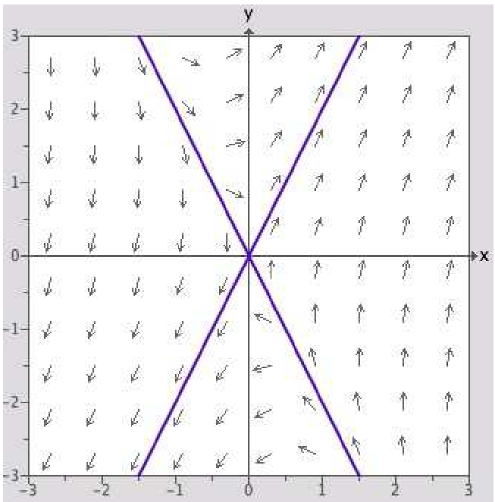
Geometry of Systems

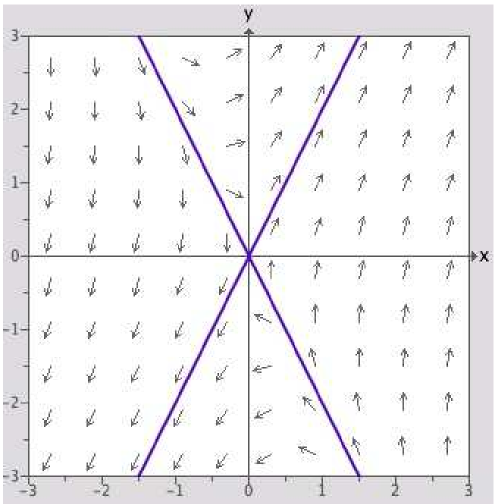
1. The differential equation $\frac{d\vec{Y}}{dt} = A\vec{Y}$ has two straight line solutions corresponding to eigenvectors $\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ that are shown on the direction field below. We denote the associated eigenvalues by λ_1 and λ_2.



We can deduce that λ_1 is

(a) positive real
(b) negative real
(c) zero
(d) complex
(e) There is not enough information

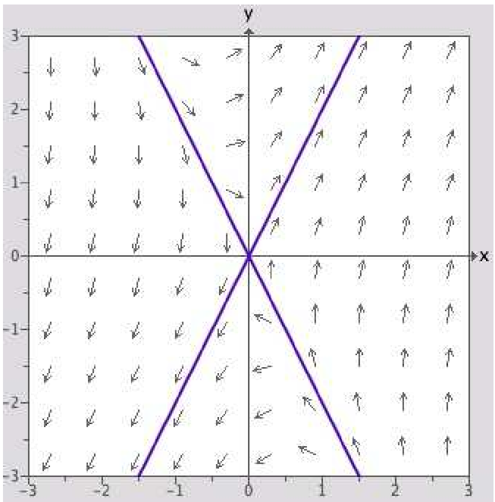
2. The differential equation $\frac{d\vec{Y}}{dt} = A\vec{Y}$ has two straight line solutions corresponding to eigenvectors $\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ that are shown on the direction field below. We denote the associated eigenvalues by λ_1 and λ_2.



We can deduce that λ_2 is

(a) positive real
(b) negative real
(c) zero
(d) complex
(e) There is not enough information

3. The differential equation $\dfrac{d\vec{Y}}{dt} = A\vec{Y}$ has two straight line solutions corresponding to eigenvectors $\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ that are shown on the direction field below.



Suppose we have a solution $\vec{Y}(t)$ to this system of differential equations which satisfies initial condition $\vec{Y}(t) = (x_0, y_0)$ where the point (x_0, y_0) is not on the line through the point $(1, -2)$. Which statement best describes the behavior of the solution as $t \to \infty$?

(a) The solution tends towards the origin.

(b) The solution moves away from the origin and asymptotically approaches the line through $< 1, 2 >$.

(c) The solution moves away from the origin and asymptotically approaches the line through $< 1, -2 >$.

(d) The solution spirals and returns to the point (x_0, y_0).

(e) There is not enough information.

4. Suppose you have a linear, homogeneous system of differential equations with constant coefficients whose coefficient matrix has the following eigensystem: eigenvalues -4 and -1 and eigenvectors $< 1, 1 >$ and $< -2, 1 >$ respectively. The function $\vec{Y}(t)$ is a solution to this system of differential equations which satisfies initial value $\vec{Y}(0) = (-15, 20)$. Which statement best describes the behavior of the solution as $t \to \infty$?

   (a) The solution tends towards the origin.

   (b) The solution moves away from the origin and asymptotically approaches the line through $< 1, 1 >$.

   (c) The solution moves away from the origin and asymptotically approaches the line through $< -2, 1 >$.

   (d) The solution spirals and returns to the point $(-15, 20)$.

   (e) There is not enough information.

5. Suppose we have a linear, homogeneous system of differential equations with constant coefficients who coefficient matrix has the following eigensystem: eigenvalues -4 and -1 and eigenvectors $< 1, 1 >$ and $< -2, 1 >$ respectively. Suppose we have a solution $\vec{Y}(t)$ which satisfies $\vec{Y}(0) = (x_0, y_0)$ where the point (x_0, y_0) is not on the line through the point $(1, 1)$. How can we best describe the manner in which the solution $\vec{Y}(t)$ approaches the origin?

   (a) The solution will approach the origin in the same manner as the line which goes through the point $(1, 1)$.

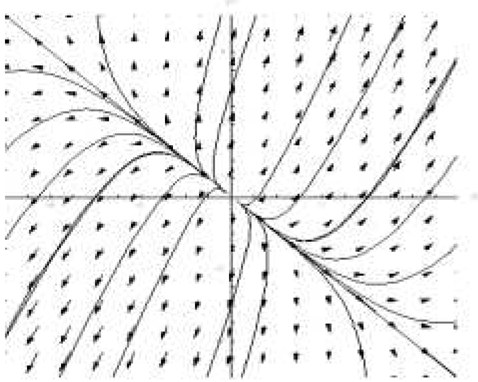
   (b) The solution will approach the origin in the same manner as the line which goes through the point $(-2, 1)$.

   (c) The solution will directly approach the origin in a straight line from the point (x_0, y_0).

   (d) The answer can vary greatly depending on what the point (x_0, y_0) is.

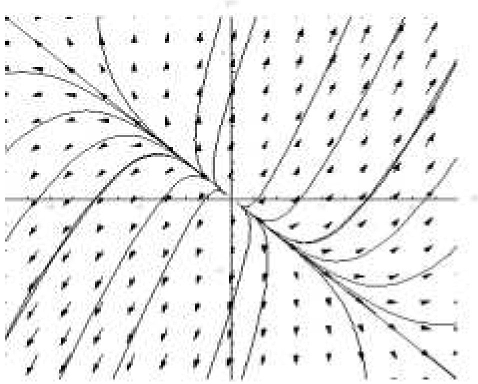
   (e) The solution doesn't approach the origin.

6. Using the phase portrait below for the system $Y' = AY$, we can deduce that the eigenvalues of the coefficient matrix A are:



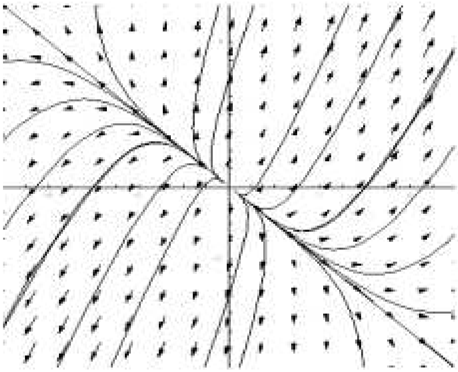
(a) both real
(b) both complex
(c) one real, one complex
(d) Not enough information is given

7. Using the phase portrait below for the system $Y' = AY$, we can deduce that the eigenvalues are:



(a) of mixed sign
(b) both negative
(c) both positive
(d) Not enough information is given

8. Using the phase portrait below for the system $Y' = AY$, we can deduce that the dominant eigenvector is:



(a) $< -1, 1 >$

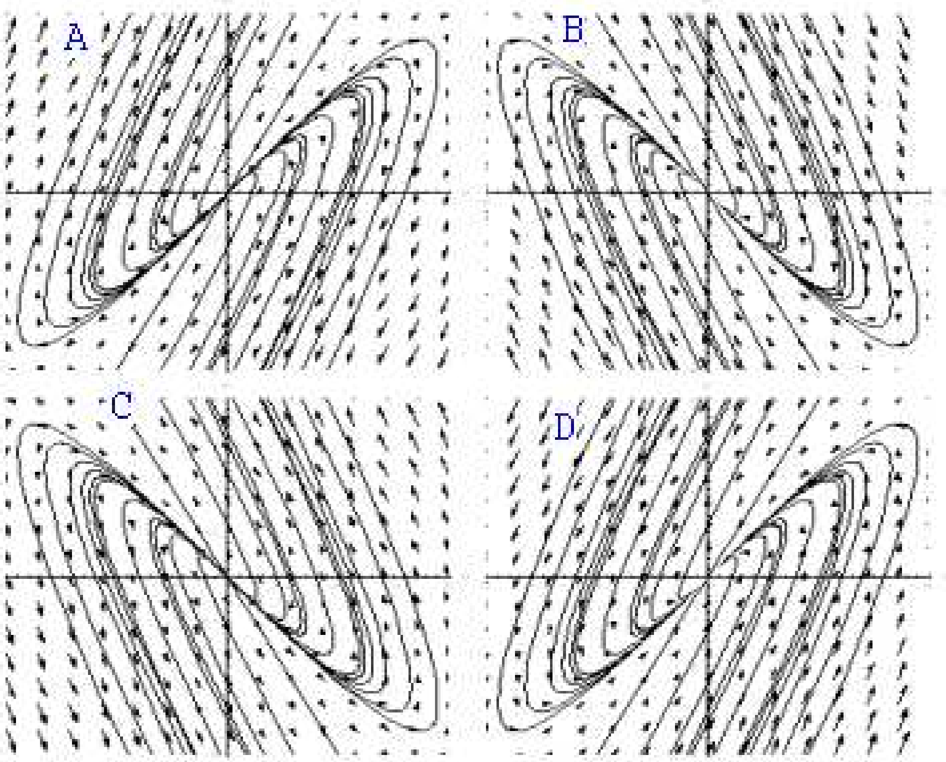
(b) $< 1, 3 >$

(c) $< 1, -2 >$

(d) There is no dominant eigenvector because there is no vector that is being approached by all of the solution curves.

(e) Not enough information is given

9. Which phase portrait below corresponds to the linear, homogeneous, system of differential equations with constant coefficients whose coefficient matrix has the following eigensystem: eigenvalues -5 and -2 and eigenvectors $< -1, 2 >$ and $< -4, 5 >$, respectively?



10. Classify the equilibrium point at the origin for the system

$$\frac{d\vec{Y}}{dt} = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix} \vec{Y}.$$

(a) Sink

(b) Source

(c) Saddle

(d) None of the above