

## Linear Combinations and Independence of Functions

- Which of the following expressions is a linear combination of the functions  $f(t)$  and  $g(t)$ ?
  - $2f(t) + 3g(t) + 4$
  - $f(t) - 2g(t) + t$
  - $2f(t)g(t) - 3f(t)$
  - $f(t) - g(t)$
  - All of the above
  - None of the above
- True or False** The function  $h(t) = 4 + 3t$  is a linear combination of the functions  $f(t) = (1 + t)^2$  and  $g(t) = 2 - t - 2t^2$ .
- True or False** The function  $h(t) = \sin(t + 2)$  is a linear combination of the functions  $f(t) = \sin t$  and  $g(t) = \cos t$ .
- True or False**  $h(t) = t^2$  is a linear combination of  $f(t) = (1 - t)^2$  and  $g(t) = (1 + t)^2$ .
- Let  $y_1(t) = \sin(2t)$ . For which of the following functions  $y_2(t)$  will  $\{y_1(t), y_2(t)\}$  be a linearly independent set?
  - $y_2(t) = \sin(t) \cos(t)$
  - $y_2(t) = 2 \sin(2t)$
  - $y_2(t) = \cos(2t - \pi/2)$
  - $y_2(t) = \sin(-2t)$
  - All of the above
  - None of the above
- Let  $y_1(t) = e^{2t}$ . For which of the following functions  $y_2(t)$  will  $\{y_1(t), y_2(t)\}$  be a linearly independent set?
  - $y_2(t) = e^{-2t}$

- (b)  $y_2(t) = te^{2t}$   
 (c)  $y_2(t) = 1$   
 (d)  $y_2(t) = e^{3t}$   
 (e) All of the above  
 (f) None of the above
7. The functions  $y_1(t)$  and  $y_2(t)$  are linearly independent on the interval  $a < t < b$  if
- (a) for some constant  $k$ ,  $y_1(t) = ky_2(t)$  for  $a < t < b$ .  
 (b) there exists some  $t_0 \in (a, b)$  and some constants  $c_1$  and  $c_2$  such that  $c_1y_1(t_0) + c_2y_2(t_0) \neq 0$ .  
 (c) the equation  $c_1y_1(t) + c_2y_2(t) = 0$  holds for all  $t \in (a, b)$  only if  $c_1 = c_2 = 0$ .  
 (d) the ratio  $y_1(t)/y_2(t)$  is a constant function.  
 (e) All of the above  
 (f) None of the above
8. The functions  $y_1(t)$  and  $y_2(t)$  are linearly dependent on the interval  $a < t < b$  if
- (a) there exist two constants  $c_1$  and  $c_2$  such that  $c_1y_1(t) + c_2y_2(t) = 0$  for all  $a < t < b$ .  
 (b) there exist two constants  $c_1$  and  $c_2$ , not both 0, such that  $c_1y_1(t) + c_2y_2(t) = 0$  for all  $a < t < b$ .  
 (c) for each  $t$  in  $(a, b)$ , there exists constants  $c_1$  and  $c_2$  such that  $c_1y_1(t) + c_2y_2(t) = 0$ .  
 (d) for some  $a < t_0 < b$ , the equation  $c_1y_1(t_0) + c_2y_2(t_0) = 0$  can only be true if  $c_1 = c_2 = 0$ .  
 (e) All of the above  
 (f) None of the above
9. The functions  $y_1(t)$  and  $y_2(t)$  are both solutions of a certain second-order linear homogeneous differential equation with continuous coefficients for  $a < t < b$ . Which of the following statements are true?
- (i) The general solution to the ODE is  $y(t) = c_1y_1(t) + c_2y_2(t)$ ,  $a < t < b$ .  
 (ii)  $y_1(t)$  and  $y_2(t)$  must be linearly independent, since they both are solutions.  
 (iii)  $y_1(t)$  and  $y_2(t)$  may be linearly dependent, in which case we do not know enough information to write the general solution.  
 (iv) The Wronskian of  $y_1(t)$  and  $y_2(t)$  must be nonzero for these functions.

- (a) Only (i) and (ii) are true.
- (b) Only (i) is true.
- (c) Only (ii) and (iv) are true.
- (d) Only (iii) is true.
- (e) None are true.

10. Can the functions  $y_1(t) = t$  and  $y_2(t) = t^2$  be a linearly independent pair of solutions for an ODE of the form

$$y'' + p(t)y' + q(t)y = 0 \quad -1 \leq t \leq 1$$

where  $p(t)$  and  $q(t)$  are continuous functions?

- (a) Yes
- (b) No

11. Which pair of functions whose graphs are shown below could be linearly independent pairs of solutions to a second-order linear homogeneous differential equation?

