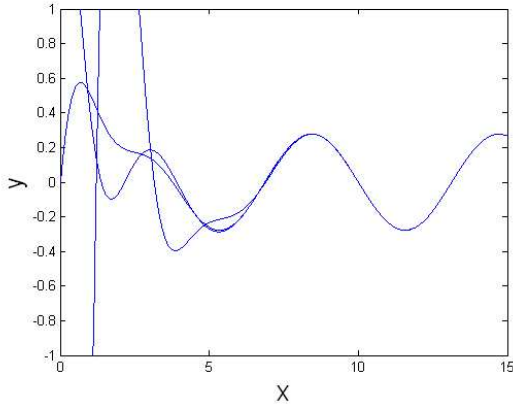


MathQuest: Differential Equations

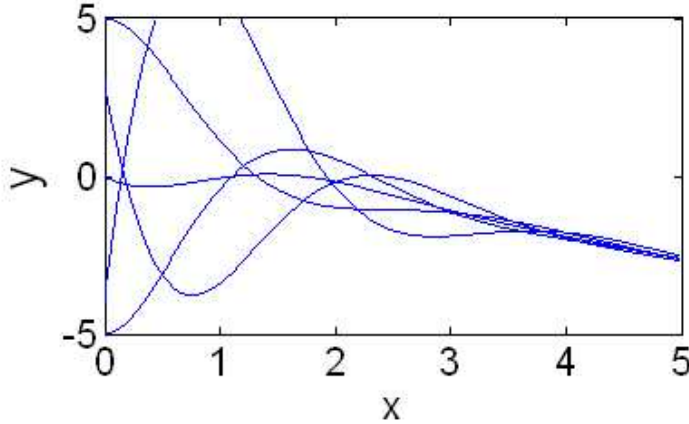
## Second Order Differential Equations: Forcing

1. The three functions plotted below are all solutions of  $y'' + ay' + 4y = \sin x$ . Is  $a$  positive or negative?



- (a)  $a$  is positive.  
(b)  $a$  is negative.  
(c)  $a = 0$ .  
(d) Not enough information is given.
2. If we conjecture the function  $y(x) = C_1 \sin 2x + C_2 \cos 2x + C_3$  as a solution to the differential equation  $y'' + 4y = 8$ , which of the constants is determined by the differential equation?
- (a)  $C_1$   
(b)  $C_2$   
(c)  $C_3$   
(d) None of them are determined.
3. What will the solutions of  $y'' + ay' + by = c$  look like if  $b$  is negative and  $a$  is positive.
- (a) Solutions will oscillate at first and level out at a constant.  
(b) Solutions will grow exponentially.  
(c) Solutions will oscillate forever.

4. The functions plotted below are solutions to which of the following differential equations?



- (a)  $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = 3 - 3x$   
 (b)  $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = 3e^{2x}$   
 (c)  $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = \sin \frac{2\pi}{9}x$   
 (d)  $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = 2x^2 + 3x - 4$   
 (e) None of the above
5. The general solution to  $f'' + 7f' + 12f = 0$  is  $f(t) = C_1e^{-3t} + C_2e^{-4t}$ . What should we conjecture as a particular solution to  $f'' + 7f' + 12f = 5e^{-2t}$ ?
- (a)  $f(t) = Ce^{-4t}$   
 (b)  $f(t) = Ce^{-3t}$   
 (c)  $f(t) = Ce^{-2t}$   
 (d)  $f(t) = C \cos 2t$   
 (e) None of the above
6. The general solution to  $f'' + 7f' + 12f = 0$  is  $f(t) = C_1e^{-3t} + C_2e^{-4t}$ . What is a particular solution to  $f'' + 7f' + 12f = 5e^{-6t}$ ?

- (a)  $f(t) = \frac{5}{6}e^{-6t}$   
 (b)  $f(t) = \frac{5}{31}e^{-6t}$   
 (c)  $f(t) = \frac{5}{20}e^{-6t}$   
 (d)  $f(t) = e^{-3t}$   
 (e) None of the above

7. To find a particular solution to the differential equation

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = \cos t,$$

we replace it with a new differential equation that has been “complexified.” What is the new differential equation to which we will find a particular solution?

- (a)  $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{2it}$
- (b)  $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{3it}$
- (c)  $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{it}$
- (d)  $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{-2it}$
- (e)  $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{-it}$
- (f) None of the above.

8. To solve the differential equation  $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{it}$ , we make a guess of  $y_p(t) = Ce^{it}$ . What equation results when we evaluate this in the differential equation?

- (a)  $-Ce^{it} + 3Cie^{it} + 2Ce^{it} = Ce^{it}$
- (b)  $-Ce^{it} + 3Cie^{it} + 2Ce^{it} = e^{it}$
- (c)  $Cie^{it} + 3Ce^{it} + 2Ce^{it} = e^{it}$
- (d)  $-Ce^{it} + 3Ce^{it} + 2Ce^{it} = Ce^{it}$

9. To solve the differential equation  $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{it}$ , we make a guess of  $y_p(t) = Ce^{it}$ . What value of  $C$  makes this particular solution work?

- (a)  $C = \frac{1 + 3i}{10}$
- (b)  $C = \frac{1 - 3i}{10}$
- (c)  $C = \frac{1 + 3i}{\sqrt{10}}$
- (d)  $C = \frac{1 - 3i}{\sqrt{10}}$

10. In order to find a particular solution to  $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = \cos t$ , do we want the real part or the imaginary part of the particular solution  $y_p(t) = Ce^{it}$  that solved the complexified equation  $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{it}$ ?

(a) Real part

(b) Imaginary part

(c) Neither, we need the whole solution to the complexified equation.

11. What is a particular solution to  $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = \cos t$ ?

(a)  $y_p(t) = \frac{3}{10} \cos t + \frac{1}{10} \sin t$

(b)  $y_p(t) = \frac{-3}{10} \cos t + \frac{1}{10} \sin t$

(c)  $y_p(t) = \frac{1}{10} \cos t + \frac{-3}{10} \sin t$

(d)  $y_p(t) = \frac{1}{10} \cos t + \frac{3}{10} \sin t$