MathQuest: Differential Equations

Mixing Models

1. The differential equation for a mixing problem is $x' + 0.08x = 4$, where x is the amount of dissolved substance, in pounds, and time is measured in minutes. What are the units of 4?

   (a) pounds
   (b) minutes
   (c) pounds/minute
   (d) minutes/pound
   (e) None of the above

2. The differential equation for a mixing problem is $x' + 0.08x = 4$, where x is the amount of dissolved substance, in pounds, and time is measured in minutes. What are the units of 0.08?

   (a) pounds
   (b) minutes
   (c) per pound
   (d) per minute
   (e) None of the above

3. The differential equation for a mixing problem is $x' + 0.08x = 4$, where x is the amount of dissolved substance, in pounds, and time is measured in minutes. What is the equilibrium value for this model?

   (a) 0.08
   (b) 50
   (c) 0
   (d) 0.02
   (e) 4
   (f) None of the above

4. In a mixing model where x' has units of pounds per minute, the equilibirum value is 80. Which of the following is a correct interpretation of the equilibrium?

   (a) In the long-run, there will be 80 pounds of contaminant in the system.
   (b) After 80 minutes, the mixture will stabilize.
   (c) The rate in is equal to the rate out when the concentration of contaminant is 80 pounds per gallon.
   (d) The rate in is equal to the rate out when the amount of contaminant is 80 pounds.
   (e) None of the above

5. A tank initially contains 60 gallons of pure water. A solution containing 3 pounds/gallon of salt is pumped into the tank at a rate of 2 gallons/minute. The mixture is stirred constantly and flows out at a rate of 2 gallons per minute. If $x(t)$ is the amount of salt in the tank at time t, which initial value problem represents this scenario?

   (a) $x'(t) = 2 - 2$, with $x(0) = 60$
   (b) $x'(t) = 6 - \frac{x}{30}$, with $x(0) = 0$
   (c) $x'(t) = 3x - 2$, with $x(0) = 60$
   (d) $x'(t) = 6 - \frac{x}{60}$, with $x(0) = 0$
   (e) None of the above

6. The solution to a mixing problem is

$$x(t) = -0.01(100 - t)^2 + (100 - t),$$

   where $x(t)$ is the amount of a contaminant in a tank of water. What is the long-term behavior of this solution?

   (a) The amount of contaminant will reach a steady-state of 100 pounds.
   (b) The amount of contaminant will increase forever.
   (c) The amount of contaminant will approach zero.
   (d) The tank will run out of water.

7. Tank A initially contains 30 gallons of pure water, and tank B initially contains 40 gallons of pure water. A solution containing 2 pounds/gallon of salt is pumped into tank A at a rate of 1.5 gallons/minute. The mixture in tank A is stirred constantly and flows into tank B at a rate of 1.5 gallons/minute. The mixture in tank B is also stirred constantly, and tank B drains at a rate of 1.5 gallons/minute. If $A(t)$ is the amount of salt in the tank A at time t and $B(t)$ is the amount of salt in tank B at time t, which initial value problem represents this scenario?

(a)

$$\begin{aligned} A'(t) &= 1.5 - \frac{A}{30} \qquad A(0) = 0 \\ B'(t) &= 1.5 - \frac{B}{40} \qquad B(0) = 0 \end{aligned}$$

(b)

$$\begin{aligned} A'(t) &= 1.5 - \frac{A}{30} \qquad A(0) = 0 \\ B'(t) &= \frac{A}{30} - \frac{B}{40} \qquad B(0) = 0 \end{aligned}$$

(c)

$$\begin{aligned} A'(t) &= 3 - \frac{A}{30} \qquad A(0) = 0 \\ B'(t) &= \frac{A}{30} - \frac{B}{40} \qquad B(0) = 0 \end{aligned}$$

(d)

$$\begin{aligned} A'(t) &= 3 - \frac{A}{20} \qquad A(0) = 0 \\ B'(t) &= \frac{A}{20} - \frac{3B}{80} \qquad B(0) = 0 \end{aligned}$$

(e) None of the above

8. Medication flows from the GI tract into the bloodstream. Suppose that A units of an antihistamine are present in the GI tract at time 0 and that the medication moves from the GI tract into the blood at a rate proportional to the amount in the GI tract, x. Assume that no further medication enters the GI tract, and that the kidneys and liver clear the medication from the blood at a rate proportional to the amount currently in the blood, y. If k_1 and k_2 are positive constants, which initial value problem models this scenario?

(a)

$$\begin{aligned} \frac{dx}{dt} &= k_1 x - k_2 y \qquad x(0) = A \\ \frac{dy}{dt} &= k_2 y \qquad y(0) = 0 \end{aligned}$$

(b)

$$\begin{aligned} \frac{dx}{dt} &= k_1 x - k_2 y \qquad x(0) = 0 \\ \frac{dy}{dt} &= k_2 y \qquad y(0) = 0 \end{aligned}$$

(c)

$$\begin{aligned} \frac{dx}{dt} &= -k_1 x \qquad x(0) = A \\ \frac{dy}{dt} &= k_1 x - k_2 y \qquad y(0) = 0 \end{aligned}$$

(d)

$$\begin{aligned} \frac{dx}{dt} &= k_1 x - k_2 y \qquad x(0) = 0 \\ \frac{dy}{dt} &= k_1 x + k_2 y \qquad y(0) = 0 \end{aligned}$$

9. Referring to the antihistamine model developed in the previous question, if time is measured in hours and the quantity of antihistamine is measured in milligrams, what are the units of k_1?

   (a) hours
   (b) hours per milligram
   (c) milligrams per hour
   (d) 1/hours
   (e) milligrams
   (f) None of the above

10. Referring to the antihistamine model discussed in the previous questions, what is the effect of increasing k_2?

    (a) The blood will be cleaned faster.
    (b) Antihistamine will be removed from the GI tract at a faster rate.
    (c) It will take longer for the antihistamine to move from the GI tract into the blood.
    (d) Antihistamine will accumulate in the blood at a faster rate and the patient will end up with an overdose.
    (e) None of the above

11. Referring to the antihistamine model discussed in the previous questions, describe how the amount of antihistamine in the blood changes with time.

    (a) It decreases and asymptotically approaches zero.
    (b) It levels off at some nonzero value.
    (c) It increases indefinitely.
    (d) It increases, reaches a peak, and then decreases, asymptotically approaching zero.
    (e) None of the above