{3}{2}$

(c) $C_1 = 3$

(d) $C_1 = 2$

(e) Any value of C_1 will work.

(f) No value of C_1 will work.

6. When we have $y' = 7y + 2x$ we should conjecture $y = C_0e^{7x} + C_1x + C_2$. Why do we add the C_2?

(a) Because the $7y$ becomes a constant 7 when we take the derivative and we need a term to cancel this out.

(b) Because when we take the derivative of C_1x we get a constant C_1 and we need a term to cancel this out.

(c) Because this will allow us to match different initial conditions.

(d) This does not affect the equation because it goes away when we take the derivative.

7. We have the equation $y' = 2y + \sin 3t$. What should be our conjecture?

(a) $y = C_0e^{2t} + \sin 3t$

(b) $y = C_0e^{2t} + \sin 3t + \cos 3t$

(c) $y = C_0e^{2t} + C_1 \sin 3t$

(d) $y = C_0e^{2t} + C_1 \sin 3t + C_2 \cos 3t$

(e) $y = C_0e^{2t} + C_1e^{-2t} + C_2 \sin 3t + C_3 \cos 3t$

(f) None of the above

8. Consider $\frac{dg}{dz} = ag + b\cos cz$, where a, b, and c are all positive parameters. What will be the long term behavior of this system?

   (a) It will grow exponentially.

   (b) It will converge to an equilibrium.

   (c) It will oscillate.

   (d) Different behaviors are possible depending on the values of a, b, and c.

9. A bookstore is constantly discarding a certain percentage of its unsold inventory and also receiving new books from its supplier so that the rate of change of the number of books in inventory is $B'(t) = -0.02B + 400 + 0.05t$, where B is the number of books and t is in months. In the long run, what will happen to the number of books in inventory, according to this model?

   (a) The number of books will approach zero.

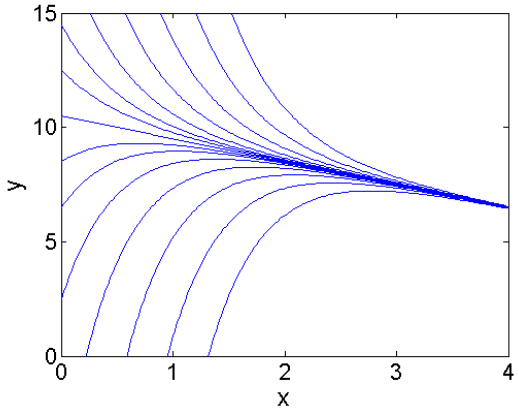
   (b) The number of books will approach a stable equilibrium.

   (c) The number of books will exponentially diverge from an unstable equilibrium.

   (d) The number of books will grow linearly.

   (e) None of the above

10. The figure below shows several functions that solve the differential equation $y' = ay + bx + c$. What could be the values of a, b, and c?



   (a) $a = 2$, $b = 2$, $c = 20$

   (b) $a = -2$, $b = -2$, $c = -20$

   (c) $a = 2$, $b = -2$, $c = -20$

(d) $a = -2$, $b = -2$, $c = 20$

(e) $a = -2$, $b = 2$, $c = 20$

(f) Not enough information is given.

11. It is currently 10 degrees outside and your furnace goes out, so the temperature of your house will follow $\frac{dT}{dt} = 0.1(10 - T)$. You find an old heater which will add heat to your house at a rate of $h(t) = 3 + 2\sin 0.1t$ degrees per hour. What should you conjecture as a function to describe the temperature of your house?

(a) $T(t) = Ae^{-0.1t} + B$

(b) $T(t) = A\sin 0.1t + B$

(c) $T(t) = Ae^{-0.1t} + B\sin 0.1t + C$

(d) $T(t) = Ae^{-0.1t} + B\sin 0.1t + C\cos 0.1t + D$

(e) $T(t) = Ae^{0.1t} + Be^{-0.1t} + C\sin 0.1t + D\cos 0.1t + E$

(f) None of the above.

12. Which of the following is not a solution to $y'(t) = 5y + 3t$?

(a) $y = 8e^{5t}$

(b) $y = -\frac{3}{5}t - \frac{3}{25}$

(c) $y = 8e^{5t} - \frac{3}{5}t - \frac{3}{25}$

(d) All are solutions.

(e) More than one of (a) - (c) are not solutions.