

MathQuest: Differential Equations

Second Order Differential Equations as Systems

1. A standard approach to converting second order equations such as $x'' = x' - 2x + 4$ is to introduce a new variable, y , such that:

(a) $y' = x$

(b) $y = x'$

(c) $y = x$

(d) $y' = x'$

2. A first-order system equivalent to the second order differential equation $x'' + 2x' + x = 2$ is:

(a)

$$\begin{aligned}x' &= y \\y' &= x - 2x' + 2\end{aligned}$$

(b)

$$\begin{aligned}x' &= y \\y' &= -2x + y + 2\end{aligned}$$

(c)

$$\begin{aligned}x' &= y \\y' &= -x - 2y + 2\end{aligned}$$

(d)

$$\begin{aligned}x' &= y \\y' &= -x + 2y + 2\end{aligned}$$

3. Which second-order differential equation is equivalent to the first-order system below?

$$\begin{aligned}x' &= y \\ y' &= 2x + 4y\end{aligned}$$

- (a) $y' = 2x + 4x'$
- (b) $x'' - 4x' - 2x = 0$
- (c) $x'' + 4x' + 2x = 0$
- (d) None of the above

4. Which second-order differential equation is equivalent to the first-order system below?

$$\begin{aligned}x' &= -3x + y \\ y' &= x - 2y\end{aligned}$$

- (a) $x'' + 5x' + 5x = 0$
- (b) $x' = -3x + x' + 3x$
- (c) $y'' + 5y' + 5y = 0$
- (d) $y' = -2y - (y' + 2y)$
- (e) This system can not be converted to a second-order equation.

5. In the spring mass system described by $x'' = -2x' - 2x$, what does the variable x represent?

- (a) The spring's displacement from equilibrium
- (b) The mass's displacement from equilibrium
- (c) The spring's velocity
- (d) The mass's velocity
- (e) None of the above

6. The spring mass system described by $x'' = -2x' - 2x$, can be converted to a first-order system by introducing the new variable $y = x'$. What does y represent?

- (a) The mass's displacement from equilibrium
- (b) The mass's velocity
- (c) The mass's acceleration

(d) None of the above

7. A first-order system equivalent to the spring mass system $x'' = -2x' - 2x$ is:

(a)

$$\begin{aligned}x' &= y \\y' &= -2x - 2x'\end{aligned}$$

(b)

$$\begin{aligned}x' &= y \\y' &= -2y + 2x\end{aligned}$$

(c)

$$\begin{aligned}x' &= y \\y' &= -2y - 2x\end{aligned}$$

(d)

$$\begin{aligned}x' &= y \\y' &= 2y - 2x\end{aligned}$$

8. The solution to the spring mass system $x'' = -2x' - 2x$ is:

(a) $\begin{bmatrix} x \\ y \end{bmatrix} = c_1 e^{-2t} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + c_2 e^t \begin{bmatrix} 3 \\ -2 \end{bmatrix}$

(b) $\begin{bmatrix} y \\ x \end{bmatrix} = c_1 e^{-2t} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + c_2 e^t \begin{bmatrix} 3 \\ -2 \end{bmatrix}$

(c) $\begin{bmatrix} x \\ y \end{bmatrix} = e^{-t} (c_1 \cos t + c_2 \sin t) \begin{bmatrix} 1 \\ 1 \end{bmatrix} + e^{-t} (-c_1 \sin t + c_2 \cos t) \begin{bmatrix} -2 \\ 0 \end{bmatrix}$

(d) $\begin{bmatrix} x \\ y \end{bmatrix} = e^{-t} (c_1 \cos t + c_2 \sin t) \begin{bmatrix} 1 \\ -2 \end{bmatrix} + e^{-t} (-c_1 \sin t + c_2 \cos t) \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

9. The position of the mass in the spring mass system $x'' = -2x' - 2x$ is given by:

(a) $y = -2c_1 e^{-t} \cos t - 2c_2 e^{-t} \sin t$

(b) $y = c_1 e^{-t} (\cos t - \sin t) + c_2 e^{-t} (\cos t + \sin t)$

(c) $x = c_1 e^{-t} (\cos t - \sin t) + c_2 e^{-t} (\cos t + \sin t)$

(d) $x = -2c_1 e^{-t} \cos t - 2c_2 e^{-t} \sin t$