What is a Differential Equation?

1. Which of the following is not a differential equation?

   (a) $y' = 3y$

   (b) $2x^2y + y^2 = 6$

   (c) $tx\frac{dx}{dt} = 2$

   (d) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 7y + 8x = 0$

   (e) All are differential equations.

   Answer: (b). This is not a differential equation, because it does not contain the derivative of a function.

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   WH GC MAT345 S08: 0/**90**/10/0/0
   CC HZ MA232 S08: 16/**61**/3/0/19 time 1:00
   HHS JG MA232 S08: 0/**50**/0/0/50
   HC AS MA304 S08: 3/**69**/0/0/28

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2. Which of the following is not a differential equation?

   (a) $6\frac{dy}{dx} + 3xy$

   (b) $8 = \frac{y'}{y}$

   (c) $2\frac{d^2f}{dt^2} + 7\frac{df}{dt} = f$

   (d) $h(x) + 2h'(x) = g(x)$

   (e) All are differential equations.

   Answer: (a). This is not a differential equation because it is has no equal sign, so it is not an equation.

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   WH GC MAT345 S08: **30**/0/0/0/70
   HHS JG MA232 S08: **0**/0/0/0/100
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3. Which of the following couldn't be the solution of a differential equation?

   (a) $z(t) = 6$
   (b) $y = 3x^2 + 7$
   (c) $x = 0$
   (d) $y = 3x + y'$
   (e) All could be solutions of a differential equation.

   Answer: (d). This is a differential equation, because it contains a derivative. The solution of a differential equation is a function, and all the other options are functions. The post-vote discussion usually brings out issues that students have with answers (a) and (c). Some do not think (a) can be a solution because it is just a constant, and many believe that $x = 0$ is not a function because the graph is a vertical line. Students forget that x could be the dependent variable.

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   CHS DP MA232 S08: 0/0/77/**23**/0
   HHS JG MA232 S08: 0/0/0/**100**/0

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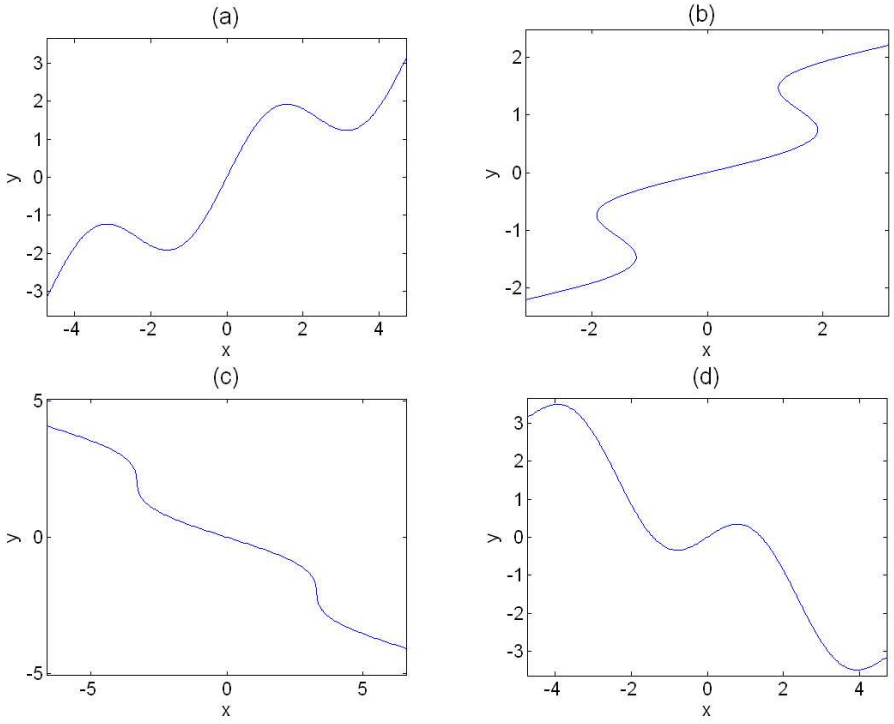
4. Which of the following could not be a solution of a differential equation?



Answer: (b). A solution of a differential equation is a function, and this is not a function because it has multiple y values for a given x value. It is possible that this graph might contain multiple branches of solutions for some differential equation, however in this case it still does not satisfy our requirement that this be 'a solution of a differential equation.'

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WH GC MAT345 S08: 0/**90**/0/10
HHS JG MA232 S08: 17/**67**/0/17
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5. Which of the following could not be a solution of a differential equation?

   (a) $f = 2y + 7$
   (b) $q(d) = 2d^2 - 6e^d$
   (c) $6y^2 + 2yx = \sqrt{x}$
   (d) $y = 4 \sin 8\pi z$
   (e) All could be a solution of a differential equation.

   Answer: (c). A solution of a differential equation is a function, and this is not a function. A function returns one number as output from a given input. This expression could be solved for y, and hence we could say that functions are defined implicitly by this equation. However as this equation is quadratic in y, the solution has two branches, and so there are two functions that are defined by this equation, and so it does not satisfy our requirement that this be 'a solution of a differential equation.'

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   HHS JG MA232 S08: 0/0/**0**/0/100
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6. True or False? A differential equation is a type of function.

   (a) True
   (b) False

   Answer: (b). False. A function is a mathematical process where we get a single number as output from a given input. An equation is a mathematical expression involving an equal sign, and a differential equation involves the derivative of a function.

   HHS JG MA232 S08: 67/**33**

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7. The amount of a chemical in a lake is decreasing at a rate of 30% per year. If $p(t)$ is the total amount of the chemical in the lake as a function of time t (in years), which differential equation models this situation?

   (a) $p'(t) = -30$

   (b) $p'(t) = -0.30$

   (c) $p'(t) = p - 30$

   (d) $p'(t) = -0.3p$

   (e) $p'(t) = 0.7p$

   Answer: (d). The derivative of the amount of pollution of the lake is equal to 30% of the total amount, and is negative because the amount of pollution is decreasing. Many students like answer (b) because they view this as a constant rate of change of -30%.

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   WH GC MAT345 S08: 0/20/0/**60**/20
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   CHS DP MA232 S08: 7/64/0/**29**/0
   HHS JG MA232 S08: 0/50/0/**33**/17
   HC AS MA304 S08: 0/3/0/**93**/3

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8. The evolution of the temperature of a hot cup of coffee cooling off in a room is described by $\frac{dT}{dt} = -0.01T + 0.6$, where T is in °F and t is in hours. What are the units of the numbers -0.01 and 0.6?

   (a) -0.01 °F, and 0.6 °F

   (b) -0.01 per hour, and 0.6 °F per hour

   (c) -0.01 °F per hour, and 0.6 °F

   (d) neither number has units

   Answer: (b). Since T has units of °F, $\frac{dT}{dt}$ has units of °F per hour, so 0.6 must have these units as well and -0.01 has units of 'per hour' in order to give the product the same units.

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CHS DP MA232 S08: 8/**0**/92/0
HHG JG MA232 S08: 0/**33**/67/0

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9. We want to test the function $z(x) = 4\sin 3x$ to see if it solves $z'' + 2z' + 4z = 0$, by substituting the function into the differential equation. What is the resulting equation before simplification?

   (a) $-36\sin 3x + 24\cos 3x + 16\sin 3x = 0$
   (b) $4\sin 3x + 8\sin 3x + 16\sin 3x = 0$
   (c) $-36\sin 3x + 12\cos 3x + 4\sin 3x = 0.$
   (d) $4\sin 3x + 8\cos 3x + 4\sin 3x = 0$
   (e) none of the above

   Answer: (a). We take derivatives of the function, finding that $z' = 12\cos 3x$ $z'' = -36\sin 3x$. Then we substitute z, z', and z'' into the differential equation, and we get answer (a).

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   HHS JG MA232 S08: **100**/0/0/0/0

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10. If we test the function $f(x) = ae^{bx}$ to see if it could solve $\frac{df}{dx} = cf^2$, which equation is the result?

    (a) $\frac{df}{dx} = ca^2e^{2bx}$
    (b) $abe^{bx} = cf^2$
    (c) $ae^{bx} = ca^2e^{(bx)^2}$
    (d) $abe^{bx} = ca^2e^{2bx}$
    (e) $abe^{bx} = cae^{bx}$
    (f) None of the above

Answer: (d). $\frac{df}{dx} = abe^{bx}$ and $f^2 = a^2e^{2bx}$, so we subsitute both of these into the differential equation. Note that both (a) and (b) are mathematically correct, but because we have only substituted in either the function (a) or its derivative (b), but not both, they are not useful in testing whether the function solves the differential equation.

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11. We want to test the function $f(x) = 3e^{2x} + 6x$ to see if it solves the differential equation $\frac{df}{dx} = 2f + 3x$, so we insert the function and its derivative, getting $6e^{2x} + 6 = 2(3e^{2x} + 6x) + 3x$. This means that:

    (a) This function is a solution.

    (b) This function is a solution if $x = 2/5$.

    (c) This function is not a solution.

    (d) Not enough information is given.

    Answer: (c). In order to be a solution, the resulting equation must be true for a range of values of the independent variable x. The equation we got is true only for $x = 2/5$, so this means that our function is not a solution to this differential equation.

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    CC HZ MA232 S08: 6/55/**39**/0 time 2:30
    CHS DP MA232 S08: 0/31/**62**/8

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12. A bookstore is constantly discarding a certain percentage of its unsold inventory and also receiving new books from its supplier so that the rate of change of the number of books in inventory is $B'(t) = -0.02B + 400 + 0.05t$, where B is the number of books and t is in months. If the store begins with 10,000 books in inventory, at what rate is it receiving books from its supplier at $t = 0$?

    (a) 200 books per month

    (b) 400 books per month

    (c) -200 books per month

(d) 900 books per month

Answer: (b). At $t = 0$ the bookstore is receiving books at a rate of 400 books per month from its supplier. The total rate of change of the books in inventory is $-0.02(10,000) + 400 + 0.05(0) = 200$ books per month, however we were only asking for the rate at which books are being received from the suppler, which is given by the $400 + 0.05t$ terms.

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