

## MathQuest: Differential Equations

### Testing Solutions to Linear Systems

1. We want to test the solution  $x_1 = -e^{-2t}$  and  $x_2 = e^{-2t}$  in the system

$$\begin{aligned}x_1' &= x_1 + 3x_2 \\x_2' &= 3x_1 + x_2\end{aligned}$$

What equations result from substituting the solution into the equation?

(a)

$$\begin{aligned}-e^{-2t} &= -e^{-2t} + 3e^{-2t} \\e^{-2t} &= -3e^{-2t} + e^{-2t}\end{aligned}$$

(b)

$$\begin{aligned}-e^{-2t} &= e^{-2t} - 3e^{-2t} \\e^{-2t} &= 3e^{-2t} - e^{-2t}\end{aligned}$$

(c)

$$\begin{aligned}2e^{-2t} &= -e^{-2t} + 3e^{-2t} \\-2e^{-2t} &= -3e^{-2t} + e^{-2t}\end{aligned}$$

(d)

$$\begin{aligned}-2e^{-2t} &= e^{-2t} - 3e^{-2t} \\2e^{-2t} &= 3e^{-2t} - e^{-2t}\end{aligned}$$

(e) None of the above

2. Is  $x_1 = x_2 = x_3 = e^t$  a solution to the system below?

$$\begin{aligned}x_1' &= 3x_1 - x_2 + x_3 \\x_2' &= 2x_1 - x_3 \\x_3' &= x_1 - x_2 + x_3\end{aligned}$$

- (a) Yes, it is a solution.
- (b) No, it is not a solution because it does not satisfy  $x'_1 = 3x_1 - x_2 + x_3$ .
- (c) No, it is not a solution because it does not satisfy  $x'_2 = 2x_1 - x_3$ .
- (d) No, it is not a solution because it does not satisfy  $x'_3 = x_1 - x_2 + x_3$ .
- (e) No, it is not a solution because it doesn't work in any equation for all values of  $t$ .

3. Which of the following are solutions to the system below?

$$\begin{aligned}x'_1 &= 4x_1 - x_2 \\x'_2 &= 2x_1 + x_2\end{aligned}$$

(a)

$$\begin{aligned}x_1 &= e^{2t} \\x_2 &= e^{2t}\end{aligned}$$

(b)

$$\begin{aligned}x_1 &= e^{2t} \\x_2 &= 2e^{2t}\end{aligned}$$

(c)

$$\begin{aligned}x_1 &= e^{3t} \\x_2 &= e^{-3t}\end{aligned}$$

- (d) None of the above.
- (e) All of the above.

4. Since we know that both  $x_1 = x_2 = e^{3t}$  and  $x_1 = e^{-t}, x_2 = -e^{-t}$  are solutions to the system

$$\begin{aligned}x'_1 &= x_1 + 2x_2 \\x'_2 &= 2x_1 + x_2\end{aligned}$$

Which of the following are also solutions?

(a)

$$\begin{aligned}x_1 &= 3e^{3t} - e^{-t} \\x_2 &= 3e^{3t} + e^{-t}\end{aligned}$$

(b)

$$\begin{aligned}x_1 &= -e^{3t} - e^{-t} \\x_2 &= -e^{3t} + e^{-t}\end{aligned}$$

(c)

$$\begin{aligned}x_1 &= 2e^{3t} + 4e^{-t} \\x_2 &= -4e^{-t} + 2e^{3t}\end{aligned}$$

(d)

$$\begin{aligned}x_1 &= 0 \\x_2 &= 0\end{aligned}$$

(e) None of the above.

(f) All of the above.

5. Consider the system of differential equations,

$$y'(t) = \begin{bmatrix} 14 & 0 & -4 \\ 2 & 13 & -8 \\ -3 & 0 & 25 \end{bmatrix} y(t)$$

Which of the following functions solve this system?

(a)  $y(t) = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} e^{-4t}$

(b)  $y(t) = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} e^{6t}$

(c)  $y(t) = \begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix} e^{13t}$

(d) None of the above.

(e) All of the above.