

MathQuest: Differential Equations

Bifurcations

1. How many equilibria does the differential equation $y' = y^2 + a$ have?
 - (a) Zero
 - (b) One
 - (c) Two
 - (d) Three
 - (e) Not enough information is given.

2. A *bifurcation* occurs if the number of equilibria of a system changes when we change the value of a parameter. For the differential equation $\frac{df}{dx} = bf^2 - 2$, a bifurcation occurs at what value of b ?
 - (a) $b = 0$
 - (b) $b = 2$
 - (c) $b = -2$
 - (d) $b = 2/f^2$
 - (e) Not enough information is given.

3. $x'(t) = \frac{1}{2}x^2 + bx + 8$. If $b = 5$ what are the equilibria of the system?
 - (a) $x = -5$
 - (b) $x = -3, 3$
 - (c) $x = -8, -2$
 - (d) No equilibria exist and all solutions are increasing.
 - (e) No equilibria exist and all solutions are decreasing.

4. $x'(t) = \frac{1}{2}x^2 + bx + 8$. If $b = 2$ what are the equilibria of the system?
 - (a) $x = -8$
 - (b) $x = -2 \pm \sqrt{12}$
 - (c) $x = 2$

- (d) No equilibria exist and all solutions are increasing.
- (e) No equilibria exist and all solutions are decreasing.

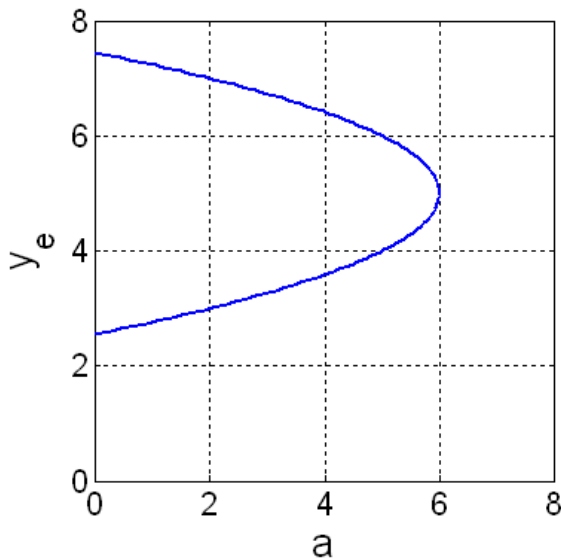
5. $x'(t) = \frac{1}{2}x^2 + bx + 8$. A bifurcation occurs where?

- (a) $x = -b \pm \sqrt{b^2 - 16}$
- (b) $b = 0$
- (c) $b = \frac{1}{2}$
- (d) $b = 4$
- (e) $b = 8$
- (f) Not enough information is given.

6. $\frac{dg}{dt} = g^3 + cg$. How many equilibria does this system have?

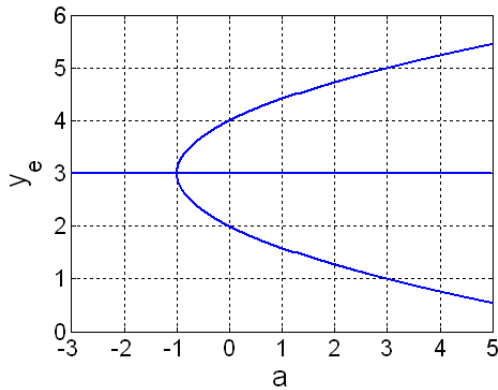
- (a) Two
- (b) One or two
- (c) One or three
- (d) Two or three
- (e) Three
- (f) Not enough information is given

7. A *bifurcation diagram* plots a system's equilibria on the y axis and the value of a parameter on the x axis. Consider the bifurcation diagram below. When our parameter is $a = 5$, what are the equilibria of the system?



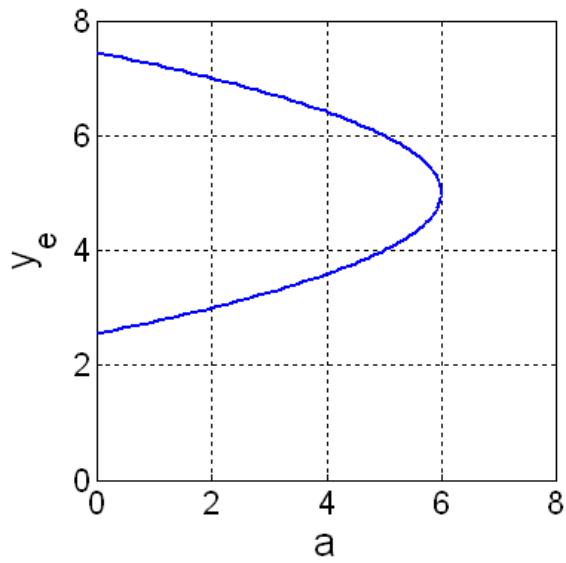
- (a) $y = 0$ and $y = 3$
- (b) $y = 5$
- (c) $y = 4$ and $y = 6$
- (d) Not enough information is given.

8. Consider the bifurcation diagram below. If our system has equilibria at $y = 1$, $y = 3$ and $y = 5$ what is the value of the parameter a ?



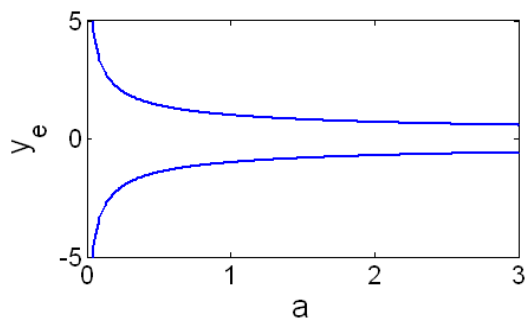
- (a) $a = -1$
- (b) $a = 0$
- (c) $a = 1$
- (d) $a = 3$
- (e) $a = 5$
- (f) Not enough information is given.

9. Consider the bifurcation diagram below. At what value of a does the system have a bifurcation?



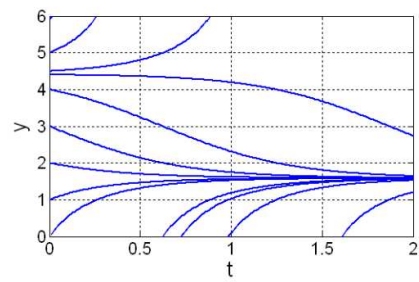
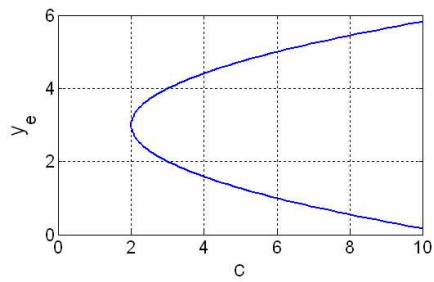
- (a) $a = 0$
- (b) $a = 2$
- (c) $a = 4$
- (d) $a = 6$
- (e) $a = 8$
- (f) Not enough information is given.

10. Which of the following differential equations is represented by the bifurcation diagram below?



- (a) $y' = y^2 + a$
- (b) $y' = ay^2 - 1$
- (c) $y' = ay$
- (d) $y' = y^2 + ay + 2$

11. The figure on the left is a bifurcation diagram, and the figure on the right plots several solution functions of this system for one specific value of the parameter. The figure on the right corresponds to what value of the parameter?



- (a) $c = 0$
- (b) $c = 2$
- (c) $c = 4$
- (d) $c = 6$
- (e) $c = 8$