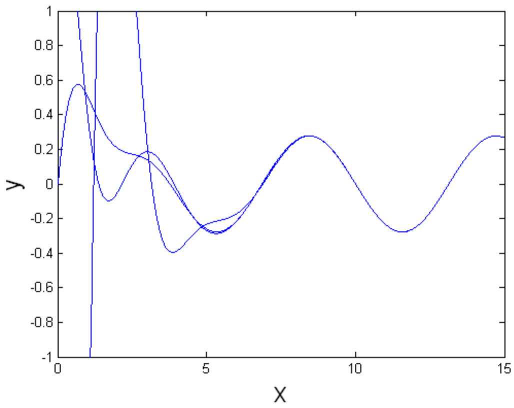
MathQuest: Differential Equations

Second Order Differential Equations: Forcing

1. The three functions plotted below are all solutions of $y''+ay'+4y = \sin x$. Is a positive or negative?



   (a) a is positive.
   (b) a is negative.
   (c) $a = 0$.
   (d) Not enough information is given.

2. If we conjecture the function $y(x) = C_1 \sin 2x + C_2 \cos 2x + C_3$ as a solution to the differential equation $y''+4y = 8$, which of the constants is determined by the differential equation?

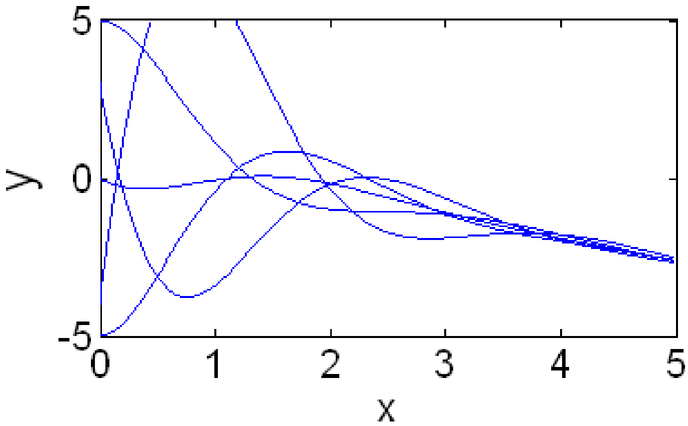
   (a) C_1
   (b) C_2
   (c) C_3
   (d) None of them are determined.

3. What will the solutions of $y'' + ay' + by = c$ look like if b is negative and a is positive.

   (a) Solutions will oscillate at first and level out at a constant.
   (b) Solutions will grow exponentially.
   (c) Solutions will oscillate forever.

(d) Solutions will decay exponentially.

4. The functions plotted below are solutions to which of the following differential equations?



(a) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = 3 - 3x$
(b) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = 3e^{2x}$
(c) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = \sin\frac{2\pi}{9}x$
(d) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = 2x^2 + 3x - 4$
(e) None of the above

5. The general solution to $f'' + 7f' + 12f = 0$ is $f(t) = C_1e^{-3t} + C_2e^{-4t}$. What should we conjecture as a particular solution to $f'' + 7f' + 12f = 5e^{-2t}$?

(a) $f(t) = Ce^{-4t}$
(b) $f(t) = Ce^{-3t}$
(c) $f(t) = Ce^{-2t}$
(d) $f(t) = C\cos 2t$
(e) None of the above

6. The general solution to $f'' + 7f' + 12f = 0$ is $f(t) = C_1e^{-3t} + C_2e^{-4t}$. What is a particular solution to $f'' + 7f' + 12f = 5e^{-6t}$?

(a) $f(t) = \frac{5}{6}e^{-6t}$
(b) $f(t) = \frac{5}{31}e^{-6t}$
(c) $f(t) = \frac{5}{20}e^{-6t}$

(d) $f(t) = e^{-3t}$

(e) None of the above

7. To find a particular solution to the differential equation

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = \cos t,$$

we replace it with a new differential equation that has been "complexified." What is the new differential equation to which we will find a particular solution?

(a) $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{2it}$

(b) $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{3it}$

(c) $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{it}$

(d) $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{-2it}$

(e) $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{-it}$

(f) None of the above.

8. To solve the differential equation $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{it}$, we make a guess of $y_p(t) = Ce^{it}$. What equation results when we evaluate this in the differential equation?

(a) $-Ce^{it} + 3Cie^{it} + 2Ce^{it} = Ce^{it}$

(b) $-Ce^{it} + 3Cie^{it} + 2Ce^{it} = e^{it}$

(c) $Cie^{it} + 3Ce^{it} + 2Ce^{it} = e^{it}$

(d) $-Ce^{it} + 3Ce^{it} + 2Ce^{it} = Ce^{it}$

9. To solve the differential equation $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{it}$, we make a guess of $y_p(t) = Ce^{it}$. What value of C makes this particular solution work?

(a) $C = \frac{1+3i}{10}$

(b) $C = \frac{1-3i}{10}$

(c) $C = \frac{1+3i}{\sqrt{10}}$

(d) $C = \frac{1-3i}{\sqrt{10}}$

10. In order to find a particular solution to $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = \cos t$, do we want the real part or the imaginary part of the particular solution $y_p(t) = Ce^{it}$ that solved the complexified equation $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{it}$?

    (a) Real part
    (b) Imaginary part
    (c) Neither, we need the whole solution to the complexified equation.

11. What is a particular solution to $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = \cos t$?

    (a) $y_p(t) = \frac{3}{10}\cos t + \frac{1}{10}\sin t$
    (b) $y_p(t) = \frac{-3}{10}\cos t + \frac{1}{10}\sin t$
    (c) $y_p(t) = \frac{1}{10}\cos t + \frac{-3}{10}\sin t$
    (d) $y_p(t) = \frac{1}{10}\cos t + \frac{3}{10}\sin t$