

MathQuest: Differential Equations

Second Order Differential Equations: Damping

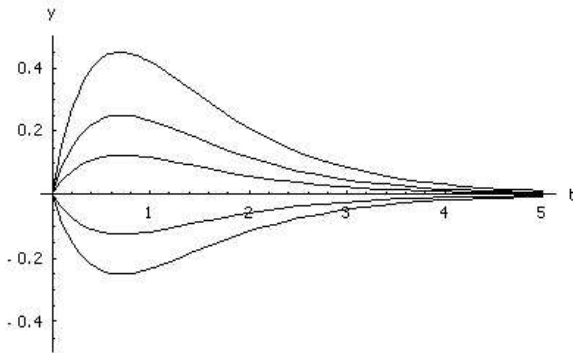
- Which of the following equations is not equivalent to $y'' + 3y' + 2y = 0$?
 - $2y'' + 6y' + 4y = 0$
 - $y'' = 3y' + 2y$
 - $-12y'' = 36y' + 24y$
 - $3y'' = -9y' - 6y$
 - All are equivalent.
- Which of the following equations is equivalent to $y'' + \frac{2}{t}y' + \frac{3}{t^2}y = 0$?
 - $t^2y'' + 2ty' + 3y = 0$
 - $y'' + 2y' + 3y = 0$
 - $t^2y'' + \frac{t}{2}y' + \frac{1}{3}y = 0$
 - None are equivalent.
- The motion of a child bouncing on a trampoline is modeled by the equation $p''(t) + 3p'(t) + 8p(t) = 0$ where p is in feet and t is in seconds. What will the child's acceleration be if he is 3 feet below equilibrium and moving up at 6 ft/s?
 - 6 ft/s² up
 - 6 ft/s² down
 - 42 ft/s² up
 - 42 ft/s² down
 - 39 ft/s² down
 - None of the above
- The motion of a child on a trampoline is modeled by the equation $p''(t) + 2p'(t) + 3p(t) = 0$ where p is in feet and t is in seconds. Suppose we want the position function to be in inches instead of feet. How does this change the differential equation?
 - There is no change
 - $p''(t) + 24p'(t) + 36p(t) = 0$

- (c) $12p''(t) + 2p'(t) + 36p(t) = 0$
- (d) $144p''(t) + 24p'(t) + 3p(t) = 0$
- (e) None of the above
5. A hydrogen atom is bound to a large molecule, and its distance from the molecule follows the equation $d'' + 4d' + 8d - 6 = 0$ where d is in picometers. At what distance from the molecule will the atom reach equilibrium?
- (a) $d = 6$ pm.
- (b) $d = \frac{3}{4}$ pm.
- (c) $d = \frac{6}{13}$ pm
- (d) No equilibrium exists.
6. When the space shuttle re-enters the Earth's atmosphere, the air resistance produces a force proportional to its velocity squared, and gravity produces an approximately constant force. Which of the following equations might model its position $p(t)$ if a and b are positive constants?
- (a) $p'' + a(p')^2 + b = 0$
- (b) $p'' - a(p')^2 + b = 0$
- (c) $p'' + a(p')^2 + bp = 0$
- (d) $p'' - a(p')^2 + bp = 0$
- (e) None of the above
7. The differential equation $m\frac{d^2y}{dt^2} + \gamma\frac{dy}{dt} + ky = 0$ with positive parameters m , γ , and k is often used to model the motion of a mass on a spring with a damping force. If γ was negative, what would this mean?
- (a) This would be like a negative friction, making the oscillations speed up over time.
- (b) This would be like a negative spring, that would push the object farther and farther from equilibrium.
- (c) This would be like a negative mass, so that the object would accelerate in the opposite direction that the forces were pushing.
- (d) None of the above
8. Test the following functions to see which is a solution to $y'' + 4y' + 3y = 0$.
- (a) $y = e^{2t}$

- (b) $y = e^t$
 (c) $y = e^{-t}$
 (d) $y = e^{-2t}$
 (e) None of these are solutions.
 (f) All are solutions.
9. Test the following functions to see which is a solution to $\frac{d^2g}{dx^2} + 2\frac{dg}{dx} + 2g = 0$.
- (a) $g = e^x$
 (b) $g = \sin x$
 (c) $g = e^{-x} \sin x$
 (d) None of these are solutions.
10. Suppose we want to solve the differential equation $y'' + by' + cy = 0$ and we conjecture that our solution is of the form $y = Ce^{rt}$. What equation do we get if we test this solution and simplify the result?
- (a) $1 + br + cr^2 = 0$
 (b) $C^2r^2 + Cr + c = 0$
 (c) $Ce^{rt} + bCe^{rt} + cCe^{rt} = 0$
 (d) $r^2 + br + c = 0$
 (e) None of the above
11. Suppose we want to solve the differential equation $\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 4y = 0$ and we conjecture that our solution is of the form $y = Ce^{rt}$. Solve the characteristic equation to determine what values of r satisfy the differential equation.
- (a) $r = -2, -8$
 (b) $r = -1, -4$
 (c) $r = -3/2, +3/2$
 (d) $r = 1, 4$
 (e) None of the above
12. Find the general solution to $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = 0$.
- (a) $y(t) = C_1e^{-t/2} + C_2e^{t/2}$

- (b) $y(t) = C_1e^{-2t} + C_2e^{-t}$
- (c) $y(t) = C_1e^{-2t} + C_2e^t$
- (d) $y(t) = -2C_1e^{-2t} - C_2e^{-t}$
- (e) None of the above

13. The graph below has five trajectories, call the top one y_1 , the one below it y_2 , down to the lowest one y_5 . Which of these could be a solution of $y'' + 3y' + 2y = 0$ with $y(0) = 0$ and $y'(0) = 1$?



- (a) y_1
 - (b) y_2
 - (c) y_3
 - (d) y_4
 - (e) y_5
14. What is the general solution to $f'' + 2f' + 2f = 0$?
- (a) $f(x) = C_1e^{-x/2} \cos x + C_2e^{-x/2} \sin x$
 - (b) $f(x) = C_1e^{-x} \cos x + C_2e^{-x} \sin x$
 - (c) $f(x) = C_1e^{-x} \cos \frac{x}{2} + C_2e^{-x} \sin \frac{x}{2}$
 - (d) $f(x) = C_1 + C_2e^{-2x}$
 - (e) None of the above
15. The harmonic oscillator modeled by $mx'' + bx' + kx = 0$ with parameters $m = 1$, $k = 2$, and $b = 1$ is underdamped and thus oscillates. What is the period of the oscillations?

- (a) $2\pi/\sqrt{7}$
- (b) $4\pi/\sqrt{7}$

- (c) $\sqrt{7}/2\pi$
(d) $\sqrt{7}/4\pi$
(e) None of the above.
16. A harmonic oscillator is modeled by $mx'' + bx' + kx = 0$. If we increase the parameter m slightly, what happens to the period of oscillation?
- (a) The period gets larger.
(b) The period gets smaller.
(c) The period stays the same.
17. A harmonic oscillator is modeled by $mx'' + bx' + kx = 0$. If we increase the parameter k slightly, what happens to the period of oscillation?
- (a) The period gets larger.
(b) The period gets smaller.
(c) The period stays the same.
18. A harmonic oscillator is modeled by $mx'' + bx' + kx = 0$. If we increase the parameter b slightly, what happens to the period of oscillation?
- (a) The period gets larger.
(b) The period gets smaller.
(c) The period stays the same.

19. Classify the harmonic oscillator described below:

$$3\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + y = 0.$$

- (a) underdamped
(b) overdamped
(c) critically damped
(d) no damping
20. Does the harmonic oscillator described below oscillate?

$$3\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + y = 0.$$

- (a) Yes.
(b) No.