

## MathQuest: Differential Equations

### Second Order Differential Equations: Damping

- Which of the following equations is not equivalent to  $y'' + 3y' + 2y = 0$ ?
  - $2y'' + 6y' + 4y = 0$
  - $y'' = 3y' + 2y$
  - $-12y'' = 36y' + 24y$
  - $3y'' = -9y' - 6y$
  - All are equivalent.
- Which of the following equations is equivalent to  $y'' + \frac{2}{t}y' + \frac{3}{t^2}y = 0$ ?
  - $t^2y'' + 2ty' + 3y = 0$
  - $y'' + 2y' + 3y = 0$
  - $t^2y'' + \frac{t}{2}y' + \frac{1}{3}y = 0$
  - None are equivalent.
- The motion of a child bouncing on a trampoline is modeled by the equation  $p''(t) + 3p'(t) + 8p(t) = 0$  where  $p$  is in feet and  $t$  is in seconds. What will the child's acceleration be if he is 3 feet below equilibrium and moving up at 6 ft/s?
  - 6 ft/s<sup>2</sup> up
  - 6 ft/s<sup>2</sup> down
  - 42 ft/s<sup>2</sup> up
  - 42 ft/s<sup>2</sup> down
  - 39 ft/s<sup>2</sup> down
  - None of the above
- The motion of a child on a trampoline is modeled by the equation  $p''(t) + 2p'(t) + 3p(t) = 0$  where  $p$  is in feet and  $t$  is in seconds. Suppose we want the position function to be in inches instead of feet. How does this change the differential equation?
  - There is no change
  - $p''(t) + 24p'(t) + 36p(t) = 0$

- (c)  $12p''(t) + 2p'(t) + 36p(t) = 0$
- (d)  $144p''(t) + 24p'(t) + 3p(t) = 0$
- (e) None of the above

5. A hydrogen atom is bound to a large molecule, and its distance from the molecule follows the equation  $d'' + 4d' + 8d - 6 = 0$  where  $d$  is in picometers. At what distance from the molecule will the atom reach equilibrium?

- (a)  $d = 6$  pm.
- (b)  $d = \frac{3}{4}$  pm.
- (c)  $d = \frac{6}{13}$  pm
- (d) No equilibrium exists.

6. When the space shuttle re-enters the Earth's atmosphere, the air resistance produces a force proportional to its velocity squared, and gravity produces an approximately constant force. Which of the following equations might model its position  $p(t)$  if  $a$  and  $b$  are positive constants?

- (a)  $p'' + a(p')^2 + b = 0$
- (b)  $p'' - a(p')^2 + b = 0$
- (c)  $p'' + a(p')^2 + bp = 0$
- (d)  $p'' - a(p')^2 + bp = 0$
- (e) None of the above

7. The differential equation  $m\frac{d^2y}{dt^2} + \gamma\frac{dy}{dt} + ky = 0$  with positive parameters  $m$ ,  $\gamma$ , and  $k$  is often used to model the motion of a mass on a spring with a damping force. If  $\gamma$  was negative, what would this mean?

- (a) This would be like a negative friction, making the oscillations speed up over time.
- (b) This would be like a negative spring, that would push the object farther and farther from equilibrium.
- (c) This would be like a negative mass, so that the object would accelerate in the opposite direction that the forces were pushing.
- (d) None of the above

8. Test the following functions to see which is a solution to  $y'' + 4y' + 3y = 0$ .

- (a)  $y = e^{2t}$

- (b)  $y = e^t$
- (c)  $y = e^{-t}$
- (d)  $y = e^{-2t}$
- (e) None of these are solutions.
- (f) All are solutions.

9. Test the following functions to see which is a solution to  $\frac{d^2g}{dx^2} + 2\frac{dg}{dx} + 2g = 0$ .

- (a)  $g = e^x$
- (b)  $g = \sin x$
- (c)  $g = e^{-x} \sin x$
- (d) None of these are solutions.

10. Suppose we want to solve the differential equation  $y'' + ay' + by = 0$  and we conjecture that our solution is of the form  $y = Ce^{rt}$ . What equation do we get if we test this solution and simplify the result?

- (a)  $1 + ar + br^2 = 0$
- (b)  $C^2r^2 + Cr + c = 0$
- (c)  $Ce^{rt} + aCe^{rt} + bCe^{rt} = 0$
- (d)  $r^2 + ar + b = 0$
- (e) None of the above

11. Suppose we want to solve the differential equation  $\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 4y = 0$  and we conjecture that our solution is of the form  $y = Ce^{rt}$ . Solve the characteristic equation to determine what values of  $r$  satisfy the differential equation.

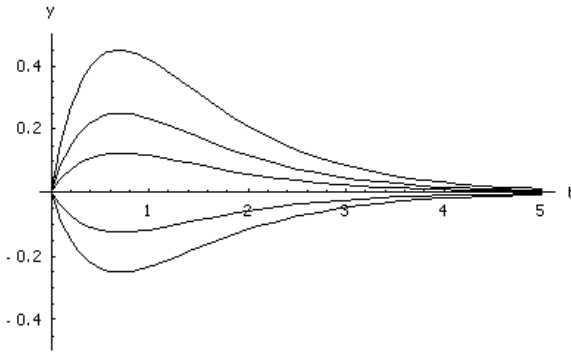
- (a)  $r = -2, -8$
- (b)  $r = -1, -4$
- (c)  $r = -3/2, +3/2$
- (d)  $r = 1, 4$
- (e) None of the above

12. Find the general solution to  $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = 0$ .

- (a)  $y(t) = C_1e^{-t/2} + C_2e^{t/2}$

- (b)  $y(t) = C_1e^{-2t} + C_2e^{-t}$
- (c)  $y(t) = C_1e^{-2t} + C_2e^t$
- (d)  $y(t) = -2C_1e^{-2t} - C_2e^{-t}$
- (e) None of the above

13. The graph below has five trajectories, call the top one  $y_1$ , the one below it  $y_2$ , down to the lowest one  $y_5$ . Which of these could be a solution of  $y'' + 3y' + 2y = 0$  with  $y(0) = 0$  and  $y'(0) = 1$ ?



- (a)  $y_1$
- (b)  $y_2$
- (c)  $y_3$
- (d)  $y_4$
- (e)  $y_5$

14. What is the general solution to  $f'' + 2f' + 2f = 0$ ?

- (a)  $f(x) = C_1e^{-x/2} \cos x + C_2e^{-x/2} \sin x$
- (b)  $f(x) = C_1e^{-x} \cos x + C_2e^{-x} \sin x$
- (c)  $f(x) = C_1e^{-x} \cos \frac{x}{2} + C_2e^{-x} \sin \frac{x}{2}$
- (d)  $f(x) = C_1 + C_2e^{-2x}$
- (e) None of the above

15. The harmonic oscillator modeled by  $mx'' + bx' + kx = 0$  with parameters  $m = 1$ ,  $k = 2$ , and  $b = 1$  is underdamped and thus oscillates. What is the period of the oscillations?

- (a)  $2\pi/\sqrt{7}$
- (b)  $4\pi/\sqrt{7}$

- (c)  $\sqrt{7}/2\pi$
- (d)  $\sqrt{7}/4\pi$
- (e) None of the above.

16. A harmonic oscillator is modeled by  $mx'' + bx' + kx = 0$ . If we increase the parameter  $m$  slightly, what happens to the period of oscillation?

- (a) The period gets larger.
- (b) The period gets smaller.
- (c) The period stays the same.

17. A harmonic oscillator is modeled by  $mx'' + bx' + kx = 0$ . If we increase the parameter  $k$  slightly, what happens to the period of oscillation?

- (a) The period gets larger.
- (b) The period gets smaller.
- (c) The period stays the same.

18. A harmonic oscillator is modeled by  $mx'' + bx' + kx = 0$ . If we increase the parameter  $b$  slightly, what happens to the period of oscillation?

- (a) The period gets larger.
- (b) The period gets smaller.
- (c) The period stays the same.

19. Classify the harmonic oscillator described below:

$$3\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + y = 0.$$

- (a) underdamped
- (b) overdamped
- (c) critically damped
- (d) no damping

20. Does the harmonic oscillator described below oscillate?

$$3\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + y = 0.$$

- (a) Yes.
- (b) No.