Classroom Voting in Mathematics

Kelly S. Cline

How do you get your students to take an active role during a mathematics class? For instance, how do you get them to form opinions and to participate in discussions about difficult concepts? There is a large body of education research demonstrating how active learning methods can be very effective, especially in comparison to traditional lectures (e.g., Bonwell and Eison 1991; Davidson 1990; Dees 1991; Hagelgans et al. 1995; Norwood 1995; Springer, Stanne, and Donovan 1999). Even when supplemented with demonstrations and PowerPoint, lectures encourage students to be passive observers, and passive students rarely learn. Classroom voting is a powerful technique promoting active learning. It engages every student in the material, and it can easily be incorporated into an otherwise traditional class. This technique breaks students out of the passive-receptive mode and requires them to participate, creating a more effective learning environment.

The basic idea is to integrate into the class period a series of voting events in which the teacher poses a multiple-choice or true/false question to the class. After a brief period for consideration and informal discussion (usually about two minutes), the students vote on the correct answer, either by holding up a colored index card (a = blue, b = green, etc.) or by using an electronic "clicker" device much like a television remote control. The results of card voting can be assessed with a quick glance around the room. Electronic votes are received and tabulated on a computer at the front of the class; after the vote, a bar graph of the results is projected for the entire class to see. The teacher can then guide the class through a discussion of the concepts involved.

In my calculus class, for example, after discussing how to find the tangent line to a function at a specific point, I put the following question on an overhead and read it aloud:

> What is the equation of the line tangent to the function $f(x) = |x|$ at the point (0, 0)?
> (a) The equation of the tangent line at this point is $y = 0$.
> (b) There are two tangent lines, with equations $y = -x$ and $y = x$.
> (c) This function has no tangent line at this point.
> (d) This function has infinitely many tangent lines at this point.

Then I start a two-minute timer and ask students to compare their thinking with at least one other person's. The classroom is usually quiet for about 15 seconds as students consider the question; then they form small groups for discussions. When the two minutes are up, I ask students to get out their cards and decide how they are going to vote. After a count of three, I have them all hold up their cards. If there is a near consensus on the right answer, I briefly comment on why this was correct and move on. A good question usually provokes a diversity of answers, however, and so I announce the results: "It looks like an even split among (a), (b), and (c)." Then I ask, "Which of the voters for (a) is willing to share his or her thoughts?" A student may explain that because the slope changes from −1 to +1 at this point, it flattens out, so the average slope is 0 and the line must be $y = 0$. Then I ask for a volunteer to explain why answer (b) is correct, and we might hear a student point out that because the lines $y = -x$ and $y = x$ go through this point, they are both tangent lines. Finally I ask for a discussion of (c), and a student might explain that there is no slope at this point because the limit

$$\lim_{x\to 0}\frac{f(x)-0}{x-0}$$

approaches −1 from the left and +1 from the right, and since that limit fails to exist, there is no tangent. Often when someone has the right idea, students around the room react as they grasp the logic of the situation. Other times, the solution is not so clear, and I have to explain it carefully. Depending on the results, I can then decide whether to move on or to ask a similar question to see if student understanding has improved.

ADVANTAGES OF CLASSROOM VOTING

This technique is useful for a variety of reasons:

- It requires each student in the class to consider a question, to form an opinion, and to make a decision by voting. This prevents students from sitting passively through class and encourages them to participate actively and to discuss mathematics with the people around them.
- It provides immediate feedback to the teacher: Is this a concept the students understand, or does it need to be reviewed? If a large majority gets the question right, then the teacher can move on to the next topic. However, if there is controversy, then the teacher knows right then and there that something must be done.
- It provides immediate feedback to students as well. If students learn a technique incorrectly in a lecture situation, they often do not realize the error until they do a problem wrong on a homework assignment and get their paper back several days later. Classroom voting is one teaching method that allows students to discover and to correct their mistake before they leave class.
- It also provides an efficient springboard for creating more fruitful classroom discussions. Students may be reluctant to voice opinions in a mathematics class, afraid of being incorrect in front of their peers. Because voting allows students time to think and decide on a vote, however, and because they see others voting the same way, students are often more willing to speak out after a vote. This prevents class discussions from being dominated by a few students and engages a larger, more diverse population in the process.
- Perhaps most important, classroom voting is fun. Students like the act of participating and enjoy the "game show" atmosphere of the process. Several of my students have commented that when voting, the class "goes faster" than regular classes, and they complain if we skip the voting for a day. When students have fun in class, when they look forward to mathematics as one of the highlights of the day, their minds will be awake and involved, and they will be ready to learn. If we can create an environment in which our students take pleasure in doing substantial mathematics, then our students will learn.

DRILL OR CONCEPTUAL QUESTIONS?

What sorts of questions work best? This technique can be used with any question that can be asked in a multiple-choice or true/false format. One strategy is to begin with fundamental drill questions and then move to more advanced questions that probe the difficult conceptual issues.

Some of the most effective questions are designed to elicit common errors and misconceptions. Knowing where many students will go wrong, the teacher can use voting to confront and deal with their mistakes from the outset. For example, knowing the sorts of problems that students in my calculus classes often have taking the derivative of functions with negative exponents, I ask:

What is the derivative of $f(x)=\frac{2}{x^3}$?

(a) $f'(x) = 2x^{-3}$
(b) $f'(x) = -3x^{-4}$
(c) $f'(x) = 6x^{-3}$
(d) $f'(x) = -6x^{-4}$
(e) $f'(x) = -6x^{-2}$

Classroom voting with drill problems allows students to practice each new mathematical technique in a safe environment, where they can get help from their peers or from the teacher, so that later they will be thoroughly prepared to do homework assignments by themselves.

On the other hand, the goal of this technique is to get students to engage with the material, to provoke a discussion that forces them to grapple with the complexities of mathematics. It can be challenging to create multiple-choice questions of this nature, but it is far from impossible. For example:

Suppose $\int_0^6 f(x)dx = 10.$

What does this tell us about

$$\int_0^3 f(2x)dx?$$

(a) $\int_0^3 f(2x)dx = 10$

(b) $\int_0^3 f(2x)dx = 5$

(c) $\int_0^3 f(2x)dx = 20$

(d) Not enough information is given.

A question like this, which requires careful discussion and analysis of graphs, can challenge student understanding. They can see that the integral of $f(2x)$ will simply compress the function f by a factor of 2, resulting in half as much area as before. Posing a conceptually challenging question like this on a graded homework assignment may demoralize and frustrate some students, because they may not be sure how to begin. The classroom voting method allows the teacher to pose deeper questions like this in a nongraded situation, so students are not worried about losing points and can focus on ideas.

PRE-VOTE DISCUSSIONS?

There are several ways to conduct class during the vote itself. First, putting a specific amount of time on the clock (e.g., two minutes) lets students know they have time to work but not to dawdle. Another approach is to have complete silence after the question is initially projected to the class. This requires students to work individually, so that each person comes up with an answer and no one copies a neighbor's vote. I think it is much more effective, however, to encourage students to discuss each question informally with their classmates. I often tell them to compare their answer with at least one other person's and discuss any differences before voting. Small group discussions can make students engage, think through things from different perspectives, and explore various viewpoints on each problem. The discussion is effectively motivated because each student must vote individually. It gives students a specific and immediate reason to ask opinions of their peers, to listen carefully, and to consider contrasting views. Combining these techniques or using other cooperative learning strategies such as think-pair-share might also be effective.

LET GO OF THE CLASSROOM

Effectively using classroom voting can be difficult because it involves giving up a measure of control and embracing a new atmosphere. Many students expect that in school they need to be quiet and listen attentively. Not all students are used to spending class time discussing mathematical questions with the people around them, voicing their opinions through the voting procedure, and describing their thinking to the rest of the class. As a result, it is important to explain to your class what you are doing, and why. They will learn more if they spend time talking, discussing, voting, and debating, rather than listening passively for the entire period. Some students may have trouble coping with the new atmosphere, but most will enjoy the process. As a result, I have found that students speak out in class more readily and occasionally engage in small group discussions even when not asked to do so.

Although this technique has been used successfully in lecture classes with hundreds of students at large universities, my colleagues and I have found classroom voting to be even more useful in our small classrooms of twenty or thirty. Shifting gears between lecture segments and voting questions is much easier in smaller classes, and more of the students are able to participate in a post-vote discussion.

ELECTRONIC VERSUS NONELECTRONIC VOTING

As described above, classroom voting can be done with no technology at all. One only needs a few packages of colored index cards and some transparencies. We have found, however, that using electronic clickers and a computer with a receiver can make this technique more effective. The technology makes it easy to know exactly which student voted for which answer. This way, if no one volunteers to explain an answer, I can glance down at the screen and find someone to do so. The technology also makes it easier to be sure that everyone participates. With a quick headcount I know how many votes I should receive, and I can keep the voting open until everyone has voted. Another advantage of technology is that the act of voting is more private if clickers are used. With colored card voting, it is not too difficult for a student to quickly glance around and see what color is winning; but with the clickers, no one knows until the vote is closed and the bar graph appears.

The main drawbacks of the technology are cost and ease of use. Our department paid approximately $1,450 for each package containing a receiver, the software, and thirty-two clickers, which were needed in addition to a laptop and a projector. The electronic system requires a few minutes to boot up the laptop, warm up the projector, and mount the receiver on the wall. (We have found that a Velcro patch is an effective way of securing a receiver where it can "see" all the clickers.) The effort is minimal, but time is precious, and the more involved classroom voting becomes, the more likely I am to skip it when running late.

RESOURCES

Many books and articles discuss classroom voting and different teachers' experience with this technique. Sometimes called "peer instruction," this approach was pioneered in introductory physics classes by Eric Mazur at Harvard (Crouch and Mazur 2001; Fagen, Crouch, and Mazur 2002; Mazur 1997) and has been applied in a wide variety of fields, including astronomy (Green 2003) and chemistry (Landis et al. 2000). In the past few years the method has been applied to mathematics at a variety of institutions with very positive results (Butler 2005; Loman and Robinson 2004; Pilzer 2001; Schlatter 2002).

Several great sources for the multiple-choice and true/false questions have been designed specifically for classroom voting in calculus, often called "ConcepTests." The Cornell GoodQuestions project (www.math.cornell.edu/~GoodQuestions/) has created an extensive library of questions and provides a great set of resources for anyone wanting to try out classroom voting for the first time. Another set of questions comes with the instructor's resource package for the Harvard Consortium Calculus Text (Hughes-Hallett et al. 2002). The majority of their ConcepTests are multiple-choice questions ready for classroom voting, which they intersperse with other, more free-response style questions. For multivariable calculus, Mark Schlatter of Centenary College has created his own set of voting questions (personal.centenary.edu/~mschlat/conceptests.pdf).

Three major companies currently sell electronic classroom voting systems of this type that can be run from a computer with a projector. The system we purchased is called the InterWrite PRS (Personal Response System) and is sold by GTO CalComp (formerly Educue; more information from www.gtcocalcomp.com/interwriteprs.htm). This system has worked quite well for us, as long as we use a laptop with a parallel port. The PRS receiver will not plug into a USB port without a special adapter, which posed a problem when we tried to attach it to a Compaq laptop. Other electronic classroom voting systems are from Hyper-Interactive Teaching Technology (www.h-itt.com) and the CPS classroom performance system from eInstruction (www.einstruction.com).

CONCLUSION

The mathematics teachers at our college have been amazed at the power of classroom voting and have quickly incorporated it into our classrooms. Our department has purchased four sets of clickers/laptop receivers that are currently being used by five teachers in all six sections of calculus and both sections of multivariable calculus. To build on this success, we are working to create voting questions for our courses in linear algebra and differential equations.

Give classroom voting a try. It takes just a few minutes to hand out the packages of colored index cards and to create transparencies with a few interesting questions. If our experience is any indication, you will be surprised at how quickly your students become engaged and how much fun they have learning mathematics together.

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KELLY CLINE, kcline@carroll.edu, teaches mathematics and astronomy at Carroll College, Helena, MT 59625. He is interested in mathematical modeling and how technology can be used to foster active learning. Photograph by Jamie Harmon; all rights reserved