MathQuest: Differential Equations

Euler's Method and Systems of Equations

1. We have the system of differential equations $x' = 3x - 2y$ and $y' = 4y^2 - 7x$. If we know that $x(0) = 2$ and $y(0) = 1$, estimate the values of x and y at $t = 0.1$.

   (a) $x(0.1) = 4$, $y(0.1) = -10$

   (b) $x(0.1) = 6$, $y(0.1) = -9$

   (c) $x(0.1) = 2.4$, $y(0.1) = 0$

   (d) $x(0.1) = 0.4$, $y(0.1) = -1$

   (e) None of the above

2. We have the system of differential equations $x' = x(-x-2y+5)$ and $y' = y(-x-y+10)$. If we know that $x(4.5) = 3$ and $y(4.5) = 2$, estimate the values of x and y at $t = 4$.

   (a) $x(4) = 0$, $y(4) = -3$

   (b) $x(4) = 6$, $y(4) = 10$

   (c) $x(4) = 6$, $y(4) = 7$

   (d) None of the above

3. We have a system of differential equations for $\frac{dx}{dt}$ and $\frac{dy}{dt}$, along with the initial conditions that $x(0) = 5$ and $y(0) = 7$. We want to know the value of these functions when $t = 5$. Using Euler's method and $\Delta t = 1$ we get the result that $x(5) \approx 14.2$ and $y(5) \approx 23.8$. Next, we use Euler's method again with $\Delta t = 0.5$ and find that $x(5) \approx 14.6$ and $y(5) \approx 5.3$. Finally we use $\Delta t = 0.25$, finding that $x(5) \approx 14.8$ and $y(5) \approx -3.7$. What does this mean?

   (a) Fewer steps means fewer opportunities for error, so $(x(5), y(5)) \approx (14.2, 23.8)$.

   (b) Smaller stepsize means smaller errors, so $(x(5), y(5)) \approx (14.8, -3.7)$.

   (c) We have no way of knowing whether any of these estimates is anywhere close to the true values of $(x(5), y(5))$.

   (d) At these step sizes we can conclude that $x(5) \approx 15$, but we can only conclude that $y(5) < -3.7$.

   (e) Results like this are impossible: We must have made an error in our calculations.