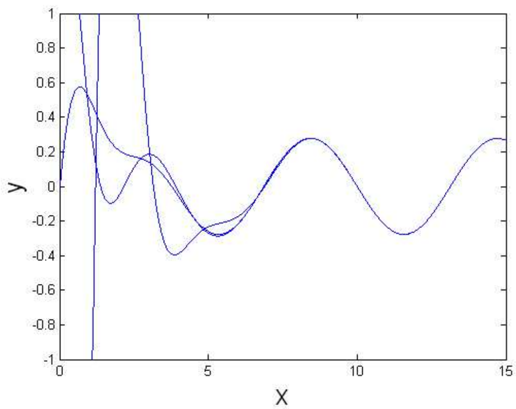
MathQuest: Differential Equations

Second Order Differential Equations: Forcing

1. The three functions plotted below are all solutions of $y'' + ay' + 4y = \sin x$. Is a positive or negative?



   (a) a is positive.
   (b) a is negative.
   (c) $a = 0$.
   (d) Not enough information is given.

   Answer: (a). a is positive. All three functions start with different initial conditions but converge to the same behavior. This means that the initial conditions die out, leaving only motion controlled by the driving function $\sin(x)$. The characteristic equation must have negative real roots, so there must be positive damping in the equation.

   CC KC MA334 S08: **83**/0/0/17 time 3:00 ($a = 0$ was not an option)
   CC KC MA334 S09: **21**/12/25/42 time 3:00

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   DEQ.10.16.010

2. If we conjecture the function $y(x) = C_1 \sin 2x + C_2 \cos 2x + C_3$ as a solution to the differential equation $y'' + 4y = 8$, which of the constants is determined by the differential equation?

   (a) C_1
   (b) C_2

(c) C_3

(d) None of them are determined.

Answer: (c). C_3 is determined. If we take two derivatives of this function we get $-4C_1 \sin 2x - 4C_2 \cos 2x$. If we put this into the differential equation, we find that all the sine and cosine terms cancel out, allowing us to solve for $C_3 = 2$.

WH GC MAT345 S08: 0/0/**100**/0

CC KC MA334 S08: 0/0/**100**/0 time 2:00

AU CS MA244 S08: 0/0/**92**/8

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DEQ.10.16.020

3. What will the solutions of $y'' + ay' + by = c$ look like if b is negative and a is positive.

   (a) Solutions will oscillate at first and level out at a constant.
   (b) Solutions will grow exponentially.
   (c) Solutions will oscillate forever.
   (d) Solutions will decay exponentially.

   Answer: (b). Solutions will grow exponentially. If b was positive, our solutions would contain sines and cosines, but because b is negative, our solutions are growing and decaying exponentials. Except for a very special set of initial conditions, most solutions will contain both a growing and a decaying part, with the growing part dominating the long term behavior.

   WH GC MAT345 S08: 80/**20**/0
   CC KC MA334 S09: 45/**52**/0 (option d added)

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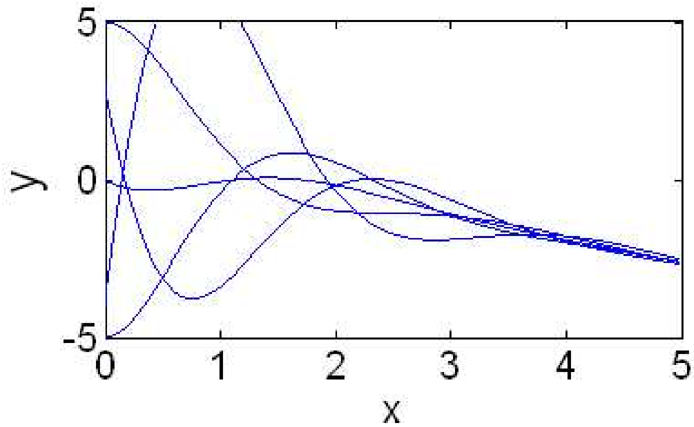
   DEQ.10.16.030

4. The functions plotted below are solutions to which of the following differential equations?



(a) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = 3 - 3x$

(b) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = 3e^{2x}$

(c) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = \sin \frac{2\pi}{9}x$

(d) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = 2x^2 + 3x - 4$

(e) None of the above

Answer: (a). The left side of the differential equation can be thought to represent a damped harmonic oscillator, and the terms on the right side are a forcing function. Over time, solution trajectories will converge to a solution with the form of the forcing function. These solutions are converging to a linear function, so they must be responding to a linear forcing function. This is question is intended to be given before we have learned the precise method of solving this type of equation.

CC KC MA334 S08: **100**/0/0/0/0 AU CS MA244 S08: **19**/25/25/31/0
CC KC MA334 S09: **73**/4/19/0/4 time 2:00

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DEQ.10.16.040

5. The general solution to $f'' + 7f' + 12f = 0$ is $f(t) = C_1e^{-3t} + C_2e^{-4t}$. What should we conjecture as a particular solution to $f'' + 7f' + 12f = 5e^{-2t}$?

(a) $f(t) = Ce^{-4t}$

(b) $f(t) = Ce^{-3t}$

(c) $f(t) = Ce^{-2t}$

(d) $f(t) = C\cos 2t$

(e) None of the above

Answer: (c). Our particular solution should generally have the same form as the driving term.

AU CS MA244 S08: 0/0/**93**/7/0
CC KC MA334 S09: 0/4/**89**/4/4 time 1:20

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DEQ.10.16.050

6. The general solution to $f'' + 7f' + 12f = 0$ is $f(t) = C_1 e^{-3t} + C_2 e^{-4t}$. What is a particular solution to $f'' + 7f' + 12f = 5e^{-6t}$?

   (a) $f(t) = \frac{5}{6}e^{-6t}$
   (b) $f(t) = \frac{5}{31}e^{-6t}$
   (c) $f(t) = \frac{5}{20}e^{-6t}$
   (d) $f(t) = e^{-3t}$
   (e) None of the above

   Answer: (a). We conjecture that a particular solution should be of the form $f(t) = Ce^{-6t}$. We take derivatives finding that $f'(t) = -6Ce^{-6t}$ and $f''(t) = 36Ce^{-6t}$. We put these into the differential equation and find that $36C - 42C + 12C = 5$ and thus $C = \frac{5}{6}$.

   CC KC MA334 S08: **61**/0/11/6/22 time 4:00
   AU CS MA244 S08: **100**/0/0/0/0
   CC KC MA334 S09: **88**/0/8/4/0 time 3:20

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   DEQ.10.16.060

7. To find a particular solution to the differential equation

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = \cos t,$$

   we replace it with a new differential equation that has been "complexified." What is the new differential equation to which we will find a particular solution?

   (a) $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{2it}$
   (b) $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{3it}$

(c) $\dfrac{d^2y}{dt^2} + 3\dfrac{dy}{dt} + 2y = e^{it}$

(d) $\dfrac{d^2y}{dt^2} + 3\dfrac{dy}{dt} + 2y = e^{-2it}$

(e) $\dfrac{d^2y}{dt^2} + 3\dfrac{dy}{dt} + 2y = e^{-it}$

(f) None of the above.

Answer: (c).

AU CS MA244 S08: 0/6/**88**/0/6/0

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DEQ.10.16.070 cks24

8. To solve the differential equation $\dfrac{d^2y}{dt^2}+3\dfrac{dy}{dt}+2y = e^{it}$, we make a guess of $y_p(t) = Ce^{it}$. What equation results when we evaluate this in the differential equation?

   (a) $-Ce^{it} + 3Cie^{it} + 2Ce^{it} = Ce^{it}$

   (b) $-Ce^{it} + 3Cie^{it} + 2Ce^{it} = e^{it}$

   (c) $Cie^{it} + 3Ce^{it} + 2Ce^{it} = e^{it}$

   (d) $-Ce^{it} + 3Ce^{it} + 2Ce^{it} = Ce^{it}$

   Answer: (b).

   AU CS MA244 S08: 0/**100**/0/0

   by Christopher Storm

   DEQ.10.16.080 cks25

9. To solve the differential equation $\dfrac{d^2y}{dt^2}+3\dfrac{dy}{dt}+2y = e^{it}$, we make a guess of $y_p(t) = Ce^{it}$. What value of C makes this particular solution work?

   (a) $C = \dfrac{1+3i}{10}$

   (b) $C = \dfrac{1-3i}{10}$

   (c) $C = \dfrac{1+3i}{\sqrt{10}}$

   (d) $C = \dfrac{1-3i}{\sqrt{10}}$

Answer: (b).

AU CS MA244 S08: 0/**100**/0/0

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DEQ.10.16.090 cks26

10. In order to find a particular solution to $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = \cos t$, do we want the real part or the imaginary part of the particular solution $y_p(t) = Ce^{it}$ that solved the complexified equation $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{it}$?

    (a) Real part
    (b) Imaginary part
    (c) Neither, we need the whole solution to the complexified equation.

    Answer: (a).

    AU CS MA244 S08: **85**/15/0

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    DEQ.10.16.100 cks27

11. What is a particular solution to $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = \cos t$?

    (a) $y_p(t) = \frac{3}{10}\cos t + \frac{1}{10}\sin t$
    (b) $y_p(t) = \frac{-3}{10}\cos t + \frac{1}{10}\sin t$
    (c) $y_p(t) = \frac{1}{10}\cos t + \frac{-3}{10}\sin t$
    (d) $y_p(t) = \frac{1}{10}\cos t + \frac{3}{10}\sin t$

    Answer: (d).

    AU CS MA244 S08: 0/0/13/**87**
    CC KC MA334 S09: 4/16/48/**32** time 6:00

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    DEQ.10.16.110 cks28