

MathQuest: Differential Equations

Nonlinear Systems

1. The nonlinear system of differential equations given below has an equilibrium point at $(0, 0)$. Identify the system which represents a linear approximation of the nonlinear system around this point.

$$\begin{aligned}\frac{dx}{dt} &= y + x^2 \\ \frac{dy}{dt} &= -2y + \sin x\end{aligned}$$

(a)

$$\begin{aligned}\frac{dx}{dt} &= y + 2x \\ \frac{dy}{dt} &= -2y\end{aligned}$$

(b)

$$\begin{aligned}\frac{dx}{dt} &= y \\ \frac{dy}{dt} &= -2y\end{aligned}$$

(c)

$$\begin{aligned}\frac{dx}{dt} &= y + 2x \\ \frac{dy}{dt} &= -2y + x\end{aligned}$$

(d)

$$\begin{aligned}\frac{dx}{dt} &= y \\ \frac{dy}{dt} &= -2y + x\end{aligned}$$

2. For the nonlinear system given below, compute the Jacobian $J(x, y)$ that we associate to it.

$$\begin{aligned}\frac{dx}{dt} &= x + 2xy \\ \frac{dy}{dt} &= -2y + x^2\end{aligned}$$

(a)
$$J(x, y) = \begin{pmatrix} 1 + 2x & 2y \\ 2x & -2 \end{pmatrix}$$

(b)
$$J(x, y) = \begin{pmatrix} 1 + 2y & 2x \\ 2x & -2 \end{pmatrix}$$

(c)
$$J(x, y) = \begin{pmatrix} 2x & 1 + 2y \\ -2 & 2x \end{pmatrix}$$

(d)
$$J(x, y) = \begin{pmatrix} 2x & -2 \\ 1 + 2y & 2x \end{pmatrix}$$

3. The nonlinear system given below has an equilibrium point at $(0, 0)$. Classify this point.

$$\begin{aligned} \frac{dx}{dt} &= x + 2xy \\ \frac{dy}{dt} &= -2y + x^2 \end{aligned}$$

- (a) Sink
- (b) Source
- (c) Saddle
- (d) Spiral Sink
- (e) Spiral Source
- (f) Center