

MathQuest: Differential Equations

Introduction to Partial Differential Equations

1. Which of the following functions satisfies the equation $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = f$?
 - (a) $f = e^{x+y}$
 - (b) $f = 2x + 3y + 5$
 - (c) $f = x^{0.2}y^{0.8}$
 - (d) $f = x^2y^3$
 - (e) More than one of the above

2. Which of the following functions satisfies the equation $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = f$?
 - (a) $f = 18\sqrt{xy}$
 - (b) $f = 3x^{0.2}y^{0.8}$
 - (c) $f = 5x^{0.4}y^{0.6}$
 - (d) $f = -37x^{0.9}y^{0.1}$
 - (e) All of the above

3. Does the function $f = 3x$ satisfy the partial differential equation $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = f$?
 - (a) Yes
 - (b) No

4. Consider the heat diffusion equation: $\frac{\partial T}{\partial t} = \kappa\frac{\partial^2 T}{\partial y^2}$ where $T(y, t)$ is a function which describes the temperature T (in °F) at position y (in meters) at time t (in hours). What are the units of the constant κ , usually called the thermal diffusivity?
 - (a) meters squared per °F hour
 - (b) meters squared per hour
 - (c) °F per hour
 - (d) °F per meters squared

5. At time $t = 0$ the temperature of a concrete slab is described by the function $T(x) = 10 \cos \frac{\pi}{10}x + 3x + 40$ where $x = -10$ at one end of the slab and $x = +10$ at the other end. The way the temperature will change over time is controlled by the heat diffusion equation: $\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial x^2}$. At what rate is the temperature changing at $x = 0$ and $t = 0$ if $\kappa = 2$ meters squared per second?
- (a) -2π °F per second
 - (b) $-\frac{\pi^2}{5}$ °F per second
 - (c) $\frac{\pi}{50}$ °F per second
 - (d) $-\frac{\pi^2}{10}$ °F per second
 - (e) None of the above
6. For what value of c does the function $T(y, t) = 3e^{ct} \sin 2\pi y$ satisfy the heat diffusion equation $\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial y^2}$ where $\kappa = 4$?
- (a) $c = -16\pi^2$
 - (b) $c = 4$
 - (c) $c = 12\pi^2$
 - (d) $c = -6\pi$
 - (e) $c = -4\pi^2$
 - (f) None of the above
7. The function $T(y, t) = 2e^{-4t} \sin 5y$ is a solution of the heat diffusion equation $\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial y^2}$ where κ is in feet² per second. What are the units of the constants 2, 4, and 5?
- (a) 2 feet² per second, 4 inverse seconds, 5 radians
 - (b) 2° F per second, 4 seconds, 5 inverse feet
 - (c) 2° F, 4 inverse seconds, 5 radians per foot
 - (d) 2° F, 4 seconds, 5 feet per radian
 - (e) None of the above
8. We have solved the heat diffusion equation to find a function that describes the temperature of a concrete slab that extends from $x = -10$ to $x = +10$, finding that $T = Ae^{kt} \cos(Bx) + Cx + D$, where $A = -25$, $B = \frac{\pi}{20}$, $C = 80$, and $D = -2$. Where is the slab warmest?
- (a) In the center
 - (b) At $x = -10$

- (c) At $x = +10$
- (d) At another location
9. We want to find a function that describes the temperature evolution of a concrete slab that extends from $x = -15$ to $x = +15$, of the form $T = Ae^{kt} \cos(Bx) + C$. We know that initially the slab is hottest in the center, and that temperature decreases outward until it reaches the edges, which are both held at a fixed temperature. What value must B have?
- (a) $B = \frac{\pi}{60}$
- (b) $B = 15\pi^2$
- (c) $B = \frac{\pi}{30}$
- (d) $B = 30\pi$
- (e) None of the above
10. We are solving the heat diffusion equation to find a function that describes how the temperature of a concrete slab changes over time, and find that the function is of the form $T = Ae^{kt} \sin(Bx) + C$. We know that the ends of the slab (at $x = 0$ and $x = 20$ meters) are both always held at a temperature of 90 degrees, and the center of the slab starts out at a temperature of 50 degrees before it starts to warm up. What are the values of A , B , and C ?
- (a) $A = 40, B = \pi/10, C = 90$
- (b) $A = -40, B = \pi/20, C = 90$
- (c) $A = -20, B = 20\pi, C = 70$
- (d) $A = -40, B = \pi/10, C = 50$
11. The temperature of a concrete slab must satisfy the heat diffusion equation $\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial x^2}$ where $\kappa = 2$ meters squared per hour, with the boundary conditions that the temperature is always $60^\circ F$ at the origin and at $x = 10$ meters. Which of the following could be $T(x, t)$?
- (a) $T = 7e^{-t\pi^2 2/25} \sin \frac{\pi}{5}x + 60$
- (b) $T = -18e^{-t\pi^2 8/25} \sin \frac{2\pi}{5}x + 60$
- (c) $T = 60$
- (d) $T = 3.34e^{-t\pi^2 18/25} \sin \frac{3\pi}{5}x + 60$
- (e) All of the above

12. Suppose the function $T_0(x, t)$ solves the heat diffusion equation $\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial x^2}$. Which of the following transformations of this function would NOT also be a solution?
- (a) $T_1 = 3T_0$
 - (b) $T_2 = (T_0)^2$
 - (c) $T_3 = T_0 + 50$
 - (d) $T_4 = -T_0$
 - (e) All are solutions
13. Suppose we have two different functions $f(x, t)$ and $g(x, t)$ which each solve the heat diffusion equation $\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial x^2}$. Which of the following would NOT also be a solution?
- (a) $T = f + g$
 - (b) $T = 2f - 3g$
 - (c) $T = f + g + 10$
 - (d) $T = \frac{f}{g}$
 - (e) All are solutions
14. The wave equation $\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{C^2} \frac{\partial^2 \psi}{\partial t^2}$ allows us to understand the motion of water waves, sound waves, or light waves. If the function ψ gives us the height of the wave in inches as a function of position x in feet, and time t in seconds, what are the units of the constant C ?
- (a) feet squared per second squared
 - (b) seconds per foot
 - (c) feet per second
 - (d) inches per second
 - (e) None of the above
15. We conjecture that the function $\psi = A \sin(kx + \omega t)$ is a solution to the wave equation $\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{C^2} \frac{\partial^2 \psi}{\partial t^2}$. If $A = 10$, $k = 3$ and $C = 5$, what value could ω have?
- (a) $\omega = 15$
 - (b) $\omega = 150$
 - (c) $\omega = 3/5$
 - (d) $\omega = 9/25$
 - (e) $\omega = 225$

16. The function $\psi = A \sin(kx + \omega t)$ is a solution to the wave equation $\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{C^2} \frac{\partial^2 \psi}{\partial t^2}$. If $A = 10$, $k = 3$ and $\omega = -5$, which direction is the wave traveling?
- (a) In the $+x$ direction
 - (b) In the $-x$ direction
 - (c) Not enough information is given.