

MathQuest: Differential Equations

Second Order Differential Equations: Oscillations

1. A branch sways back and forth with position $f(t)$. Studying its motion you find that its acceleration is proportional to its position, so that when it is 8 cm to the right, it will accelerate to the left at a rate of 2 cm/s^2 . Which differential equation describes the motion of the branch?

(a) $\frac{d^2 f}{dt^2} = 8f$

(b) $\frac{d^2 f}{dt^2} = -4f$

(c) $\frac{d^2 f}{dt^2} = -2$

(d) $\frac{d^2 f}{dt^2} = \frac{f}{4}$

(e) $\frac{d^2 f}{dt^2} = -\frac{f}{4}$

2. The differential equation $\frac{d^2 f}{dt^2} = -0.1f + 3$ is solved by a function $f(t)$ where f is in feet and t is in minutes. What units does the number 3 have?

(a) feet

(b) minutes

(c) per minute

(d) per minute²

(e) feet per minute²

(f) no units

3. The differential equation $y'' = 7y$ is solved by a function $y(t)$ where y is in meters and t is in seconds. What units does the number 7 have?

(a) meters

(b) seconds

(c) per second

(d) per second²

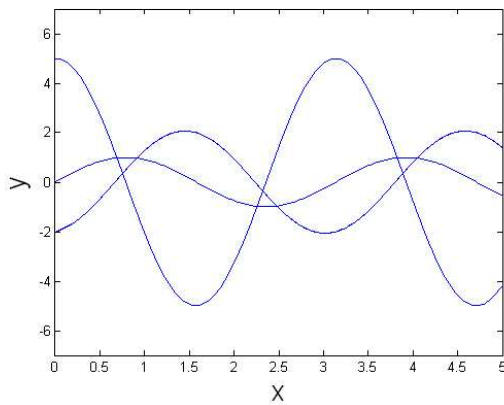
(e) meters per second²

(f) no units

4. A differential equation is solved by the function $y(t) = 3 \sin 2t$ where y is in meters and t is in seconds. What units do the numbers 3 and 2 have?

- (a) 3 is in meters, 2 is in seconds
- (b) 3 is in meters, 2 is in per second
- (c) 3 is in meters per second, 2 has no units
- (d) 3 is in meters per second, 2 is in seconds

5. Three different functions are plotted below. Could these all be solutions of the same second order differential equation?



- (a) Yes
- (b) No
- (c) Not enough information is given.

6. Which of the following is not a solution of $y'' + ay = 0$ for some value of a ?

- (a) $y = 4 \sin 2t$
- (b) $y = 8 \cos 3t$
- (c) $y = 2e^{2t}$
- (d) all are solutions

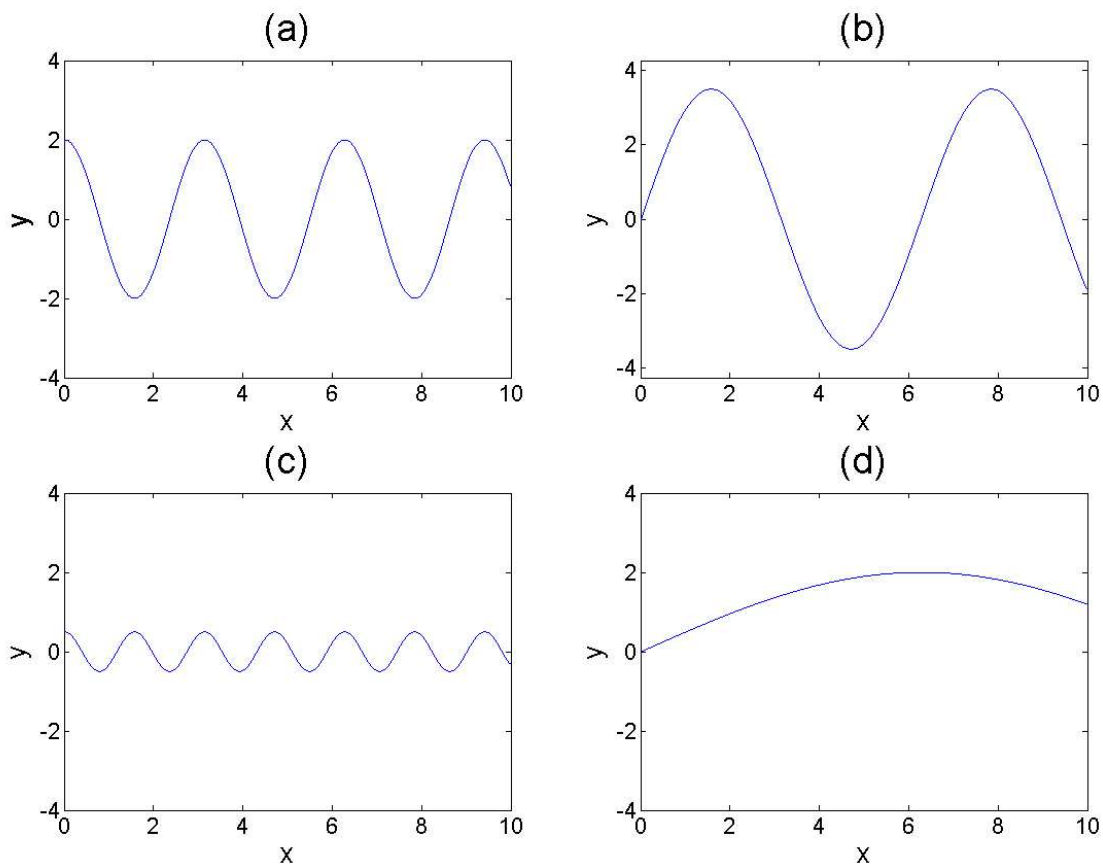
7. The functions below are solutions of $y'' + ay = 0$ for different values of a . Which represents the largest value of a ?

- (a) $y(t) = 100 \sin 2\pi t$
- (b) $y(t) = 25 \cos 6\pi t$

- (c) $y(t) = 0.1 \sin 50t$
 (d) $y(t) = 3 \sin 2t + 8 \cos 2t$
8. Each of the differential equations below represents the motion of a mass on a spring. If the mass is the same in each case, which spring is stiffer?
- (a) $s'' + 8s = 0$
 (b) $s'' + 2s = 0$
 (c) $2s'' + s = 0$
 (d) $8s'' + s = 0$
9. The motion of a mass on a spring follows the equation $mx'' = -kx$ where the displacement of the mass is given by $x(t)$. Which of the following would result in the highest frequency motion?
- (a) $k = 6, m = 2$
 (b) $k = 4, m = 4$
 (c) $k = 2, m = 6$
 (d) $k = 8, m = 6$
 (e) All frequencies are equal
10. Each of the differential equations below represents the motion of a mass on a spring. Which system has the largest maximum velocity?
- (a) $2s'' + 8s = 0, s(0) = 5, s'(0) = 0$
 (b) $2s'' + 4s = 0, s(0) = 7, s'(0) = 0$
 (c) $s'' + 4s = 0, s(0) = 10, s'(0) = 0$
 (d) $8s'' + s = 0, s(0) = 20, s'(0) = 0$
11. Which of the following is not a solution of $\frac{d^2y}{dt^2} = -ay$ for some positive value of a ?
- (a) $y = 2 \sin 6t$
 (b) $y = 4 \cos 5t$
 (c) $y = 3 \sin 2t + 8 \cos 2t$
 (d) $y = 2 \sin 3t + 2 \cos 5t$

12. Which function does not solve both $z' = 3z$ and $z'' = 9z$?
- (a) $z = 7e^{3t}$
 - (b) $z = 0$
 - (c) $z = 12e^{-3t}$
 - (d) $z = -6e^{3t}$
 - (e) all are solutions to both
13. How are the solutions of $y'' = \frac{1}{4}y$ different from solutions of $y' = \frac{1}{2}y$?
- (a) The solutions of $y'' = \frac{1}{4}y$ grow half as fast as solutions of $y' = \frac{1}{2}y$.
 - (b) The solutions of $y'' = \frac{1}{4}y$ include decaying exponentials.
 - (c) The solutions of $y'' = \frac{1}{4}y$ include sines and cosines.
 - (d) None of the above
14. How are the solutions of $y'' = -\frac{1}{4}y$ different from solutions of $y'' = -\frac{1}{2}y$?
- (a) The solutions of $y'' = -\frac{1}{4}y$ oscillate twice as fast as the solutions of $y'' = -\frac{1}{2}y$.
 - (b) The solutions of $y'' = -\frac{1}{4}y$ have a period which is twice as long as the solutions of $y'' = -\frac{1}{2}y$.
 - (c) The solutions of $y'' = -\frac{1}{4}y$ have a smaller maximum value than the solutions of $y'' = -\frac{1}{2}y$.
 - (d) More than one of the above is true.
 - (e) None of the above are true.
15. What function solves the equation $y'' + 10y = 0$?
- (a) $y = 10 \sin 10t$
 - (b) $y = 60 \cos \sqrt{10}t$
 - (c) $y = \sqrt{10}e^{-10t}$
 - (d) $y = 20e^{\sqrt{10}t}$
 - (e) More than one of the above
16. We know that the solution of a differential equation is of the form $y = A \sin 3x + B \cos 3x$. Which of the following would tell us that $A = 0$?
- (a) $y(0) = 0$

- (b) $y'(0) = 0$
(c) $y(1) = 0$
(d) none of the above
17. We know that the solutions to a differential equation are of the form $y = Ae^{3x} + Be^{-3x}$. If we know that $y = 0$ when $x = 0$, this means that
- (a) $A = 0$
(b) $B = 0$
(c) $A = -B$
(d) $A = B$
(e) none of the above
18. An ideal spring produces an acceleration that is proportional to the displacement, so $my'' = -ky$ for some positive constant k . In the lab, we find that a mass is held on an imperfect spring: As the mass gets farther from equilibrium, the spring produces a force stronger than an ideal spring. Which of the following equations could model this scenario?
- (a) $my'' = ky^2$
(b) $my'' = -k\sqrt{y}$
(c) $my'' = -k|y|$
(d) $my'' = -ky^3$
(e) $my'' = -ke^{-y}$
(f) None of the above
19. The functions plotted below are solutions of $y'' = -ay$ for different values of a . Which case corresponds to the largest value of a ?



20. The motion of a child bouncing on a trampoline is modeled by the equation $p''(t) + 3p(t) = 6$ where p is in inches and t is in seconds. Suppose we want the position function to be in feet instead of inches. How does this change the differential equation?

- (a) There is no change
- (b) $p''(t) + 3p(t) = 0.5$
- (c) $p''(t) + 3p(t) = 72$
- (d) $144p''(t) + 3p(t) = 3$
- (e) $p''(t) + 36p(t) = 3$
- (f) $144p''(t) + 36p(t) = 3$

21. A float is bobbing up and down on a lake, and the distance of the float from the lake floor follows the equation $2d'' + 5d - 30 = 0$, where $d(t)$ is in feet and t is in seconds. At what distance from the lake floor could the float reach equilibrium?

- (a) 2 feet
- (b) 5 feet
- (c) 30 feet

- (d) 6 feet
- (e) 15 feet
- (f) No equilibrium exists.