

MathQuest: Differential Equations

Modeling with Systems

1. In the predator - prey population model

$$\begin{aligned}\frac{dx}{dt} &= ax - \frac{ax^2}{N} - bxy \\ \frac{dy}{dt} &= cy + kxy\end{aligned}$$

with $a > 0$, $b > 0$, $c > 0$, $N > 0$, and $k > 0$,
which variable represents the predator population?

- (a) x
- (b) $\frac{dx}{dt}$
- (c) y
- (d) $\frac{dy}{dt}$

2. In which of the following predator - prey population models does the prey have the highest intrinsic reproduction rate?

- (a)

$$\begin{aligned}P' &= 2P - 3Q * P \\ Q' &= -Q + 1/2Q * P\end{aligned}$$

- (b)

$$\begin{aligned}P' &= P(1 - 4Q) \\ Q' &= Q(-2 + 3P)\end{aligned}$$

- (c)

$$\begin{aligned}P' &= P(3 - 2Q) \\ Q' &= Q(-1 + P)\end{aligned}$$

(d)

$$\begin{aligned}P' &= 4P(1/2 - Q) \\Q' &= Q(-1.5 + 2P)\end{aligned}$$

3. For which of the following predator - prey population models is the predator most successful at catching prey?

(a)

$$\begin{aligned}\frac{dx}{dt} &= 2x - 3x * y \\ \frac{dy}{dt} &= -y + 1/2x * y\end{aligned}$$

(b)

$$\begin{aligned}\frac{dx}{dt} &= x(1 - 4y) \\ \frac{dy}{dt} &= y(-2 + 3x)\end{aligned}$$

(c)

$$\begin{aligned}\frac{dx}{dt} &= x(3 - 2y) \\ \frac{dy}{dt} &= y(-1 + x)\end{aligned}$$

(d)

$$\begin{aligned}\frac{dx}{dt} &= 4x(1/2 - y) \\ \frac{dy}{dt} &= 2y(-1/2 + x)\end{aligned}$$

4. In this predator - prey population model

$$\begin{aligned}\frac{dx}{dt} &= -ax + bxy \\ \frac{dy}{dt} &= cy - dxy\end{aligned}$$

with $a > 0$, $b > 0$, $c > 0$, and $d > 0$,

does the prey have limits to its population other than that imposed by the predator?

- (a) Yes
- (b) No
- (c) Can not tell

5. In this predator - prey population model

$$\begin{aligned}\frac{dx}{dt} &= ax - \frac{ax^2}{N} - bxy \\ \frac{dy}{dt} &= cy + kxy\end{aligned}$$

with $a > 0$, $b > 0$, $c > 0$, and $k > 0$,

if the prey becomes extinct, will the predator survive?

- (a) Yes
- (b) No
- (c) Can not tell

6. In this predator - prey population model

$$\begin{aligned}\frac{dx}{dt} &= ax - \frac{ax^2}{N} - bxy \\ \frac{dy}{dt} &= cy + kxy\end{aligned}$$

with $a > 0$, $b > 0$, $c > 0$, $N > 0$, and $k > 0$,

are there any limits on the prey's population other than the predator?

- (a) Yes
- (b) No
- (c) Can not tell

7. On Komodo Island we have three species: Komodo dragons (K), deer (D), and a variety of plant (P). The dragons eat the deer and the deer eat the plant. Which of the following systems of differential equations could represent this scenario?

- (a)

$$\begin{aligned}K' &= aK - bKD \\ D' &= cD + dKD - eDP \\ P' &= -fP + gDP\end{aligned}$$

(b)

$$\begin{aligned}K' &= -aK + bKD \\D' &= -cD - dKD + eDP \\P' &= fP - gDP\end{aligned}$$

(c)

$$\begin{aligned}K' &= aK - bKD + KP \\D' &= cD + dKD - eDP \\P' &= -fP + gDP - hKP\end{aligned}$$

(d)

$$\begin{aligned}K' &= -aK + bKD - KP \\D' &= -cD - dKD + eDP \\P' &= fP - gDP + hKP\end{aligned}$$

8. In the two species population model

$$\begin{aligned}R' &= 2R - bFR \\F' &= -F + 2FR\end{aligned}$$

for what value of the parameter b will the system have a stable equilibrium?

(a) $b < 0$

(b) $b = 0$

(c) $b > 0$

(d) For no value of b

9. Two forces are fighting one another. x and y are the number of soldiers in each force. Let a and b be the fighting efficiencies of x and y , respectively. Assume that forces are lost only to combat, and no reinforcements are brought in. What system represents this scenario?

(a)

$$\begin{aligned}\frac{dx}{dt} &= -ay \\ \frac{dy}{dt} &= -bx\end{aligned}$$

(b)

$$\begin{aligned}\frac{dx}{dt} &= -by \\ \frac{dy}{dt} &= -ax\end{aligned}$$

(c)

$$\begin{aligned}\frac{dx}{dt} &= y - a \\ \frac{dy}{dt} &= x - b\end{aligned}$$

(d)

$$\begin{aligned}\frac{dx}{dt} &= y - b \\ \frac{dy}{dt} &= x - a\end{aligned}$$

10. Two forces, x and y , are fighting one another. Let a and b be the fighting efficiencies of x and y , respectively. Assume that forces are lost only to combat, and no reinforcements are brought in. How does the size of the y army change with respect to the size of the x army?

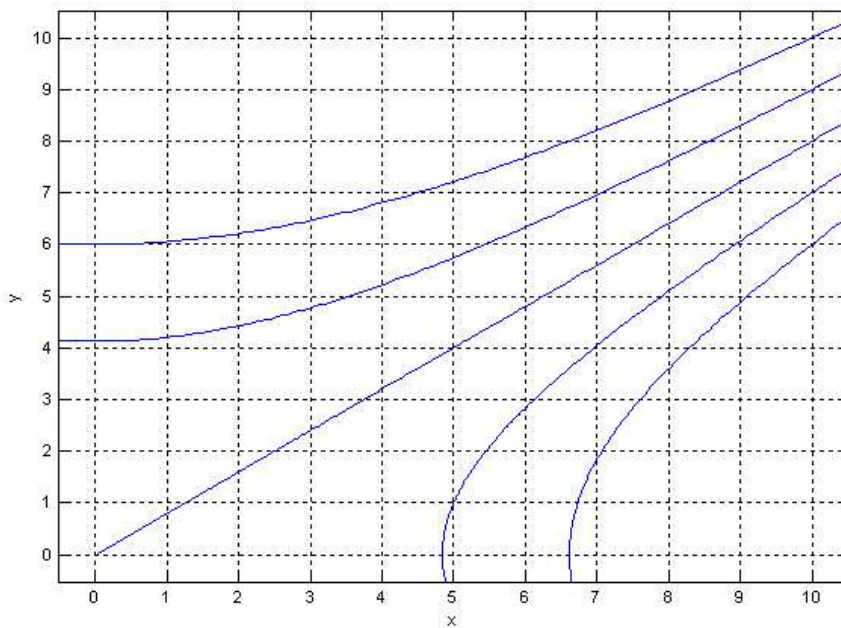
(a) $\frac{dy}{dx} = \frac{ax}{by}$

(b) $\frac{dy}{dx} = \frac{x}{y}$

(c) $\frac{dy}{dx} = \frac{y}{x}$

(d) $\frac{dy}{dx} = -by - ax$

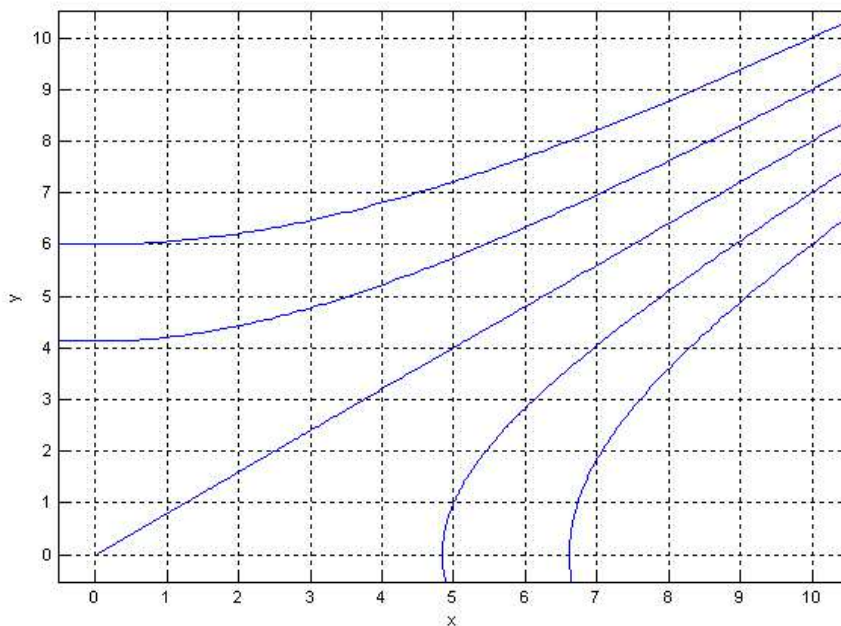
11. Two forces, x and y , are fighting one another. Assume that forces are lost only to combat, and no reinforcements are brought in. Based on the phase plane below, if $x(0) = 10$ and $y(0) = 7$, who wins?



1

- (a) x wins
- (b) y wins
- (c) They tie.
- (d) Neither wins - both armies grow, and the battles escalate forever.

12. Two forces, x and y , are fighting one another. Assume that forces are lost only to combat, and no reinforcements are brought in. Based on the phase plane below, which force has a greater fighting efficiency?



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- (a) x has the greater fighting efficiency.
- (b) y has the greater fighting efficiency.
- (c) They have the same fighting efficiencies.

13. Two forces, x and y , are fighting one another. Assume that forces are lost only to combat, and no reinforcements are brought in. You are the x -force, and you want to improve your chance of winning. Assuming that it would be possible, would you rather double your fighting efficiency or double your number of soldiers?

- (a) Double the fighting efficiency
- (b) Double the number of soldiers
- (c) These would both have the same effect