

# MathQuest: Differential Equations

## Mixing Models

1. The differential equation for a mixing problem is  $x' + 0.08x = 4$ , where  $x$  is the amount of dissolved substance, in pounds, and time is measured in minutes. What are the units of 4?
  - (a) pounds
  - (b) minutes
  - (c) pounds/minute
  - (d) minutes/pound
  - (e) None of the above
  
2. The differential equation for a mixing problem is  $x' + 0.08x = 4$ , where  $x$  is the amount of dissolved substance, in pounds, and time is measured in minutes. What are the units of 0.08?
  - (a) pounds
  - (b) minutes
  - (c) per pound
  - (d) per minute
  - (e) None of the above
  
3. The differential equation for a mixing problem is  $x' + 0.08x = 4$ , where  $x$  is the amount of dissolved substance, in pounds, and time is measured in minutes. What is the equilibrium value for this model?
  - (a) 0.08
  - (b) 50
  - (c) 0
  - (d) 0.02
  - (e) 4
  - (f) None of the above

4. In a mixing model where  $x'$  has units of pounds per minute, the equilibrium value is 80. Which of the following is a correct interpretation of the equilibrium?
- (a) In the long-run, there will be 80 pounds of contaminant in the system.
  - (b) After 80 minutes, the mixture will stabilize.
  - (c) The rate in is equal to the rate out when the concentration of contaminant is 80 pounds per gallon.
  - (d) The rate in is equal to the rate out when the amount of contaminant is 80 pounds.
  - (e) None of the above
5. A tank initially contains 60 gallons of pure water. A solution containing 3 pounds/gallon of salt is pumped into the tank at a rate of 2 gallons/minute. The mixture is stirred constantly and flows out at a rate of 2 gallons per minute. If  $x(t)$  is the amount of salt in the tank at time  $t$ , which initial value problem represents this scenario?
- (a)  $x'(t) = 2 - 2$ , with  $x(0) = 60$
  - (b)  $x'(t) = 6 - \frac{x}{30}$ , with  $x(0) = 0$
  - (c)  $x'(t) = 3x - 2$ , with  $x(0) = 60$
  - (d)  $x'(t) = 6 - \frac{x}{60}$ , with  $x(0) = 0$
  - (e) None of the above
6. The solution to a mixing problem is

$$x(t) = -0.01(100 - t)^2 + (100 - t),$$

where  $x(t)$  is the amount of a contaminant in a tank of water. What is the long-term behavior of this solution?

- (a) The amount of contaminant will reach a steady-state of 100 pounds.
  - (b) The amount of contaminant will increase forever.
  - (c) The amount of contaminant will approach zero.
  - (d) The tank will run out of water.
7. Tank  $A$  initially contains 30 gallons of pure water, and tank  $B$  initially contains 40 gallons of pure water. A solution containing 2 pounds/gallon of salt is pumped into tank  $A$  at a rate of 1.5 gallons/minute. The mixture in tank  $A$  is stirred constantly and flows into tank  $B$  at a rate of 1.5 gallons/minute. The mixture in tank  $B$  is also stirred constantly, and tank  $B$  drains at a rate of 1.5 gallons/minute. If  $A(t)$  is the amount of salt in the tank  $A$  at time  $t$  and  $B(t)$  is the amount of salt in tank  $B$  at time  $t$ , which initial value problem represents this scenario?

(a)

$$\begin{aligned}A'(t) &= 1.5 - \frac{A}{30} & A(0) &= 0 \\B'(t) &= 1.5 - \frac{B}{40} & B(0) &= 0\end{aligned}$$

(b)

$$\begin{aligned}A'(t) &= 1.5 - \frac{A}{30} & A(0) &= 0 \\B'(t) &= \frac{A}{30} - \frac{B}{40} & B(0) &= 0\end{aligned}$$

(c)

$$\begin{aligned}A'(t) &= 3 - \frac{A}{30} & A(0) &= 0 \\B'(t) &= \frac{A}{30} - \frac{B}{40} & B(0) &= 0\end{aligned}$$

(d)

$$\begin{aligned}A'(t) &= 3 - \frac{A}{20} & A(0) &= 0 \\B'(t) &= \frac{A}{20} - \frac{3B}{80} & B(0) &= 0\end{aligned}$$

(e) None of the above

8. Medication flows from the GI tract into the bloodstream. Suppose that  $A$  units of an antihistamine are present in the GI tract at time 0 and that the medication moves from the GI tract into the blood at a rate proportional to the amount in the GI tract,  $x$ . Assume that no further medication enters the GI tract, and that the kidneys and liver clear the medication from the blood at a rate proportional to the amount currently in the blood,  $y$ . If  $k_1$  and  $k_2$  are positive constants, which initial value problem models this scenario?

(a)

$$\begin{aligned}\frac{dx}{dt} &= k_1x - k_2y & x(0) &= A \\ \frac{dy}{dt} &= k_2y & y(0) &= 0\end{aligned}$$

(b)

$$\begin{aligned}\frac{dx}{dt} &= k_1x - k_2y & x(0) &= 0 \\ \frac{dy}{dt} &= k_2y & y(0) &= 0\end{aligned}$$

(c)

$$\begin{aligned}\frac{dx}{dt} &= -k_1x & x(0) &= A \\ \frac{dy}{dt} &= k_1x - k_2y & y(0) &= 0\end{aligned}$$

(d)

$$\begin{aligned}\frac{dx}{dt} &= k_1x - k_2y & x(0) &= 0 \\ \frac{dy}{dt} &= k_1x + k_2y & y(0) &= 0\end{aligned}$$

9. Referring to the antihistamine model developed in the previous question, if time is measured in hours and the quantity of antihistamine is measured in milligrams, what are the units of  $k_1$ ?

- (a) hours
- (b) hours per milligram
- (c) milligrams per hour
- (d) 1/hours
- (e) milligrams
- (f) None of the above

10. Referring to the antihistamine model discussed in the previous questions, what is the effect of increasing  $k_2$ ?

- (a) The blood will be cleaned faster.
- (b) Antihistamine will be removed from the GI tract at a faster rate.
- (c) It will take longer for the antihistamine to move from the GI tract into the blood.
- (d) Antihistamine will accumulate in the blood at a faster rate and the patient will end up with an overdose.
- (e) None of the above

11. Referring to the antihistamine model discussed in the previous questions, describe how the amount of antihistamine in the blood changes with time.

- (a) It decreases and asymptotically approaches zero.
- (b) It levels off at some nonzero value.
- (c) It increases indefinitely.
- (d) It increases, reaches a peak, and then decreases, asymptotically approaching zero.
- (e) None of the above