

## MathQuest: Differential Equations

### Euler's Method and Systems of Equations

1. We have the system of differential equations  $x' = 3x - 2y$  and  $y' = 4y^2 - 7x$ . If we know that  $x(0) = 2$  and  $y(0) = 1$ , estimate the values of  $x$  and  $y$  at  $t = 0.1$ .
  - (a)  $x(0.1) = 4, y(0.1) = -10$
  - (b)  $x(0.1) = 6, y(0.1) = -9$
  - (c)  $x(0.1) = 2.4, y(0.1) = 0$
  - (d)  $x(0.1) = 0.4, y(0.1) = -1$
  - (e) None of the above
  
2. We have the system of differential equations  $x' = x(-x - 2y + 5)$  and  $y' = y(-x - y + 10)$ . If we know that  $x(4.5) = 3$  and  $y(4.5) = 2$ , estimate the values of  $x$  and  $y$  at  $t = 4$ .
  - (a)  $x(4) = 0, y(4) = -3$
  - (b)  $x(4) = 6, y(4) = 10$
  - (c)  $x(4) = 6, y(4) = 7$
  - (d) None of the above
  
3. We have a system of differential equations for  $\frac{dx}{dt}$  and  $\frac{dy}{dt}$ , along with the initial conditions that  $x(0) = 5$  and  $y(0) = 7$ . We want to know the value of these functions when  $t = 5$ . Using Euler's method and  $\Delta t = 1$  we get the result that  $x(5) \approx 14.2$  and  $y(5) \approx 23.8$ . Next, we use Euler's method again with  $\Delta t = 0.5$  and find that  $x(5) \approx 14.6$  and  $y(5) \approx 5.3$ . Finally we use  $\Delta t = 0.25$ , finding that  $x(5) \approx 14.8$  and  $y(5) \approx -3.7$ . What does this mean?
  - (a) Fewer steps means fewer opportunities for error, so  $(x(5), y(5)) \approx (14.2, 23.8)$ .
  - (b) Smaller stepsize means smaller errors, so  $(x(5), y(5)) \approx (14.8, -3.7)$ .
  - (c) We have no way of knowing whether any of these estimates is anywhere close to the true values of  $(x(5), y(5))$ .
  - (d) At these step sizes we can conclude that  $x(5) \approx 15$ , but we can only conclude that  $y(5) < -3.7$ .
  - (e) Results like this are impossible: We must have made an error in our calculations.