MathQuest: Differential Equations

Second Order Differential Equations as Systems

1. A standard approach to converting second order equations such as $x'' = x' - 2x + 4$ is to introduce a new variable, y, such that:

   (a) $y' = x$

   (b) $y = x'$

   (c) $y = x$

   (d) $y' = x'$

2. A first-order system equivalent to the second order differential equation $x''+2x'+x = 2$ is:

   (a)

   $$\begin{aligned} x' &= y \\ y' &= x - 2x' + 2 \end{aligned}$$

   (b)

   $$\begin{aligned} x' &= y \\ y' &= -2x + y + 2 \end{aligned}$$

   (c)

   $$\begin{aligned} x' &= y \\ y' &= -x - 2y + 2 \end{aligned}$$

   (d)

   $$\begin{aligned} x' &= y \\ y' &= -x + 2y + 2 \end{aligned}$$

3. Which second-order differential equation is equivalent to the first-order system below?

$$\begin{aligned} x' &= y \\ y' &= 2x + 4y \end{aligned}$$

(a) $y' = 2x + 4x'$

(b) $x'' - 4x' - 2x = 0$

(c) $x'' + 4x' + 2x = 0$

(d) None of the above

4. Which second-order differential equation is equivalent to the first-order system below?

$$\begin{aligned} x' &= -3x + y \\ y' &= x - 2y \end{aligned}$$

(a) $x'' + 5x' + 5x = 0$

(b) $x' = -3x + x' + 3x$

(c) $y'' + 5y' + 5y = 0$

(d) $y' = -2y - (y' + 2y)$

(e) This system can not be converted to a second-order equation.

5. In the spring mass system described by $x'' = -2x' - 2x$, what does the variable x represent?

(a) The spring's displacement from equilibrium

(b) The mass's displacement from equilibrium

(c) The spring's velocity

(d) The mass's velocity

(e) None of the above

6. The spring mass system described by $x'' = -2x' - 2x$, can be converted to a first-order system by introducing the new variable $y = x'$. What does y represent?

(a) The mass's displacement from equilibrium

(b) The mass's velocity

(c) The mass's acceleration

(d) None of the above

7. A first-order system equivalent to the spring mass system $x'' = -2x' - 2x$ is:

(a)
$$\begin{aligned} x' &= y \\ y' &= -2x - 2x' \end{aligned}$$

(b)
$$\begin{aligned} x' &= y \\ y' &= -2y + 2x \end{aligned}$$

(c)
$$\begin{aligned} x' &= y \\ y' &= -2y - 2x \end{aligned}$$

(d)
$$\begin{aligned} x' &= y \\ y' &= 2y - 2x \end{aligned}$$

8. The solution to the spring mass system $x'' = -2x' - 2x$ is:

(a) $\begin{bmatrix} x \\ y \end{bmatrix} = c_1 e^{-2t} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + c_2 e^{t} \begin{bmatrix} 3 \\ -2 \end{bmatrix}$

(b) $\begin{bmatrix} y \\ x \end{bmatrix} = c_1 e^{-2t} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + c_2 e^{t} \begin{bmatrix} 3 \\ -2 \end{bmatrix}$

(c) $\begin{bmatrix} x \\ y \end{bmatrix} = e^{-t}\left(c_1 \cos t + c_2 \sin t\right) \begin{bmatrix} 1 \\ 1 \end{bmatrix} + e^{-t}\left(-c_1 \sin t + c_2 \cos t\right) \begin{bmatrix} -2 \\ 0 \end{bmatrix}$

(d) $\begin{bmatrix} x \\ y \end{bmatrix} = e^{-t}\left(c_1 \cos t + c_2 \sin t\right) \begin{bmatrix} 1 \\ -2 \end{bmatrix} + e^{-t}\left(-c_1 \sin t + c_2 \cos t\right) \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

9. The position of the mass in the spring mass system $x'' = -2x' - 2x$ is given by:

(a) $y = -2c_1 e^{-t} \cos t - 2c_2 e^{-t} \sin t$

(b) $y = c_1 e^{-t}\left(\cos t - \sin t\right) + c_2 e^{-t}\left(\cos t + \sin t\right)$

(c) $x = c_1 e^{-t}\left(\cos t - \sin t\right) + c_2 e^{-t}\left(\cos t + \sin t\right)$

(d) $x = -2c_1 e^{-t} \cos t - 2c_2 e^{-t} \sin t$