MathQuest: Differential Equations

Laplace Tranforms

1. **True or False** The Laplace transform method is the only way to solve some types of differential equations.

   (a) True, and I am very confident
   (b) True, but I am not very confident
   (c) False, but I am not very confident
   (d) False, and I am very confident

2. Which of the following differential equations would be impossible to solve using the Laplace transform?

   (a) $\frac{dc}{dr} = 12c + 3\sin(2r) + 8\cos^2(3r + 2)$
   (b) $\frac{d^2f}{dx^2} - 100\frac{df}{dx} = \frac{18}{ln(x)}$
   (c) $g'(b) = \frac{12}{g}$
   (d) $p''(q) = \frac{4}{q} + 96p' - 12p$

3. Suppose we know that zero is an equilibrium value for a certain homogeneous linear differential equation with constant coefficients. Further, suppose that if we begin at equilibrium and we add the nonhomogenous driving term $f(t)$ to the equation, then the solution will be the function $y(t)$. **True or False**: If instead we add the nonhomogeous driving term $2f(t)$, then the solution will be the function $2y(t)$.

   (a) True, and I am very confident
   (b) True, but I am not very confident
   (c) False, but I am not very confident
   (d) False, and I am very confident

4. Suppose we know that zero is an equilibrium value for a certain homogeneous linear differential equation with constant coefficients. Further, suppose that if we begin at equilibrium and we add a nonhomogenous driving term $f(t)$ to the equation that acts only for an instant, at $t = t_1$, then the solution will be the function $y(t)$. **True or False**: Even if we know nothing about the differential equation itself, using our knowledge of $f(t)$ and $y(t)$ we can infer how the system must behave for any other nonhomogeneous driving function $g(t)$ and for any other initial condition.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

5. The Laplace transform of a function $y(t)$ is defined to be a function $Y(s)$ so that $\mathcal{L}[y(t)] = Y(s) = \int_0^\infty y(t)e^{-st}dt$. If we have the function $y(t) = e^{5t}$, then what is its Laplace transform $\mathcal{L}[y(t)] = Y(s)$?

   (a) $\mathcal{L}[y(t)] = \frac{1}{s-5}$ if $s > 5$.
   (b) $\mathcal{L}[y(t)] = \frac{1}{s-5}$ if $s < 5$.
   (c) $\mathcal{L}[y(t)] = \frac{1}{5-s}$ if $s > 5$.
   (d) $\mathcal{L}[y(t)] = \frac{1}{5-s}$ if $s < 5$.

6. What is the Laplace transform of the function $y(t) = 1$?

   (a) $\mathcal{L}[y(t)] = -\frac{1}{s}$ if $s > 0$.
   (b) $\mathcal{L}[y(t)] = -\frac{1}{s}$ if $s < 0$.
   (c) $\mathcal{L}[y(t)] = \frac{1}{s}$ if $s > 0$.
   (d) $\mathcal{L}[y(t)] = \frac{1}{s}$ if $s < 0$.
   (e) This function has no Laplace transform.

7. What is $\mathcal{L}[e^{3t}]$?

   (a) $\mathcal{L}[e^{3t}] = \frac{1}{3-s}$ if $s < 3$.
   (b) $\mathcal{L}[e^{3t}] = \frac{1}{s-3}$ if $s > 3$.
   (c) $\mathcal{L}[e^{3t}] = \frac{1}{s+3}$ if $s > 3$.
   (d) $\mathcal{L}[e^{3t}] = \frac{-1}{s+3}$ if $s < 3$.

8. Suppose we have the Laplace transform $Y(s) = \frac{1}{s-2}$ and we want the original function $y(t)$. What is $\mathcal{L}^{-1}\left[\frac{1}{s-2}\right]$?

   (a) $\mathcal{L}^{-1}\left[\frac{1}{s-2}\right] = e^{-2t}$
   (b) $\mathcal{L}^{-1}\left[\frac{1}{s-2}\right] = e^{2t}$
   (c) This cannot be done

9. Suppose we know that the Laplace transform of a particular function $y(t)$ is the function $Y(s)$ so that $\mathcal{L}[y(t)] = Y(s)$. Now, suppose we multiply this function by 5 and take the Laplace transform. What can we say about $\mathcal{L}[5y(t)]$?

   (a) $\mathcal{L}[5y(t)] = 5Y(s)$.
   (b) $\mathcal{L}[5y(t)] = -5Y(s)$.
   (c) $\mathcal{L}[5y(t)] = -\frac{1}{5}Y(s)$.
   (d) $\mathcal{L}[5y(t)] = \frac{1}{5}e^{-5}Y(s)$.
   (e) None of the above

10. Suppose we know that the Laplace transform of a particular function $f(t)$ is the function $W(s)$ and that the Laplace transform of another function $g(t)$ is the function $X(s)$ so that $\mathcal{L}[f(t)] = W(s)$ and $\mathcal{L}[g(t)] = X(s)$. Now, suppose we multiply these two functions together and take the Laplace transform. What can we say about $\mathcal{L}[f(t)g(t)]$?

   (a) $\mathcal{L}[f(t)g(t)] = W(s)X(s)$.
   (b) $\mathcal{L}[f(t)g(t)] = \frac{W(s)}{X(s)}$.
   (c) $\mathcal{L}[f(t)g(t)] = W(s) + X(s)$.
   (d) $\mathcal{L}[f(t)g(t)] = W(s) - X(s)$.
   (e) None of the above

11. Suppose we know that the Laplace transform of a particular function $f(t)$ is the function $W(s)$ and that the Laplace transform of another function $g(t)$ is the function $X(s)$ so that $\mathcal{L}[f(t)] = W(s)$ and $\mathcal{L}[g(t)] = X(s)$. Now, suppose we add these two functions together and take the Laplace transform. What can we say about $\mathcal{L}[f(t) + g(t)]$?

   (a) $\mathcal{L}[f(t) + g(t)] = W(s)X(s)$.
   (b) $\mathcal{L}[f(t) + g(t)] = \frac{W(s)}{X(s)}$.
   (c) $\mathcal{L}[f(t) + g(t)] = W(s) + X(s)$.
   (d) $\mathcal{L}[f(t) + g(t)] = W(s) - X(s)$.
   (e) None of the above

12. Suppose we have $Y(s) = \frac{1}{s-2} + \frac{5}{s+6}$ and we want the function $y(t)$. What is $\mathcal{L}^{-1}\left[\frac{1}{s-2} + \frac{5}{s+6}\right]$?

   (a) $\mathcal{L}^{-1}\left[\frac{1}{s-2} + \frac{5}{s+6}\right] = e^{2t} + e^{6t/5}$
   (b) $\mathcal{L}^{-1}\left[\frac{1}{s-2} + \frac{5}{s+6}\right] = e^{2t} + 5e^{-6t}$
   (c) $\mathcal{L}^{-1}\left[\frac{1}{s-2} + \frac{5}{s+6}\right] = 5e^{2t}e^{-6t}$

(d) $\mathcal{L}^{-1}\left[\frac{1}{s-2}+\frac{5}{s+6}\right]=\frac{e^{2t}e^{6t}}{5}$

(e) This cannot be done

13. Suppose we know that $\mathcal{L}[y(t)]=Y(s)$ and we want to take the Laplace transform of the derivative of $y(t)$, $\mathcal{L}\left[\frac{dy}{dt}\right]$. To get started, recall the method of integration by parts, which tells us that $\int u\,dv = uv - \int v\,du$.

    (a) $\mathcal{L}\left[\frac{dy}{dt}\right]=y(t)e^{-st}+sY(s)$.

    (b) $\mathcal{L}\left[\frac{dy}{dt}\right]=y(t)e^{-st}-sY(s)$.

    (c) $\mathcal{L}\left[\frac{dy}{dt}\right]=y(0)+sY(s)$.

    (d) $\mathcal{L}\left[\frac{dy}{dt}\right]=y(0)-sY(s)$.

    (e) $\mathcal{L}\left[\frac{dy}{dt}\right]=-y(0)+sY(s)$.

14. We know that $\mathcal{L}[y(t)]=Y(s)=\int_0^\infty y(t)e^{-st}dt$ and that $\mathcal{L}\left[\frac{dy}{dt}\right]=-y(0)+sY(s)$. What do we get when we take the Laplace transform of the differential equation $\frac{dy}{dt}=2y+3e^{-t}$ with $y(0)=2$?

    (a) $2+sY(s)=2Y(s)+\frac{3}{s+1}$

    (b) $2+sY(s)=\frac{1}{2}Y(s)+\frac{3}{s-1}$

    (c) $-2+sY(s)=2Y(s)+\frac{3}{s+1}$

    (d) $-2+sY(s)=2Y(s)-\frac{3}{s+1}$

    (e) $2-sY(s)=\frac{1}{2}Y(s)+\frac{3}{1-s}$

15. We know that $\mathcal{L}[y(t)]=Y(s)$ and that $\mathcal{L}\left[\frac{dy}{dt}\right]=-y(0)+sY(s)$. Take the Laplace transform of the differential equation $\frac{dy}{dt}=5y+2e^{-3t}$ with $y(0)=4$ and solve for $Y(s)$.

    (a) $Y(s)=\frac{2}{(-s-3)(s-5)}+\frac{4}{s-5}$

    (b) $Y(s)=\frac{2}{(-s-3)(s-5)}+\frac{4}{s+5}$

    (c) $Y(s)=\frac{2}{(s+3)(s-5)}+\frac{4}{s-5}$

    (d) $Y(s)=\frac{2}{(s+3)(s+5)}+\frac{4}{s+5}$

16. Suppose that after taking the Laplace transform of a differential equation we solve for the function $Y(s)=\frac{3}{(s+2)(s-6)}$. This is equivalent to which of the following?

    (a) $Y(s)=\frac{-3/8}{s+2}+\frac{3/8}{s-6}$

(b) $Y(s) = \frac{-8/3}{s+2} + \frac{8/3}{s-6}$

(c) $Y(s) = \frac{3/8}{s+2} + \frac{-3/8}{s-6}$

(d) $Y(s) = \frac{8/3}{s+2} + \frac{-8/3}{s-6}$

17. Find $\mathcal{L}^{-1}\left[\frac{3}{(s+2)(s-6)}\right]$.

(a) $\mathcal{L}^{-1}\left[\frac{3}{(s+2)(s-6)}\right] = 3e^{-2t}e^{6t}$

(b) $\mathcal{L}^{-1}\left[\frac{3}{(s+2)(s-6)}\right] = -\frac{3}{8}e^{-2t} + \frac{3}{8}e^{6t}$

(c) $\mathcal{L}^{-1}\left[\frac{3}{(s+2)(s-6)}\right] = \frac{3}{8}e^{2t} - \frac{3}{8}e^{-6t}$

(d) $\mathcal{L}^{-1}\left[\frac{3}{(s+2)(s-6)}\right] = e^{-3/8}e^{-2t} + e^{3/8}e^{6t}$

(e) $\mathcal{L}^{-1}\left[\frac{3}{(s+2)(s-6)}\right] = -\frac{3}{8}e^{2t} + \frac{3}{8}e^{-6t}$

18. Solve the differential equation $\frac{dy}{dt} = -4y + 3e^{2t}$ with $y(0) = 5$ using the method of Laplace transforms: First take the Laplace transform of the entire equation, then solve for $Y(s)$, use the method of partial fractions to simplify the result, and take the inverse transform to get the function $y(t)$.

(a) $y(t) = \frac{9}{2}e^{4t} + \frac{1}{2}e^{-2t}$

(b) $y(t) = \frac{7}{2}e^{4t} + \frac{3}{2}e^{-2t}$

(c) $y(t) = \frac{9}{2}e^{-4t} + \frac{1}{2}e^{2t}$

(d) $y(t) = \frac{7}{2}e^{-4t} + \frac{3}{2}e^{2t}$

19. Consider the Heaviside function, which is defined as

$$u_a(t) = \begin{cases} 0 & \text{if } t < a \\ 1 & \text{if } t \geq a \end{cases}$$

Suppose $a = 2$. What is the Laplace transform of this $\mathcal{L}[u_2(t)]$? Hint: It may be helpful to break this integral into two pieces, from 0 to 2 and from 2 to ∞.

(a) $\mathcal{L}[u_2(t)] = -\frac{1}{s}e^{2s} + \frac{1}{s}$

(b) $\mathcal{L}[u_2(t)] = \frac{1}{s}e^{-2s} - \frac{1}{s}$

(c) $\mathcal{L}[u_2(t)] = -\frac{1}{s}e^{-2s}$

(d) $\mathcal{L}[u_2(t)] = \frac{1}{s}e^{-2s}$

(e) $\mathcal{L}[u_2(t)] = \frac{1}{s}e^{2s}$

20. Find the Laplace transform of the function $u_5(t)e^{-(t-5)}$.

    (a) $\mathcal{L}[u_5(t)e^{-(t-5)}] = \frac{e^5}{s+1}$
    (b) $\mathcal{L}[u_5(t)e^{-(t-5)}] = \frac{e^{-5s}}{s+1}$
    (c) $\mathcal{L}[u_5(t)e^{-(t-5)}] = \frac{e^5 e^{-5s}}{s+1}$
    (d) $\mathcal{L}[u_5(t)e^{-(t-5)}] = \frac{1}{s+1}$

21. Find the Laplace transform of the function $u_4(t)e^{3(t-4)}$.

    (a) $\mathcal{L}[u_4(t)e^{3(t-4)}] = \frac{e^{-3s}}{s-4}$
    (b) $\mathcal{L}[u_4(t)e^{3(t-4)}] = \frac{e^{3s}}{s+4}$
    (c) $\mathcal{L}[u_4(t)e^{3(t-4)}] = \frac{e^{-4s}}{s+3}$
    (d) $\mathcal{L}[u_4(t)e^{3(t-4)}] = \frac{e^{-4s}}{s-3}$
    (e) $\mathcal{L}[u_4(t)e^{3(t-4)}] = \frac{e^{4s}}{s-3}$

22. Find $\mathcal{L}^{-1}\left[\frac{e^{-3s}}{s+2}\right]$.

    (a) $\mathcal{L}^{-1}\left[\frac{e^{-3s}}{s+2}\right] = u_2(t)e^{-2(t-3)}$
    (b) $\mathcal{L}^{-1}\left[\frac{e^{-3s}}{s+2}\right] = u_2(t)e^{-3(t-2)}$
    (c) $\mathcal{L}^{-1}\left[\frac{e^{-3s}}{s+2}\right] = u_2(t)e^{-3(t-3)}$
    (d) $\mathcal{L}^{-1}\left[\frac{e^{-3s}}{s+2}\right] = u_3(t)e^{-2(t-3)}$
    (e) $\mathcal{L}^{-1}\left[\frac{e^{-3s}}{s+2}\right] = u_3(t)e^{-2(t-2)}$
    (f) $\mathcal{L}^{-1}\left[\frac{e^{-3s}}{s+2}\right] = u_3(t)e^{-3(t-2)}$

23. Use Laplace transforms to solve the differential equation $\frac{dy}{dt} = -2y + 4u_2(t)$ with $y(0) = 1$.

    (a) $y(t) = e^{-2t} + 2u_2(t)e^{-(t-2)} - 2u_2(t)e^{-2(t-2)}$
    (b) $y(t) = e^{-2t} + 4u_2(t)e^{-(t-2)} - 4u_2(t)e^{-2(t-2)}$
    (c) $y(t) = e^{-2t} + 2u_2(t) - 2u_2(t)e^{-2(t-2)}$
    (d) $y(t) = e^{-2t} + 4u_2(t) - 4u_2(t)e^{-2(t-2)}$

24. What is the Laplace transform of the second derivative, $\mathcal{L}\left[\frac{d^2y}{dt^2}\right]$?

(a) $\mathcal{L}\left[\frac{d^2y}{dt^2}\right] = -y'(0) + s\mathcal{L}[y]$

(b) $\mathcal{L}\left[\frac{d^2y}{dt^2}\right] = -y'(0) - sy(0) + s^2\mathcal{L}[y]$

(c) $\mathcal{L}\left[\frac{d^2y}{dt^2}\right] = -y'(0) - sy(0) + s\mathcal{L}[y]$

(d) $\mathcal{L}\left[\frac{d^2y}{dt^2}\right] = -y'(0) + sy(0) - s^2\mathcal{L}[y]$

25. We know that the differential equation $y'' = -y$ with the initial condition $y(0) = 0$ and $y'(0) = 1$ is solved by the function $y = \sin t$. By taking the Laplace transform of this equation find an expression for $\mathcal{L}[\sin t]$.

(a) $\mathcal{L}[\sin t] = \frac{1}{s^2+1}$

(b) $\mathcal{L}[\sin t] = \frac{1}{s^2-1}$

(c) $\mathcal{L}[\sin t] = \frac{s}{s^2+1}$

(d) $\mathcal{L}[\sin t] = \frac{s}{s^2-1}$

26. Find an expression for $\mathcal{L}[\sin \omega t]$, by first using your knowledge of the second order differential equation and initial conditions $y(0)$ and $y'(0)$ that are solved by this function, and then taking the Laplace transform of this equation.

(a) $\mathcal{L}[\sin \omega t] = \frac{\omega}{s^2+1}$

(b) $\mathcal{L}[\sin \omega t] = \frac{1}{s^2\omega^2+\omega^2}$

(c) $\mathcal{L}[\sin \omega t] = \frac{1}{s^2+\omega^2}$

(d) $\mathcal{L}[\sin \omega t] = \frac{\omega}{s^2+\omega}$

(e) $\mathcal{L}[\sin \omega t] = \frac{\omega}{s^2+\omega^2}$

27. Find an expression for $\mathcal{L}[\cos \omega t]$, by first using your knowledge of the second order differential equation and initial conditions $y(0)$ and $y'(0)$ that are solved by this function, and then taking the Laplace transform of this equation.

(a) $\mathcal{L}[\cos \omega t] = \frac{s}{s^2+\omega^2}$

(b) $\mathcal{L}[\cos \omega t] = \frac{s\omega}{s^2+\omega^2}$

(c) $\mathcal{L}[\cos \omega t] = \frac{s\omega}{s^2-\omega^2}$

(d) $\mathcal{L}[\cos \omega t] = \frac{\omega}{s^2+\omega^2}$

(e) $\mathcal{L}[\cos \omega t] = \frac{\omega}{s^2-\omega^2}$

28. Find $\mathcal{L}^{-1}\left[\frac{3s}{4+s^2}\right]$.

(a) $\mathcal{L}^{-1}\left[\frac{3s}{4+s^2}\right] = 3\cos 2t$.
(b) $\mathcal{L}^{-1}\left[\frac{3s}{4+s^2}\right] = 3\cos 4t$.
(c) $\mathcal{L}^{-1}\left[\frac{3s}{4+s^2}\right] = 4\cos 3t$.
(d) $\mathcal{L}^{-1}\left[\frac{3s}{4+s^2}\right] = 3\sin 2t$.
(e) $\mathcal{L}^{-1}\left[\frac{3s}{4+s^2}\right] = 3\sin 4t$.
(f) $\mathcal{L}^{-1}\left[\frac{3s}{4+s^2}\right] = 4\sin 3t$.

29. Find $\mathcal{L}^{-1}\left[\frac{5}{2s^2+6}\right]$.

(a) $\mathcal{L}^{-1}\left[\frac{5}{2s^2+6}\right] = 5\sin\sqrt{3}t$.
(b) $\mathcal{L}^{-1}\left[\frac{5}{2s^2+6}\right] = 5\sin\sqrt{6}t$.
(c) $\mathcal{L}^{-1}\left[\frac{5}{2s^2+6}\right] = \frac{5}{2}\sin\sqrt{3}t$.
(d) $\mathcal{L}^{-1}\left[\frac{5}{2s^2+6}\right] = \frac{5}{2}\sin\sqrt{6}t$.
(e) $\mathcal{L}^{-1}\left[\frac{5}{2s^2+6}\right] = \frac{5}{2\sqrt{3}}\sin\sqrt{3}t$.

30. We know that the function $f(t)$ has a Laplace transform $F(s)$, so that $\mathcal{L}[f(t)] = \int_0^\infty f(t)e^{-st}dt = F(s)$. What can we say about the Laplace transform of the product $\mathcal{L}[e^{at}f(t)]$?

(a) $\mathcal{L}[e^{at}f(t)] = e^{at}F(s)$.
(b) $\mathcal{L}[e^{at}f(t)] = e^{as}F(s)$.
(c) $\mathcal{L}[e^{at}f(t)] = F(s-a)$.
(d) $\mathcal{L}[e^{at}f(t)] = F(s+a)$.
(e) We cannot make a general statement about this Laplace transform without knowing $f(t)$.

31. Find $\mathcal{L}[3e^{4t}\sin 5t]$.

(a) $\mathcal{L}[3e^{4t}\sin 5t] = \frac{3}{(s-4)^2+25}$.
(b) $\mathcal{L}[3e^{4t}\sin 5t] = \frac{15}{(s-4)^2+25}$.
(c) $\mathcal{L}[3e^{4t}\sin 5t] = \frac{15}{(s+4)^2+25}$.
(d) $\mathcal{L}[3e^{4t}\sin 5t] = \frac{3\sqrt{5}}{(s-4)^2+5}$.

(e) $\mathcal{L}[3e^{4t}\sin 5t] = \frac{3\sqrt{5}}{(s+4)^2+5}$.

32. Find $\mathcal{L}^{-1}\left[\frac{8}{(s+3)^2+16}\right]$.

(a) $\mathcal{L}^{-1}\left[\frac{8}{(s+3)^2+16}\right] = 2e^{-3t}\sin 4t$

(b) $\mathcal{L}^{-1}\left[\frac{8}{(s+3)^2+16}\right] = 2e^{-4t}\sin 3t$

(c) $\mathcal{L}^{-1}\left[\frac{8}{(s+3)^2+16}\right] = 8e^{-3t}\sin 4t$

(d) $\mathcal{L}^{-1}\left[\frac{8}{(s+3)^2+16}\right] = 2e^{3t}\sin 4t$

33. $s^2+6s+15$ is equivalent to which of the following?

(a) $s^2+6s+15 = (s+2)^2+11$

(b) $s^2+6s+15 = (s+3)^2+6$

(c) $s^2+6s+15 = (s+5)^2-10$

(d) $s^2+6s+15 = (s+6)^2-21$

34. Find $\mathcal{L}^{-1}\left[\frac{2s+6}{s^2+6s+15}\right]$, by first completing the square in the denominator.

(a) $\mathcal{L}^{-1}\left[\frac{2s+6}{s^2+6s+15}\right] = 2e^{-3t}\sin\sqrt{6}t$

(b) $\mathcal{L}^{-1}\left[\frac{2s+6}{s^2+6s+15}\right] = 2e^{3t}\cos\sqrt{6}t$

(c) $\mathcal{L}^{-1}\left[\frac{2s+6}{s^2+6s+15}\right] = 2e^{-3t}\cos\sqrt{6}t$

(d) $\mathcal{L}^{-1}\left[\frac{2s+6}{s^2+6s+15}\right] = 2e^{3t}\sin\sqrt{6}t$

(e) $\mathcal{L}^{-1}\left[\frac{2s+6}{s^2+6s+15}\right] = e^{6t}\cos\sqrt{6}t$

(f) $\mathcal{L}^{-1}\left[\frac{2s+6}{s^2+6s+15}\right] = e^{-6t}\cos\sqrt{6}t$

35. Find $\mathcal{L}^{-1}\left[\frac{2s+8}{s^2+6s+15}\right]$, by first completing the square in the denominator.

(a) $\mathcal{L}^{-1}\left[\frac{2s+8}{s^2+6s+15}\right] = 2e^{-3t}\cos\sqrt{6}t$

(b) $\mathcal{L}^{-1}\left[\frac{2s+8}{s^2+6s+15}\right] = 2e^{-3t}\cos\sqrt{6}t + 2$

(c) $\mathcal{L}^{-1}\left[\frac{2s+8}{s^2+6s+15}\right] = 2e^{-3t}\cos\sqrt{6}t + 2e^{-3t}\sin\sqrt{6}t$

(d) $\mathcal{L}^{-1}\left[\frac{2s+8}{s^2+6s+15}\right] = 2e^{-3t}\cos\sqrt{6}t + \frac{2}{\sqrt{6}}e^{-3t}\sin\sqrt{6}t$

36. Suppose that after taking the Laplace transform of a differential equation we obtain the function $Y(s) = \frac{2s}{(s^2+16)(s^2+3)}$. This is equivalent to which of the following?

(a) $Y(s) = \frac{2s/19}{s^2+16} + \frac{2s/19}{s^2+3}$
(b) $Y(s) = \frac{-2s/19}{s^2+16} + \frac{2s/19}{s^2+3}$
(c) $Y(s) = \frac{2s/13}{s^2+16} + \frac{-2s/13}{s^2+3}$
(d) $Y(s) = \frac{-2s/13}{s^2+16} + \frac{2s/13}{s^2+3}$

37. Find $\mathcal{L}^{-1}\left[\frac{2s}{(s^2+16)(s^2+3)}\right]$.

(a) $\mathcal{L}^{-1}\left[\frac{2s}{(s^2+16)(s^2+3)}\right] = -\frac{2}{13}\cos 4t + \frac{2}{13}\cos\sqrt{3}t.$
(b) $\mathcal{L}^{-1}\left[\frac{2s}{(s^2+16)(s^2+3)}\right] = \frac{2}{13}\cos 4t - \frac{2}{13}\cos\sqrt{3}t.$
(c) $\mathcal{L}^{-1}\left[\frac{2s}{(s^2+16)(s^2+3)}\right] = -\frac{2}{52}\sin 4t + \frac{2}{13\sqrt{3}}\sin\sqrt{3}t.$
(d) $\mathcal{L}^{-1}\left[\frac{2s}{(s^2+16)(s^2+3)}\right] = \frac{2}{52}\sin 4t - \frac{2}{13\sqrt{3}}\sin\sqrt{3}t.$

38. Use Laplace transforms to solve the differential equation $y'' = -2y + 4\sin 3t$ if we know that $y(0) = y'(0) = 0$.

(a) $y = -\frac{6\sqrt{2}}{7}\sin\sqrt{2}t + \frac{4}{7}\sin 3t$
(b) $y = \frac{6\sqrt{2}}{7}\sin\sqrt{2}t - \frac{4}{7}\sin 3t$
(c) $y = \frac{12}{7}\sin\sqrt{2}t - \frac{12}{7}\sin 3t$
(d) $y = -\frac{12}{7}\cos\sqrt{2}t + \frac{12}{7}\cos 3t$

39. The Dirac delta function $\delta_a(t)$ is defined so that $\delta_a(t) = 0$ for all $t \neq a$, and if we integrate this function over any integral containing a, the result is 1. What would be $\int_5^{10} (\delta_3(t) + 2\delta_6(t) - 3\delta_8(t) + 5\delta_{11}(t))\,dt$?

(a) -1
(b) 0
(c) 4
(d) 5
(e) -30
(f) This integral cannot be determined.

40. Find $\int_5^{10} (\delta_6(t) \cdot \delta_7(t) \cdot \delta_8(t) \cdot \delta_9(t))\, dt$?

(a) -1
(b) 0
(c) 1
(d) 4
(e) 5
(f) This integral cannot be determined.

41. Find $\mathcal{L}\left[\delta_7(t)\right]$.

(a) 1
(b) 7
(c) e^{-7s}
(d) e^{-7t}
(e) This integral cannot be determined.

42. Find $\mathcal{L}\left[\delta_0(t)\right]$.

(a) 0
(b) 1
(c) e^{-s}
(d) e^{-t}
(e) This integral cannot be determined.

43. We have a system modeled as an undamped harmonic oscillator, that begins at equilibrium and at rest, so $y(0) = y'(0) = 0$, and that receives an impulse force at $t = 4$, so that it is modeled with the equation $y'' = -9y + \delta_4(t)$. Find $y(t)$.

(a) $y(t) = u_4(t) \sin 3t$
(b) $y(t) = \frac{1}{3} u_4(t) \sin 3t$
(c) $y(t) = u_4(t) \sin 3(t-4)$
(d) $y(t) = \frac{1}{3} u_4(t) \sin 3(t-4)$

44. We have a system modeled as an undamped harmonic oscillator, that begins at equilibrium and at rest, so $y(0) = y'(0) = 0$, and that receives an impulse forcing at $t = 5$, so that it is modeled with the equation $y'' = -4y + \delta_5(t)$. Find $y(t)$.

(a) $y(t) = u_5(t)\sin 2t$
(b) $y(t) = \frac{1}{2}u_5(t)\sin 2(t-5)$
(c) $y(t) = \frac{1}{2}u_2(t)\sin 2(t-5)$
(d) $y(t) = \frac{1}{5}u_2(t)\sin 5(t-2)$
(e) $y(t) = \frac{1}{5}u_5(t)\sin 2(t-2)$

45. For many differential equations, if we know how the equation responds to a nonhomogeneous forcing function that is a Dirac delta function, we can use this response function to predict how the differential equation will respond to any other forcing function and any initial conditions. For which of the following differential equations would this be impossible?

(a) $p''(q) = \frac{4}{q} + 96p' - 12p$
(b) $\frac{d^2m}{dn^2} - 100\frac{dm}{dn} + 14nm = \frac{18}{\ln(n)}$
(c) $\frac{d^2f}{dx^2} - 100\frac{df}{dx} = \frac{18}{\ln(x)}$
(d) $\frac{dc}{dr} = 12c + 3\sin(2r) + 8\cos^2(3r+2)$

46. The convolution of two functions $f * g$ is defined to be $\int_0^t f(t-u)g(u)du$. What is the convolution of the functions $f(t) = 2$ and $g(t) = e^{2t}$?

(a) $f * g = e^{2t} - 1$
(b) $f * g = 2e^{2t} - 2$
(c) $f * g = 4e^{2t} - 4$
(d) $f * g = \frac{1}{2}e^{2t} - \frac{1}{2}$
(e) This convolution cannot be computed.

47. What is the convolution of the functions $f(t) = u_2(t)$ and $g(t) = e^{-3t}$?

(a) $f * g = \frac{1}{3}e^{-3(t-2)} - \frac{1}{3}$
(b) $f * g = \frac{1}{3}e^{-3t} - \frac{1}{3}$
(c) $f * g = -\frac{1}{3}e^{-3(t-2)} + \frac{1}{3}$
(d) $f * g = -\frac{1}{3}e^{-3t} + \frac{1}{3}$
(e) This convolution cannot be computed.

48. Let $\zeta(t)$ be the solution to the initial-value problem $\frac{d^2y}{dt^2} + p\frac{dy}{dt} + qy = \delta_0(t)$, with $y(0) = y'(0) = 0^-$. Find an expression for $\mathcal{L}[\zeta]$.

(a) $\mathcal{L}[\zeta] = 0$

(b) $\mathcal{L}[\zeta] = s^2 + ps + q$

(c) $\mathcal{L}[\zeta] = \frac{1}{s^2+ps+q}$

(d) $\mathcal{L}[\zeta] = \frac{e^{-s}}{s^2+ps+q}$

49. Let $\zeta(t) = 2e^{-3t}$ be the solution to an initial-value problem $\frac{d^2y}{dt^2} + p\frac{dy}{dt} + qy = \delta_0(t)$, with $y(0) = y'(0) = 0^-$ for some specific values of p and q, so that $\mathcal{L}[\zeta] = \frac{1}{s^2+ps+q}$. What will be the solution of $\frac{d^2y}{dt^2} + p\frac{dy}{dt} + qy = 0$ if $y(0) = 0$ and $y'(0) = 5$?

(a) $y = 2e^{-3t}$

(b) $y = 10e^{-3t}$

(c) $y = \frac{2}{5}e^{-3t}$

(d) $y = 2e^{-15t}$

(e) It cannot be determined from the information given.

50. Let $\zeta(t)$ be the solution to the initial-value problem $\frac{d^2y}{dt^2} + p\frac{dy}{dt} + qy = \delta_0(t)$, with $y(0) = y'(0) = 0^-$. Now suppose we want to solve this problem for some other forcing function $f(t)$, so that we have $\frac{d^2y}{dt^2} + p\frac{dy}{dt} + qy = f(t)$. Which of the following is a correct expression for $\mathcal{L}[y]$?

(a) $\mathcal{L}[y] = 0$.

(b) $\mathcal{L}[y] = \mathcal{L}[f]\mathcal{L}[\zeta]$.

(c) $\mathcal{L}[y] = \frac{\mathcal{L}[f]}{\mathcal{L}[\zeta]}$.

(d) $\mathcal{L}[y] = \mathcal{L}[f\zeta]$.

(e) None of the above

51. Let $\zeta(t) = e^{-3t}$ be the solution to an initial-value problem $\frac{d^2y}{dt^2} + p\frac{dy}{dt} + qy = \delta_0(t)$, with $y(0) = y'(0) = 0^-$. Now suppose we want to solve this problem for the forcing function $f(t) = e^{-2t}$, so that we have $\frac{d^2y}{dt^2} + p\frac{dy}{dt} + qy = f(t)$. Find $y(t)$.

(a) $y(t) = e^{-2t} - e^{-3t}$

(b) $y(t) = e^{-3t} - e^{-2t}$

(c) $y(t) = \frac{e^{-2t} - e^{-3t}}{5}$

(d) $y(t) = \frac{e^{2t} - e^{-3t}}{5}$