MathQuest: Differential Equations

Modeling with Systems

1. In the predator - prey population model

$$\begin{aligned}\frac{dx}{dt} &= ax - \frac{ax^2}{N} - bxy\\ \frac{dy}{dt} &= cy + kxy\end{aligned}$$

with $a > 0$, $b > 0$, $c > 0$, $N > 0$, and $k > 0$,

which variable represents the predator population?

(a) x

(b) $\frac{dx}{dt}$

(c) y

(d) $\frac{dy}{dt}$

2. In which of the following predator - prey population models does the prey have the highest intrinsic reproduction rate?

(a)

$$\begin{aligned}P' &= 2P - 3Q * P\\ Q' &= -Q + 1/2Q * P\end{aligned}$$

(b)

$$\begin{aligned}P' &= P(1 - 4Q)\\ Q' &= Q(-2 + 3P)\end{aligned}$$

(c)

$$\begin{aligned}P' &= P(3 - 2Q)\\ Q' &= Q(-1 + P)\end{aligned}$$

(d)

$$\begin{aligned} P' &= 4P(1/2 - Q) \\ Q' &= Q(-1.5 + 2P) \end{aligned}$$

3. For which of the following predator - prey population models is the predator most successful at catching prey?

   (a)

$$\begin{aligned} \frac{dx}{dt} &= 2x - 3x * y \\ \frac{dy}{dt} &= -y + 1/2x * y \end{aligned}$$

   (b)

$$\begin{aligned} \frac{dx}{dt} &= x(1 - 4y) \\ \frac{dy}{dt} &= y(-2 + 3x) \end{aligned}$$

   (c)

$$\begin{aligned} \frac{dx}{dt} &= x(3 - 2y) \\ \frac{dy}{dt} &= y(-1 + x) \end{aligned}$$

   (d)

$$\begin{aligned} \frac{dx}{dt} &= 4x(1/2 - y) \\ \frac{dy}{dt} &= 2y(-1/2 + x) \end{aligned}$$

4. In this predator - prey population model

$$\begin{aligned} \frac{dx}{dt} &= -ax + bxy \\ \frac{dy}{dt} &= cy - dxy \end{aligned}$$

   with $a > 0$, $b > 0$, $c > 0$, and $d > 0$,

   does the prey have limits to its population other than that imposed by the predator?

(a) Yes
(b) No
(c) Can not tell

5. In this predator - prey population model

$$\begin{aligned}\frac{dx}{dt} &= ax - \frac{ax^2}{N} - bxy \\ \frac{dy}{dt} &= cy + kxy\end{aligned}$$

with $a > 0$, $b > 0$, $c > 0$, and $k > 0$,

if the prey becomes extinct, will the predator survive?

(a) Yes
(b) No
(c) Can not tell

6. In this predator - prey population model

$$\begin{aligned}\frac{dx}{dt} &= ax - \frac{ax^2}{N} - bxy \\ \frac{dy}{dt} &= cy + kxy\end{aligned}$$

with $a > 0$, $b > 0$, $c > 0$, $N > 0$, and $k > 0$,

are there any limits on the prey's population other than the predator?

(a) Yes
(b) No
(c) Can not tell

7. On Komodo Island we have three species: Komodo dragons (K), deer (D), and a variety of plant (P). The dragons eat the deer and the deer eat the plant. Which of the following systems of differential equations could represent this scenario?

(a)

$$\begin{aligned}K' &= aK - bKD \\ D' &= cD + dKD - eDP \\ P' &= -fP + gDP\end{aligned}$$

(b)

$$
\begin{aligned}
K' &= -aK + bKD \\
D' &= -cD - dKD + eDP \\
P' &= fP - gDP
\end{aligned}
$$

(c)

$$
\begin{aligned}
K' &= aK - bKD + KP \\
D' &= cD + dKD - eDP \\
P' &= -fP + gDP - hKP
\end{aligned}
$$

(d)

$$
\begin{aligned}
K' &= -aK + bKD - KP \\
D' &= -cD - dKD + eDP \\
P' &= fP - gDP + hKP
\end{aligned}
$$

8. In the two species population model

$$
\begin{aligned}
R' &= 2R - bFR \\
F' &= -F + 2FR
\end{aligned}
$$

for what value of the parameter b will the system have a stable equilibrium?

(a) $b < 0$
(b) $b = 0$
(c) $b > 0$
(d) For no value of b

9. Two forces are fighting one another. x and y are the number of soldiers in each force. Let a and b be the offensive fighting capacities of x and y, respectively. Assume that forces are lost only to combat, and no reinforcements are brought in. What system represents this scenario?

(a)

$$
\begin{aligned}
\frac{dx}{dt} &= -ay \\
\frac{dy}{dt} &= -bx
\end{aligned}
$$

(b)

$$\frac{dx}{dt} = -by$$
$$\frac{dy}{dt} = -ax$$

(c)

$$\frac{dx}{dt} = y - a$$
$$\frac{dy}{dt} = x - b$$

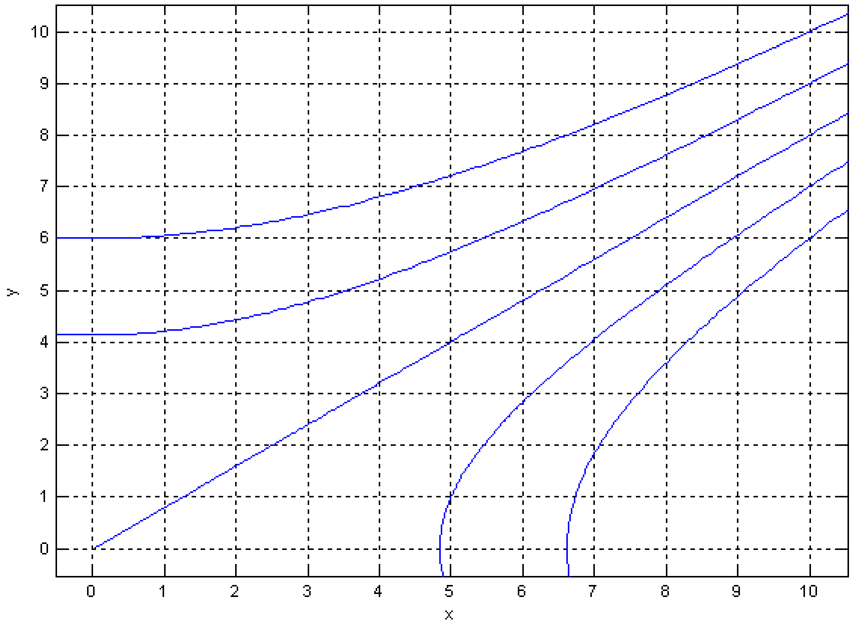
(d)

$$\frac{dx}{dt} = y - b$$
$$\frac{dy}{dt} = x - a$$

10. Two forces, x and y, are fighting one another. Let a and b be the fighting efficiences of x and y, respectively. Assume that forces are lost only to combat, and no reinforcements are brought in. How does the size of the y army change with respect to the size of the x army?

    (a) $\frac{dy}{dx} = \frac{ax}{by}$
    (b) $\frac{dy}{dx} = \frac{x}{y}$
    (c) $\frac{dy}{dx} = \frac{y}{x}$
    (d) $\frac{dy}{dx} = -by - ax$

11. Two forces, x and y, are fighting one another. Assume that forces are lost only to combat, and no reinforcements are brought in. Based on the phase plane below, if $x(0) = 10$ and $y(0) = 7$, who wins?



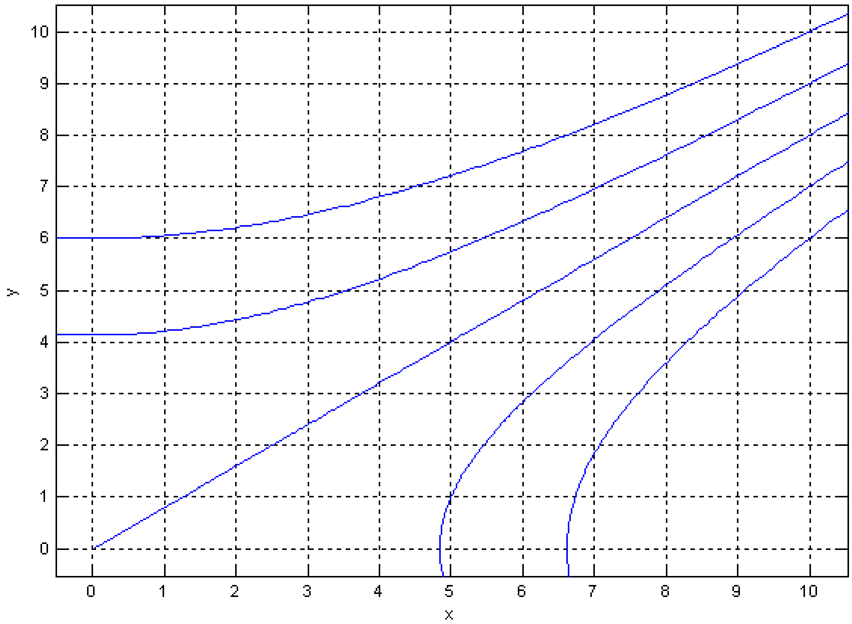
(a) x wins

(b) y wins

(c) They tie.

(d) Neither wins - both armies grow, and the battles escalate forever.

12. Two forces, x and y, are fighting one another. Assume that forces are lost only to combat, and no reinforcements are brought in. Based on the phase plane below, which force has a greater offensive fighting efficiency?



(a) x has the greater fighting efficiency.

(b) y has the greater fighting efficiency.

(c) They have the same fighting efficiencies.

13. Two forces, x and y, are fighting one another. Assume that forces are lost only to combat, and no reinforcements are brought in. You are the x-force, and you want to improve your chance of winning. Assuming that it would be possible, would you rather double your fighting efficiency or double your number of soldiers?

(a) Double the fighting efficiency

(b) Double the number of soldiers

(c) These would both have the same effect