MathQuest: Differential Equations

Bifurcations

1. How many equilibria does the differential equation $y' = y^2 + a$ have?

   (a) Zero

   (b) One

   (c) Two

   (d) Three

   (e) Not enough information is given.

   Answer: (e). The answer depends on the value of the parameter a. If a is positive, then there can be no equilibria. If a is negative then equilibria exist at $y = \pm\sqrt{-a}$.

   CC MP MA334 S07: 0/13/8/4/**75**

   CC KC MA334 S08: 0/6/67/0/**28** time 2:00

   HC AS MA304 S08: 7/15/37/0/**41**
   CC KC MA334 S09: 20/20/20/0/**40** time 2:30

   by Carroll College MathQuest

   DEQ.10.12.010

2. A *bifurcation* occurs if the number of equilibria of a system changes when we change the value of a parameter. For the differential equation $\frac{df}{dx} = bf^2 - 2$, a bifurcation occurs at what value of b?

   (a) $b = 0$

   (b) $b = 2$

   (c) $b = -2$

   (d) $b = 2/f^2$

   (e) Not enough information is given.

   Answer: (a). If b is positive, then we will have two equilibria, $f = \sqrt{2/b}$. If b is negative then there are no equilibria. Thus the bifurcation occurs at $b = 0$.

   CC MP MA334 S07: **25**/4/25/42/4
   CC KC MA334 S08: **33**/0/0/44/22 time 5:00
   CC KC MA334 S09: **20**/8/16/48/8 time 5:00

by Carroll College MathQuest

DEQ.10.12.020

3. $x'(t) = \frac{1}{2}x^2 + bx + 8$. If $b = 5$ what are the equilibria of the system?

   (a) $x = -5$

   (b) $x = -3, 3$

   (c) $x = -8, -2$

   (d) No equilibria exist and all solutions are increasing.

   (e) No equilibria exist and all solutions are decreasing.

   Answer: (c). Equilibria will occur at $x = -5 \pm 3$. Thus we have equilibria at $x = -8, -2$.

   CC MP MA334 S07: 0/0/**100**/0/0
   CC KC MA334 S08: 0/0/**100**/0/0 time 1:15
   CC KC MA334 S09: 0/0/**95**/5/0 time 1:30

by Carroll College MathQuest

DEQ.10.12.030

4. $x'(t) = \frac{1}{2}x^2 + bx + 8$. If $b = 2$ what are the equilibria of the system?

   (a) $x = -8$

   (b) $x = -2 \pm \sqrt{12}$

   (c) $x = 2$

   (d) No equilibria exist and all solutions are increasing.

   (e) No equilibria exist and all solutions are decreasing.

   Answer: (d). Equilibria will occur at $x = -2 \pm \sqrt{4 - 16}$, which has no real solutions. Thus if $b = 2$ there are no equilibria. For all values of x, $x' > 0$ and thus all solutions are increasing.

   CC MP MA334 S07: 0/0/5/**95**/0

   CC KC MA334 S08: 0/0/0/**100**/0 time 1:45

by Carroll College MathQuest

DEQ.10.12.040

5. $x'(t) = \frac{1}{2}x^2 + bx + 8$. A bifurcation occurs where?

(a) $x = -b \pm \sqrt{b^2 - 16}$

(b) $b = 0$

(c) $b = \frac{1}{2}$

(d) $b = 4$

(e) $b = 8$

(f) Not enough information is given.

Answer: (d). Equilibria will occur at $x = -b \pm \sqrt{b^2 - 16}$, and this equation has real solutions if $b \geq 4$. Thus if $b < 4$ there are no equilibria, if $b = 4$ there is one equilibrium, and if $b > 4$ there are two equilibria.

CC KC MA334 S08: 0/13/0/**87**/0/0 time 3:00
HC AS MA304 S08: 52/0/0/**35**/4/9
HC AS MA304 S08: 7/4/0/**86**/0/4 Review
CC KC MA334 S09: 4/8/0/**84**/4/0 time 3:45

by Carroll College MathQuest

DEQ.10.12.050

6. $\frac{dg}{dt} = g^3 + cg$. How many equilibria does this system have?

(a) Two

(b) One or two

(c) One or three

(d) Two or three

(e) Three

(f) Not enough information is given

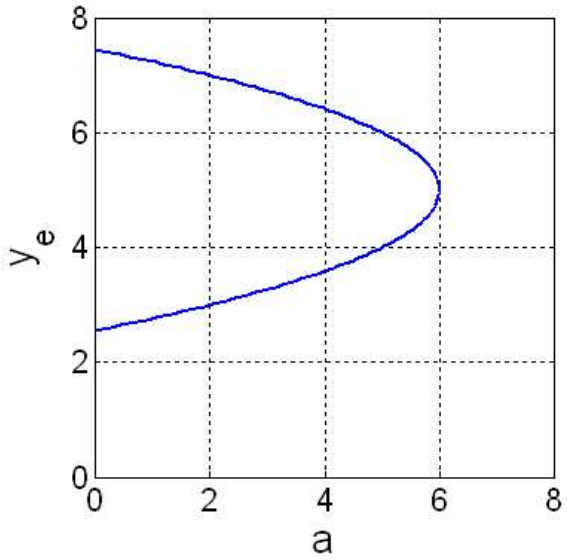
Answer: (c). $g = 0$ is an equilibrium of this system for any value of c. If c is negative then two other equilibria are $g = \pm\sqrt{-c}$. If c is positive, then there are no other equilibria. Thus the system has either one or three equilibria, depending on the value of g, and a bifurcation occurs at $g = 0$.

CC KC MA334 S08: 0/12/**53**/0/18/18 time 2:00
CC KC MA334 S09: 0/0/**44**/20/0/36 time 3:00

by Carroll College MathQuest

DEQ.10.12.060

7. A *bifurcation diagram* plots a system's equilibria on the y axis and the value of a parameter on the x axis. Consider the bifurcation diagram below. When our parameter is $a = 5$, what are the equilibria of the system?



(a) $y = 0$ and $y = 3$

(b) $y = 5$

(c) $y = 4$ and $y = 6$

(d) Not enough information is given.

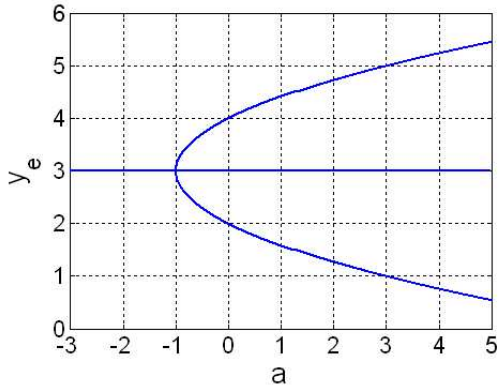
Answer: (c). On the diagram, we find $a = 5$ on the horizontal axis, which corresponds to $y = 4$ and $y = 6$ on the vertical axis.

CC MP MA334 S07: 0/0/**100**/0
CC KC MA334 S08: 0/0/**94**/6 time 1:20
CC KC MA334 S09: 0/8/**92**/0 time 1:15

by Carroll College MathQuest

DEQ.10.12.070

8. Consider the bifurcation diagram below. If our system has equilibria at $y = 1$, $y = 3$ and $y = 5$ what is the value of the parameter a?



(a) $a = -1$

(b) $a = 0$

(c) $a = 1$

(d) $a = 3$

(e) $a = 5$

(f) Not enough information is given.

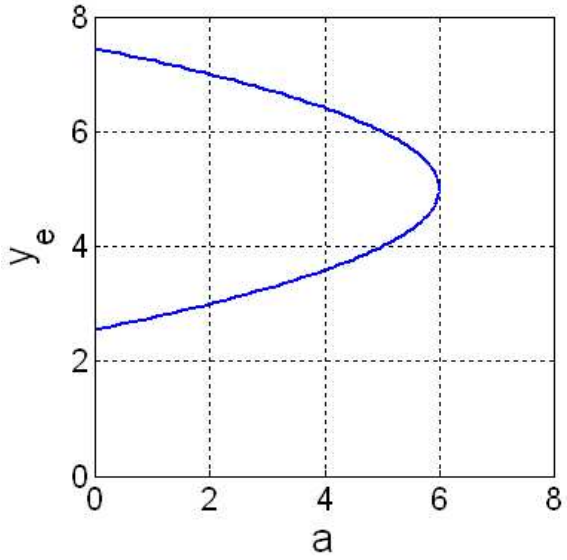
Answer: (d). We find $y = 1$, $y = 3$, and $y = 5$ on the vertical axis at the location where $a = 3$ on the horizontal axis.

CC KC MA334 S08: 0/0/6/**94**/0/0 time 1:30
CC KC MA334 S09: 0/0/12/**88**/0/0 time 1:40

by Carroll College MathQuest

DEQ.10.12.080

9. Consider the bifurcation diagram below. At what value of a does the system have a bifurcation?



(a) $a = 0$

(b) $a = 2$

(c) $a = 4$

(d) $a = 6$

(e) $a = 8$

(f) Not enough information is given.

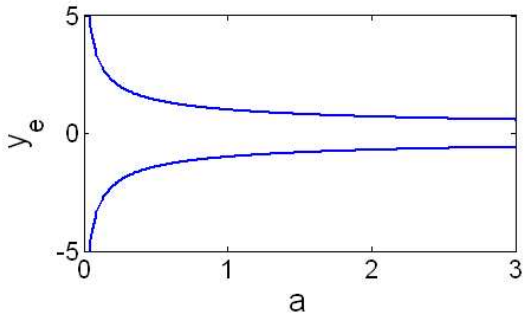
Answer: (d). For values of $a < 6$ the system has two equilibria and for values of $a > 6$ the system has no equilibria, so $a = 6$ is a bifurcation point, where the number of equilibria changes.

CC KC MA334 S08: 0/0/0/**100**/0/0 time 0:30
HC AS MA304 S08: 0/0/0/**96**/0/4
CC KC MA334 S09: 0/0/4/**92**/0/4 time 1:30

by Carroll College MathQuest

DEQ.10.12.090

10. Which of the following differential equations is represented by the bifurcation diagram below?



(a) $y' = y^2 + a$

(b) $y' = ay^2 - 1$

(c) $y' = ay$

(d) $y' = y^2 + ay + 2$

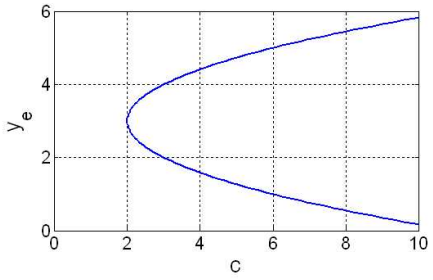
Answer: (b). The equilibria of this system are equal to $y = \pm\frac{1}{\sqrt{a}}$, which corresponds to the curves plotted in the bifurcation diagram.

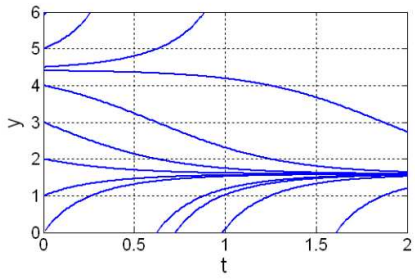
CC KC MA334 S08: 0/**77**/18/6 time 3:30
CC KC MA334 S09: 4/**81**/4/11 time 3:30

by Carroll College MathQuest

DEQ.10.12.100

11. The figure on the left is a bifurcation diagram, and the figure on the right plots several solution functions of this system for one specific value of the parameter. The figure on the right corresponds to what value of the parameter?





(a) $c = 0$

(b) $c = 2$

(c) $c = 4$

(d) $c = 6$

(e) $c = 8$

Answer: (c). The solutions trajectories are converging from a stable equilibrim at $y \approx 1.6$ and diverging from an unstable equilibrium at $y \approx 4.5$. On the bifurcation diagram, these equilibrium values correspond to a parameter of $c = 4$.

CC KC MA334 S08: 0/0/**83**/18/0 time 2:00
CC KC MA334 S09: 0/4/**85**/11/0 time 2:15

by Carroll College MathQuest

DEQ.10.12.110