MathQuest: Differential Equations

Second Order Differential Equations: Oscillations

1. A branch sways back and forth with position $f(t)$. Studying its motion you find that its acceleration is proportional to its position, so that when it is 8 cm to the right, it will accelerate to the left at a rate of 2 cm/s^2. Which differential equation describes the motion of the branch?

   (a) $\frac{d^2f}{dt^2} = 8f$
   (b) $\frac{d^2f}{dt^2} = -4f$
   (c) $\frac{d^2f}{dt^2} = -2$
   (d) $\frac{d^2f}{dt^2} = \frac{f}{4}$
   (e) $\frac{d^2f}{dt^2} = -\frac{f}{4}$

2. The differential equation $\frac{d^2f}{dt^2} = -0.1f + 3$ is solved by a function $f(t)$ where f is in feet and t is in minutes. What units does the number 3 have?

   (a) feet
   (b) minutes
   (c) per minute
   (d) per minute^2
   (e) feet per minute^2
   (f) no units

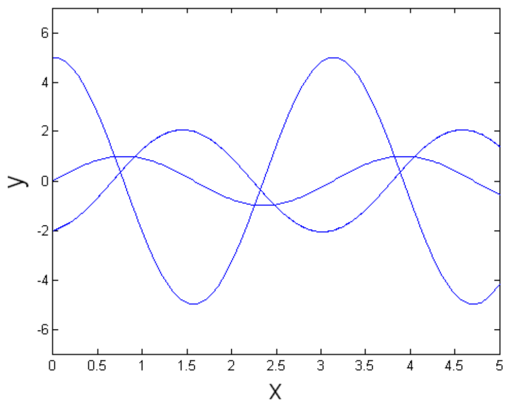
3. The differential equation $y'' = 7y$ is solved by a function $y(t)$ where y is in meters and t is in seconds. What units does the number 7 have?

   (a) meters
   (b) seconds
   (c) per second
   (d) per second^2
   (e) meters per second^2
   (f) no units

4. A differential equation is solved by the function $y(t) = 3\sin 2t$ where y is in meters and t is in seconds. What units do the numbers 3 and 2 have?

   (a) 3 is in meters, 2 is in seconds
   (b) 3 is in meters, 2 is in per second
   (c) 3 is in meters per second, 2 has no units
   (d) 3 is in meters per second, 2 is in seconds

5. Three different functions are plotted below. Could these all be solutions of the same second order differential equation?



   (a) Yes
   (b) No
   (c) Not enough information is given.

6. Which of the following is not a solution of $y'' + ay = 0$ for some value of a?

   (a) $y = 4\sin 2t$
   (b) $y = 8\cos 3t$
   (c) $y = 2e^{2t}$
   (d) all are solutions

7. The functions below are solutions of $y'' + ay = 0$ for different values of a. Which represents the largest value of a?

   (a) $y(t) = 100\sin 2\pi t$
   (b) $y(t) = 25\cos 6\pi t$

(c) $y(t) = 0.1 \sin 50t$

(d) $y(t) = 3 \sin 2t + 8 \cos 2t$

8. Each of the differential equations below represents the motion of a mass on a spring. If the mass is the same in each case, which spring is stiffer?

(a) $s'' + 8s = 0$

(b) $s'' + 2s = 0$

(c) $2s'' + s = 0$

(d) $8s'' + s = 0$

9. The motion of a mass on a spring follows the equation $mx'' = -kx$ where the displacement of the mass is given by $x(t)$. Which of the following would result in the highest frequency motion?

(a) $k = 6$, $m = 2$

(b) $k = 4$, $m = 4$

(c) $k = 2$, $m = 6$

(d) $k = 8$, $m = 6$

(e) All frequencies are equal

10. Each of the differential equations below represents the motion of a mass on a spring. Which system has the largest maximum velocity?

(a) $2s'' + 8s = 0, s(0) = 5, s'(0) = 0$

(b) $2s'' + 4s = 0, s(0) = 7, s'(0) = 0$

(c) $s'' + 4s = 0, s(0) = 10, s'(0) = 0$

(d) $8s'' + s = 0, s(0) = 20, s'(0) = 0$

11. Which of the following is not a solution of $\frac{d^2y}{dt^2} = -ay$ for some positive value of a?

(a) $y = 2 \sin 6t$

(b) $y = 4 \cos 5t$

(c) $y = 3 \sin 2t + 8 \cos 2t$

(d) $y = 2 \sin 3t + 2 \cos 5t$

12. Which function does not solve both $z' = 3z$ and $z'' = 9z$?

(a) $z = 7e^{3t}$
(b) $z = 0$
(c) $z = 12e^{-3t}$
(d) $z = -6e^{3t}$
(e) all are solutions to both

13. How are the solutions of $y'' = \frac{1}{4}y$ different from solutions of $y' = \frac{1}{2}y$?

(a) The solutions of $y'' = \frac{1}{4}y$ grow half as fast as solutions of $y' = \frac{1}{2}y$.
(b) The solutions of $y'' = \frac{1}{4}y$ include decaying exponentials.
(c) The solutions of $y'' = \frac{1}{4}y$ include sines and cosines.
(d) None of the above

14. How are the solutions of $y'' = -\frac{1}{4}y$ different from solutions of $y'' = -\frac{1}{2}y$?

(a) The solutions of $y'' = -\frac{1}{4}y$ oscillate twice as fast as the solutions of $y'' = -\frac{1}{2}y$.
(b) The solutions of $y'' = -\frac{1}{4}y$ have a period which is twice as long as the solutions of $y'' = -\frac{1}{2}y$.
(c) The solutions of $y'' = -\frac{1}{4}y$ have a smaller maximum value than the solutions of $y'' = -\frac{1}{2}y$.
(d) More than one of the above is true.
(e) None of the above are true.

15. What function solves the equation $y'' + 10y = 0$?

(a) $y = 10 \sin 10t$
(b) $y = 60 \cos \sqrt{10}t$
(c) $y = \sqrt{10}e^{-10t}$
(d) $y = 20e^{\sqrt{10}t}$
(e) More than one of the above

16. We know that the solution of a differential equation is of the form $y = A \sin 3x + B \cos 3x$. Which of the following would tell us that $A = 0$?

(a) $y(0) = 0$

(b) $y'(0) = 0$

(c) $y(1) = 0$

(d) none of the above

17. We know that the solutions to a differential equation are of the form $y = Ae^{3x} + Be^{-3x}$. If we know that $y = 0$ when $x = 0$, this means that

    (a) $A = 0$

    (b) $B = 0$

    (c) $A = -B$

    (d) $A = B$

    (e) none of the above

18. An ideal spring produces an acceleration that is proportional to the displacement, so $my'' = -ky$ for some positive constant k. In the lab, we find that a mass is held on an imperfect spring: As the mass gets farther from equilibrium, the spring produces a force stronger than an ideal spring. Which of the following equations could model this scenario?

    (a) $my'' = ky^2$

    (b) $my'' = -k\sqrt{y}$

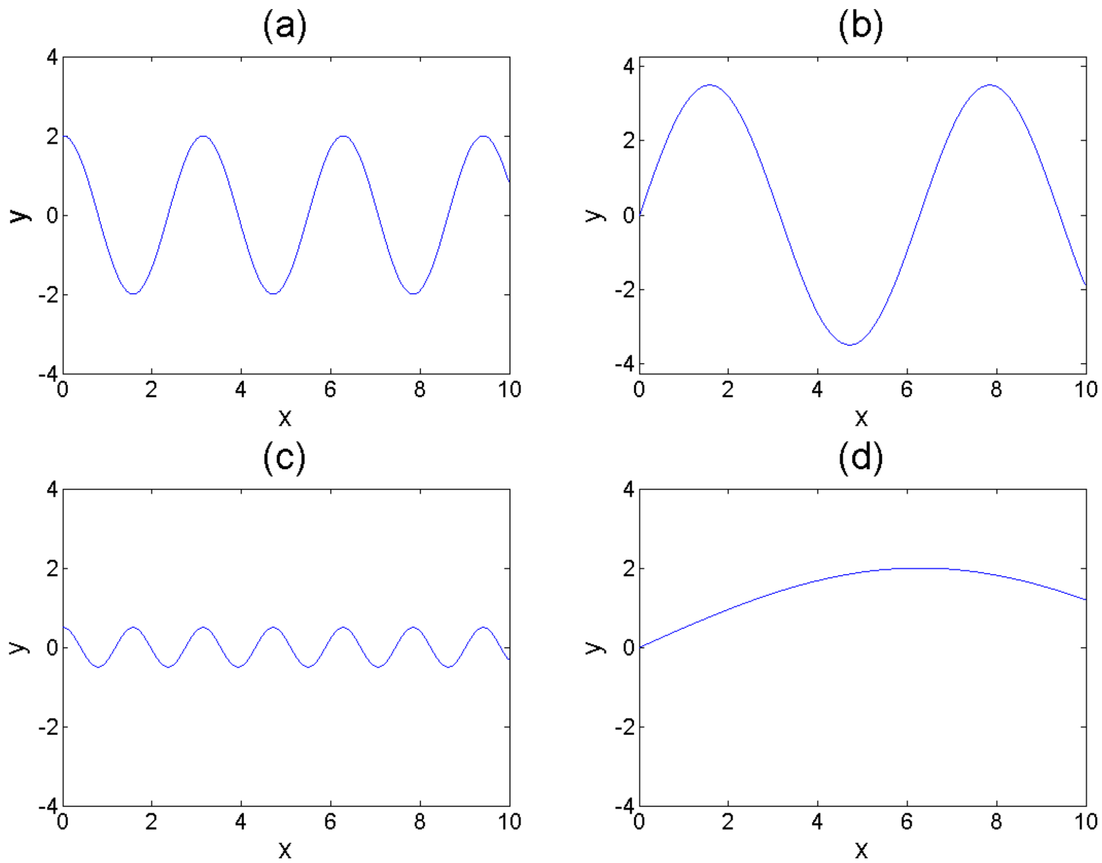
    (c) $my'' = -k|y|$

    (d) $my'' = -ky^3$

    (e) $my'' = -ke^{-y}$

    (f) None of the above

19. The functions plotted below are solutions of $y'' = -ay$ for different positive values of a. Which case corresponds to the largest value of a?



20. The motion of a child bouncing on a trampoline is modeled by the equation $p''(t) + 3p(t) = 6$ where p is in inches and t is in seconds. Suppose we want the position function to be in feet instead of inches. How does this change the differential equation?

    (a) There is no change
    (b) $p''(t) + 3p(t) = 0.5$
    (c) $p''(t) + 3p(t) = 72$
    (d) $144p''(t) + 3p(t) = 3$
    (e) $p''(t) + 36p(t) = 3$
    (f) $144p''(t) + 36p(t) = 3$

21. A float is bobbing up and down on a lake, and the distance of the float from the lake floor follows the equation $2d'' + 5d - 30 = 0$, where $d(t)$ is in feet and t is in seconds. At what distance from the lake floor could the float reach equilibrium?

    (a) 2 feet
    (b) 5 feet
    (c) 30 feet

(d) 6 feet

(e) 15 feet

(f) No equilibrium exists.