

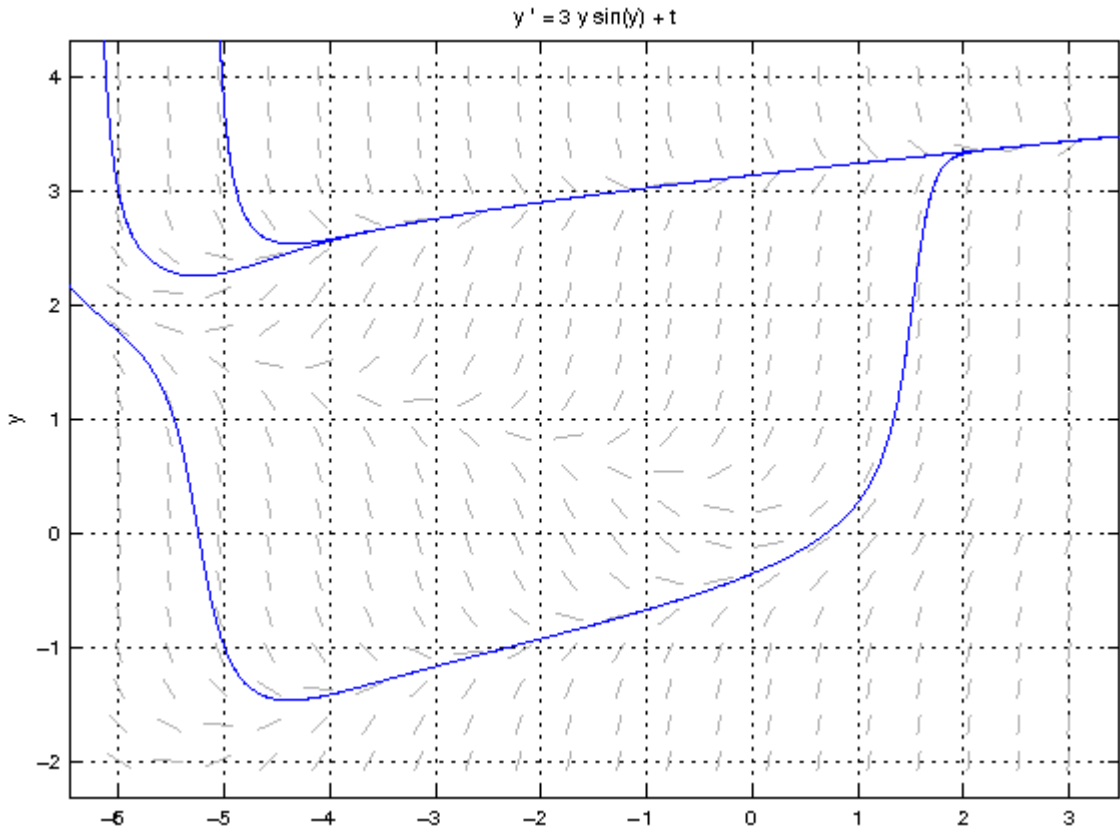
# MathQuest: Differential Equations

## Existence & Uniqueness

1. Based upon observations, Kate developed the differential equation  $\frac{dT}{dt} = -0.08(T - 72)$  to predict the temperature in her vanilla chai tea. In the equation,  $T$  represents the temperature of the chai in  $^{\circ}\text{F}$  and  $t$  is time. Kate has a cup of chai whose initial temperature is  $110^{\circ}\text{F}$  and her friend Nate has a cup of chai whose initial temperature is  $120^{\circ}\text{F}$ . According to Kate's model, will there be a point in time when the two cups of chai have exactly the same temperature?
  - (a) Yes
  - (b) No
  - (c) Can't tell with the information given
2. A bucket of water has a hole in the bottom, and so the water is slowly leaking out. The height of the water in the bucket is thus a decreasing function of time  $h(t)$  which changes according to the differential equation  $h' = -kh^{1/2}$ , where  $k$  is a positive constant that depends on the size of the hole and the bucket. If we start out a bucket with 25 cm of water in it, then according to this model, will the bucket ever be empty?
  - (a) Yes
  - (b) No
  - (c) Can't tell with the information given
3. A scientist develops the logistic population model  $P' = 0.2P(1 - \frac{P}{8.2})$  to describe her research data. This model has an equilibrium value of  $P = 8.2$ . In this model, from the initial condition  $P_0 = 4$ , the population never reaches the equilibrium value because:
  - (a) you can't have a population of 8.2.
  - (b) the population is asymptotically approaching a value of 8.2.
  - (c) the population will grow towards infinity.
  - (d) the population will drop to 0.
4. A scientist develops the logistic population model  $P' = 0.2P(1 - \frac{P}{8})$  to describe her research data. This model has an equilibrium value of  $P = 8$ . In this model, from the initial condition  $P_0 = 4$ , the population will never reach the equilibrium value because:

- (a) the population will grow toward infinity.
- (b) the population is asymptotically approaching a value of 8.
- (c) the population will drop to 0.

5. Do the solution trajectories shown below for the differential equation  $y' = 3y \sin(y) + t$  ever simultaneously reach the same value?



- (a) Yes
- (b) No
- (c) Can't tell with the information given