

## MathQuest: Differential Equations

### Introduction to Partial Differential Equations

- Which of the following functions satisfies the equation  $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = f$ ?
  - $f = e^{x+y}$
  - $f = 2x + 3y + 5$
  - $f = x^{0.2}y^{0.8}$
  - $f = x^2y^3$
  - More than one of the above
  
- Which of the following functions satisfies the equation  $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = f$ ?
  - $f = 18\sqrt{xy}$
  - $f = 3x^{0.2}y^{0.8}$
  - $f = 5x^{0.4}y^{0.6}$
  - $f = -37x^{0.9}y^{0.1}$
  - All of the above
  
- Does the function  $f = 3x$  satisfy the partial differential equation  $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = f$ ?
  - Yes
  - No
  
- Consider the heat diffusion equation:  $\frac{\partial T}{\partial t} = \kappa\frac{\partial^2 T}{\partial y^2}$  where  $T(y, t)$  is a function which describes the temperature  $T$  (in °F) at position  $y$  (in meters) at time  $t$  (in hours). What are the units of the constant  $\kappa$ , usually called the thermal diffusivity?
  - meters squared per °F hour
  - meters squared per hour
  - °F per hour
  - °F per meters squared

5. At time  $t = 0$  the temperature of a concrete slab is described by the function  $T(x) = 10 \cos \frac{\pi}{10}x + 3x + 40$  where  $x = -10$  at one end of the slab and  $x = +10$  at the other end. The way the temperature will change over time is controlled by the heat diffusion equation:  $\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial x^2}$ . At what rate is the temperature changing at  $x = 0$  and  $t = 0$  if  $\kappa = 2$  meters squared per second?
- (a)  $-2\pi$  °F per second
  - (b)  $-\frac{\pi^2}{5}$  °F per second
  - (c)  $\frac{\pi}{50}$  °F per second
  - (d)  $-\frac{\pi^2}{10}$  °F per second
  - (e) None of the above
6. For what value of  $c$  does the function  $T(y, t) = 3e^{ct} \sin 2\pi y$  satisfy the heat diffusion equation  $\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial y^2}$  where  $\kappa = 4$ ?
- (a)  $c = -16\pi^2$
  - (b)  $c = 4$
  - (c)  $c = 12\pi^2$
  - (d)  $c = -6\pi$
  - (e)  $c = -4\pi^2$
  - (f) None of the above
7. The function  $T(y, t) = 2e^{-4t} \sin 5y$  is a solution of the heat diffusion equation  $\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial y^2}$  where  $\kappa$  is in feet<sup>2</sup> per second. What are the units of the constants 2, 4, and 5?
- (a) 2 feet<sup>2</sup> per second, 4 inverse seconds, 5 radians
  - (b) 2°F per second, 4 seconds, 5 inverse feet
  - (c) 2°F, 4 inverse seconds, 5 radians per foot
  - (d) 2°F, 4 seconds, 5 feet per radian
  - (e) None of the above
8. We have solved the heat diffusion equation to find a function that describes the temperature of a concrete slab that extends from  $x = -10$  to  $x = +10$ , finding that  $T = Ae^{kt} \cos(Bx) + Cx + D$ , where  $A = -25$ ,  $B = \frac{\pi}{20}$ ,  $C = 80$ , and  $D = -2$ . Where is the slab warmest?
- (a) In the center
  - (b) At  $x = -10$

- (c) At  $x = +10$
- (d) At another location
9. We want to find a function that describes the temperature evolution of a concrete slab that extends from  $x = -15$  to  $x = +15$ , of the form  $T = Ae^{kt} \cos(Bx) + C$ . We know that initially the slab is hottest in the center, and that temperature decreases outward until it reaches the edges, which are both held at a fixed temperature. What value must  $B$  have?
- (a)  $B = \frac{\pi}{60}$
- (b)  $B = 15\pi^2$
- (c)  $B = \frac{\pi}{30}$
- (d)  $B = 30\pi$
- (e) None of the above
10. We are solving the heat diffusion equation to find a function that describes how the temperature of a concrete slab changes over time, and find that the function is of the form  $T = Ae^{kt} \sin(Bx) + C$ . We know that the ends of the slab (at  $x = 0$  and  $x = 20$  meters) are both always held at a temperature of 90 degrees, and the center of the slab starts out at a temperature of 50 degrees before it starts to warm up. What are the values of  $A$ ,  $B$ , and  $C$ ?
- (a)  $A = 40, B = \pi/10, C = 90$
- (b)  $A = -40, B = \pi/20, C = 90$
- (c)  $A = -20, B = 20\pi, C = 70$
- (d)  $A = -40, B = \pi/10, C = 50$
11. The temperature of a concrete slab must satisfy the heat diffusion equation  $\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial x^2}$  where  $\kappa = 2$  meters squared per hour, with the boundary conditions that the temperature is always  $60^\circ F$  at the origin and at  $x = 10$  meters. Which of the following could be  $T(x, t)$ ?
- (a)  $T = 7e^{-t\pi^2 2/25} \sin \frac{\pi}{5}x + 60$
- (b)  $T = -18e^{-t\pi^2 8/25} \sin \frac{2\pi}{5}x + 60$
- (c)  $T = 60$
- (d)  $T = 3.34e^{-t\pi^2 18/25} \sin \frac{3\pi}{5}x + 60$
- (e) All of the above

12. Suppose the function  $T_0(x, t)$  solves the heat diffusion equation  $\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial x^2}$ . Which of the following transformations of this function would NOT also be a solution?
- (a)  $T_1 = 3T_0$
  - (b)  $T_2 = (T_0)^2$
  - (c)  $T_3 = T_0 + 50$
  - (d)  $T_4 = -T_0$
  - (e) All are solutions
13. Suppose we have two different functions  $f(x, t)$  and  $g(x, t)$  which each solve the heat diffusion equation  $\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial x^2}$ . Which of the following would NOT also be a solution?
- (a)  $T = f + g$
  - (b)  $T = 2f - 3g$
  - (c)  $T = f + g + 10$
  - (d)  $T = \frac{f}{g}$
  - (e) All are solutions
14. The wave equation  $\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{C^2} \frac{\partial^2 \psi}{\partial t^2}$  allows us to understand the motion of water waves, sound waves, or light waves. If the function  $\psi$  gives us the height of the wave in inches as a function of position  $x$  in feet, and time  $t$  in seconds, what are the units of the constant  $C$ ?
- (a) feet squared per second squared
  - (b) seconds per foot
  - (c) feet per second
  - (d) inches per second
  - (e) None of the above
15. We conjecture that the function  $\psi = A \sin(kx + \omega t)$  is a solution to the wave equation  $\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{C^2} \frac{\partial^2 \psi}{\partial t^2}$ . If  $A = 10$ ,  $k = 3$  and  $C = 5$ , what value could  $\omega$  have?
- (a)  $\omega = 15$
  - (b)  $\omega = 150$
  - (c)  $\omega = 3/5$
  - (d)  $\omega = 9/25$
  - (e)  $\omega = 225$

16. The function  $\psi = A \sin(kx + \omega t)$  is a solution to the wave equation  $\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{C^2} \frac{\partial^2 \psi}{\partial t^2}$ . If  $A = 10$ ,  $k = 3$  and  $\omega = -5$ , which direction is the wave traveling?

- (a) In the  $+x$  direction
- (b) In the  $-x$  direction
- (c) Not enough information is given.