MathQuest: Differential Equations

Introduction to Partial Differential Equations

1. Which of the following functions satisfies the equation $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = f$?

   (a) $f = e^{x+y}$
   (b) $f = 2x + 3y + 5$
   (c) $f = x^{0.2}y^{0.8}$
   (d) $f = x^2y^3$
   (e) More than one of the above

2. Which of the following functions satisfies the equation $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = f$?

   (a) $f = 18\sqrt{xy}$
   (b) $f = 3x^{0.2}y^{0.8}$
   (c) $f = 5x^{0.4}y^{0.6}$
   (d) $f = -37x^{0.9}y^{0.1}$
   (e) All of the above

3. Does the function $f = 3x$ satisfy the partial differential equation $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = f$?

   (a) Yes
   (b) No

4. Consider the heat diffusion equation: $\frac{\partial T}{\partial t} = \kappa\frac{\partial^2 T}{\partial y^2}$ where $T(y,t)$ is a function which describes the temperature T (in °F) at position y (in meters) at time t (in hours). What are the units of the constant κ, usually called the thermal diffusivity?

   (a) meters squared per °F hour
   (b) meters squared per hour
   (c) °F per hour
   (d) °F per meters squared

5. At time $t = 0$ the temperature of a concrete slab is described by the function $T(x) = 10\cos\frac{\pi}{10}x + 3x + 40$ where $x = -10$ at one end of the slab and $x = +10$ at the other end. The way the temperature will change over time is controlled by the heat diffusion equation: $\frac{\partial T}{\partial t} = \kappa\frac{\partial^2 T}{\partial x^2}$. At what rate is the temperature changing at $x = 0$ and $t = 0$ if $\kappa = 2$ meters squared per second?

   (a) -2π °F per second
   (b) $-\frac{\pi^2}{5}$ °F per second
   (c) $\frac{\pi}{50}$ °F per second
   (d) $-\frac{\pi^2}{10}$ °F per second
   (e) None of the above

6. For what value of c does the function $T(y,t) = 3e^{ct}\sin 2\pi y$ satisfy the heat diffusion equation $\frac{\partial T}{\partial t} = \kappa\frac{\partial^2 T}{\partial y^2}$ where $\kappa = 4$?

   (a) $c = -16\pi^2$
   (b) $c = 4$
   (c) $c = 12\pi^2$
   (d) $c = -6\pi$
   (e) $c = -4\pi^2$
   (f) None of the above

7. The function $T(y,t) = 2e^{-4t}\sin 5y$ is a solution of the heat diffusion equation $\frac{\partial T}{\partial t} = \kappa\frac{\partial^2 T}{\partial y^2}$ where κ is in feet^2 per second. What are the units of the constants 2, 4, and 5?

   (a) 2 feet^2 per second, 4 inverse seconds, 5 radians
   (b) $2°F$ per second, 4 seconds, 5 inverse feet
   (c) $2°F$, 4 inverse seconds, 5 radians per foot
   (d) $2°F$, 4 seconds, 5 feet per radian
   (e) None of the above

8. We have solved the heat diffusion equation to find a function that describes the temperature of a concrete slab that extends from $x = -10$ to $x = +10$, finding that $T = Ae^{kt}\cos(Bx) + Cx + D$, where $A = -25$, $B = \frac{\pi}{20}$, $C = 80$, and $D = -2$. Where is the slab warmest?

   (a) In the center
   (b) At $x = -10$

(c) At $x = +10$

(d) At another location

9. We want to find a function that describes the temperature evolution of a concrete slab that extends from $x = -15$ to $x = +15$, of the form $T = Ae^{kt}\cos(Bx) + C$. We know that initially the slab is hottest in the center, and that temperature decreases outward until it reaches the edges, which are both held at a fixed temperature. What value must B have?

   (a) $B = \frac{\pi}{60}$

   (b) $B = 15\pi^2$

   (c) $B = \frac{\pi}{30}$

   (d) $B = 30\pi$

   (e) None of the above

10. We are solving the heat diffusion equation to find a function that describes how the temperature of a concrete slab changes over time, and find that the function is of the form $T = Ae^{kt}\sin(Bx) + C$. We know that the ends of the slab (at $x = 0$ and $x = 20$ meters) are both always held at a temperature of 90 degrees, and the center of the slab starts out at a temperature of 50 degrees before it starts to warm up. What are the values of A, B, and C?

    (a) $A = 40$, $B = \pi/10$, $C = 90$

    (b) $A = -40$, $B = \pi/20$, $C = 90$

    (c) $A = -20$, $B = 20\pi$, $C = 70$

    (d) $A = -40$, $B = \pi/10$, $C = 50$

11. The temperature of a concrete slab must satisfy the heat diffusion equation $\frac{\partial T}{\partial t} = \kappa\frac{\partial^2 T}{\partial x^2}$ where $\kappa = 2$ meters squared per hour, with the boundary conditions that the temperature is always $60°F$ at the origin and at $x = 10$ meters. Which of the following could be $T(x,t)$?

    (a) $T = 7e^{-t\pi^2 2/25}\sin\frac{\pi}{5}x + 60$

    (b) $T = -18e^{-t\pi^2 8/25}\sin\frac{2\pi}{5}x + 60$

    (c) $T = 60$

    (d) $T = 3.34e^{-t\pi^2 18/25}\sin\frac{3\pi}{5}x + 60$

    (e) All of the above

12. Suppose the function $T_0(x,t)$ solves the heat diffusion equation $\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial x^2}$. Which of the following transformations of this function would NOT also be a solution?

(a) $T_1 = 3T_0$

(b) $T_2 = (T_0)^2$

(c) $T_3 = T_0 + 50$

(d) $T_4 = -T_0$

(e) All are solutions

13. Suppose we have two different functions $f(x,t)$ and $g(x,t)$ which each solve the heat diffusion equation $\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial x^2}$. Which of the following would NOT also be a solution?

(a) $T = f + g$

(b) $T = 2f - 3g$

(c) $T = f + g + 10$

(d) $T = \frac{f}{g}$

(e) All are solutions

14. The wave equation $\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{C^2} \frac{\partial^2 \psi}{\partial t^2}$ allows us to understand the motion of water waves, sound waves, or light waves. If the function ψ gives us the height of the wave in inches as a function of position x in feet, and time t in seconds, what are the units of the constant C?

(a) feet squared per second squared

(b) seconds per foot

(c) feet per second

(d) inches per second

(e) None of the above

15. We conjecture that the function $\psi = A \sin(kx + \omega t)$ is a solution to the wave equation $\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{C^2} \frac{\partial^2 \psi}{\partial t^2}$. If $A = 10$, $k = 3$ and $C = 5$, what value could ω have?

(a) $\omega = 15$

(b) $\omega = 150$

(c) $\omega = 3/5$

(d) $\omega = 9/25$

(e) $\omega = 225$

16. The function $\psi = A\sin(kx + \omega t)$ is a solution to the wave equation $\frac{\partial^2\psi}{\partial x^2} = \frac{1}{C^2}\frac{\partial^2\psi}{\partial t^2}$. If $A = 10$, $k = 3$ and $\omega = -5$, which direction is the wave traveling?

(a) In the $+x$ direction

(b) In the $-x$ direction

(c) Not enough information is given.