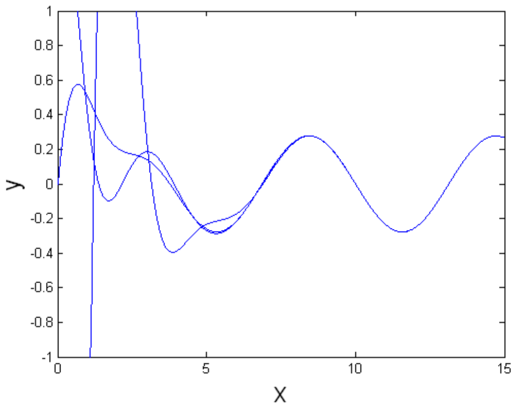
MathQuest: Differential Equations

Second Order Differential Equations: Forcing

1. The three functions plotted below are all solutions of $y''+ay'+4y=\sin x$. Is a positive or negative?



   (a) a is positive.
   (b) a is negative.
   (c) $a=0$.
   (d) Not enough information is given.

   Answer: (a). a is positive. All three functions start with different initial conditions but converge to the same behavior. This means that the initial conditions die out, leaving only motion controlled by the driving function $\sin(x)$. The characteristic equation must have negative real roots, so there must be positive damping in the equation.

   by Carroll College MathQuest

   DEQ.10.16.010
   CC KC MA334 S08: **83**/0/0/17 time 3:00 ($a=0$ was not an option)
   CC KC MA334 S09: **21**/12/25/42 time 3:00
   CC KC MA334 S13: **67**/8/0/25 time 5:00

2. If we conjecture the function $y(x)=C_1\sin 2x+C_2\cos 2x+C_3$ as a solution to the differential equation $y''+4y=8$, which of the constants is determined by the differential equation?

   (a) C_1

(b) C_2

(c) C_3

(d) None of them are determined.

Answer: (c). C_3 is determined. If we take two derivatives of this function we get $-4C_1 \sin 2x - 4C_2 \cos 2x$. If we put this into the differential equation, we find that all the sine and cosine terms cancel out, allowing us to solve for $C_3 = 2$.

by Carroll College MathQuest

DEQ.10.16.020
WH GC MAT345 S08: 0/0/**100**/0
CC KC MA334 S08: 0/0/**100**/0 time 2:00
AU CS MA244 S08: 0/0/**92**/8
CC KC MA334 S12: 11/0/**73**/16
CC KC MA334 S13: 0/4/**96**/0 time 2:00

3. What will the solutions of $y'' + ay' + by = c$ look like if b is negative and a is positive.

   (a) Solutions will oscillate at first and level out at a constant.

   (b) Solutions will grow exponentially.

   (c) Solutions will oscillate forever.

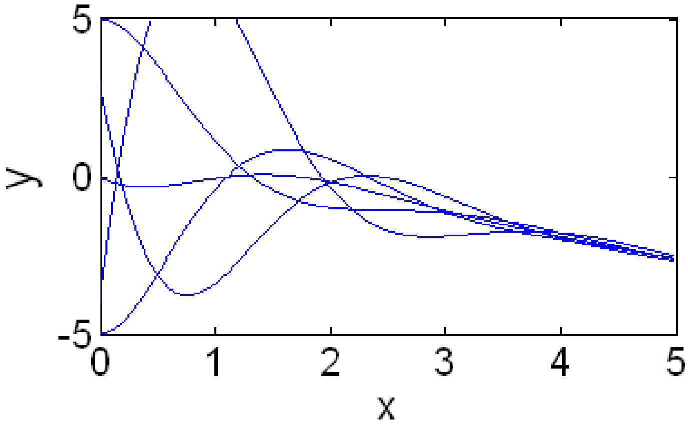
   (d) Solutions will decay exponentially.

   Answer: (b). Solutions will grow exponentially. If b was positive, our solutions would contain sines and cosines, but because b is negative, our solutions are growing and decaying exponentials. Except for a very special set of initial conditions, most solutions will contain both a growing and a decaying part, with the growing part dominating the long term behavior.

   by Carroll College MathQuest

   DEQ.10.16.030
   WH GC MAT345 S08: 80/**20**/0
   CC KC MA334 S09: 45/**52**/0 (option d added)
   CC KC MA334 S10: 0/**71**/0/29 time 3:00
   CC KC MA334 S13: 26/**55**/0/19 time 4:00

4. The functions plotted below are solutions to which of the following differential equations?



(a) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = 3 - 3x$

(b) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = 3e^{2x}$

(c) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = \sin \frac{2\pi}{9} x$

(d) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = 2x^2 + 3x - 4$

(e) None of the above

Answer: (a). The left side of the differential equation can be thought to represent a damped harmonic oscillator, and the terms on the right side are a forcing function. Over time, solution trajectories will converge to a solution with the form of the forcing function. These solutions are converging to a linear function, so they must be responding to a linear forcing function. This is question is intended to be given before we have learned the precise method of solving this type of equation.

by Carroll College MathQuest

DEQ.10.16.040
CC KC MA334 S08: **100**/0/0/0/0
AU CS MA244 S08: **19**/25/25/31/0
CC KC MA334 S09: **73**/4/19/0/4 time 2:00
CC KC MA334 S10: **67**/5/19/0/9 time 3:00
AS DH MA3335 010 S12: **0**/0/100/0/0 time 2:30
AS DH 3335 010 S13: **13**/21/67/0/0 time 3:10
CC KC MA334 S13: **82**/0/4/0/14 time 3:00
AS DH 3335 010 S14: **8**/25/21/25/21 time 3:00
AS DH 3335 010 S15: **54**/0/15/0/31 time 5:00

5. The general solution to $f'' + 7f' + 12f = 0$ is $f(t) = C_1e^{-3t} + C_2e^{-4t}$. What should we conjecture as a particular solution to $f'' + 7f' + 12f = 5e^{-2t}$?

(a) $f(t) = Ce^{-4t}$

(b) $f(t) = Ce^{-3t}$

(c) $f(t) = Ce^{-2t}$

(d) $f(t) = C\cos 2t$

(e) None of the above

Answer: (c). Our particular solution should generally have the same form as the driving term.

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DEQ.10.16.050
AU CS MA244 S08: 0/0/**93**/7/0
CC KC MA334 S09: 0/4/**89**/4/4 time 1:20
CC KC MA334 S10: 0/0/**90**/0/10 time 3:30
CC KC MA334 S12: 5/0/**90**/0/5
AS DH MA3335 010 S12: 0/0/**100**/0/0 time 2:00
AS DH 3335 010 S13: 0/0/**100**/0/0 time 2:00
CC KC MA334 S13: 0/0/**100**/0/0 time 1:30
AS DH 3335 010 S14: 0/0/**100**/0/0 time 1:30
AS DH 3335 010 S15: 0/0/**100**/0/0 time 2:00

6. The general solution to $f'' + 7f' + 12f = 0$ is $f(t) = C_1e^{-3t} + C_2e^{-4t}$. What is a particular solution to $f'' + 7f' + 12f = 5e^{-6t}$?

(a) $f(t) = \frac{5}{6}e^{-6t}$

(b) $f(t) = \frac{5}{31}e^{-6t}$

(c) $f(t) = \frac{5}{20}e^{-6t}$

(d) $f(t) = e^{-3t}$

(e) None of the above

Answer: (a). We conjecture that a particular solution should be of the form $f(t) = Ce^{-6t}$. We take derivatives finding that $f'(t) = -6Ce^{-6t}$ and $f''(t) = 36Ce^{-6t}$. We put these into the differential equation and find that $36C - 42C + 12C = 5$ and thus $C = \frac{5}{6}$.

by Carroll College MathQuest

DEQ.10.16.060
CC KC MA334 S08: **61**/0/11/6/22 time 4:00
AU CS MA244 S08: **100**/0/0/0/0
CC KC MA334 S09: **88**/0/8/4/0 time 3:20
CC KC MA334 S10: **86**/9/0/0/5 time 3:30
CC KC MA334 S12: **85**/0/0/0/15

CC KC MA334 S13: **96**/4/0/0/0 time 2:00
AS DH 3335 010 S15: 0/0/**100**/0/0 time 2:00
time 4:30

7. To find a particular solution to the differential equation

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = \cos t,$$

we replace it with a new differential equation that has been "complexified." What is the new differential equation to which we will find a particular solution?

(a) $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{2it}$
(b) $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{3it}$
(c) $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{it}$
(d) $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{-2it}$
(e) $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{-it}$
(f) None of the above.

Answer: (c).

by Christopher Storm

DEQ.10.16.070 cks24
AU CS MA244 S08: 0/6/**88**/0/6/0

8. To solve the differential equation $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{it}$, we make a guess of $y_p(t) = Ce^{it}$. What equation results when we evaluate this in the differential equation?

(a) $-Ce^{it} + 3Cie^{it} + 2Ce^{it} = Ce^{it}$
(b) $-Ce^{it} + 3Cie^{it} + 2Ce^{it} = e^{it}$
(c) $Cie^{it} + 3Ce^{it} + 2Ce^{it} = e^{it}$
(d) $-Ce^{it} + 3Ce^{it} + 2Ce^{it} = Ce^{it}$

Answer: (b).

by Christopher Storm

DEQ.10.16.080 cks25
AU CS MA244 S08: 0/**100**/0/0
HC AS MA304 S11: 0/**90**/10/0/0/0

9. To solve the differential equation $\frac{d^2y}{dt^2}+3\frac{dy}{dt}+2y = e^{it}$, we make a guess of $y_p(t) = Ce^{it}$. What value of C makes this particular solution work?

   (a) $C = \frac{1+3i}{10}$
   (b) $C = \frac{1-3i}{10}$
   (c) $C = \frac{1+3i}{\sqrt{10}}$
   (d) $C = \frac{1-3i}{\sqrt{10}}$

Answer: (b).

by Christopher Storm

DEQ.10.16.090 cks26
AU CS MA244 S08: 0/**100**/0/0

10. In order to find a particular solution to $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = \cos t$, do we want the real part or the imaginary part of the particular solution $y_p(t) = Ce^{it}$ that solved the complexified equation $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{it}$?

    (a) Real part
    (b) Imaginary part
    (c) Neither, we need the whole solution to the complexified equation.

Answer: (a).

by Christopher Storm

DEQ.10.16.100 cks27
AU CS MA244 S08: **85**/15/0

11. What is a particular solution to $\dfrac{d^2y}{dt^2} + 3\dfrac{dy}{dt} + 2y = \cos t$?

(a) $y_p(t) = \dfrac{3}{10}\cos t + \dfrac{1}{10}\sin t$

(b) $y_p(t) = \dfrac{-3}{10}\cos t + \dfrac{1}{10}\sin t$

(c) $y_p(t) = \dfrac{1}{10}\cos t + \dfrac{-3}{10}\sin t$

(d) $y_p(t) = \dfrac{1}{10}\cos t + \dfrac{3}{10}\sin t$

Answer: (d).

by Christopher Storm

DEQ.10.16.110 cks28
AU CS MA244 S08: 0/0/13/**87**
CC KC MA334 S09: 4/16/48/**32** time 6:00
CC KC MA334 S10: 0/0/0/**100** time 4:00
CC KC MA334 S12: 0/8/42/**50**
AS DH MA3335 010 S12: 0/0/0/**100** time 4:00
AS DH 3335 010 S13: 0/21/17/**63** time 5:30
CC KC MA334 S13: 12/0/44/**44** time 6:00
AS DH 3335 010 S15: 0/7/93/ time 0