Classroom Voting Patterns in Differential Calculus

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Abstract: We study how different sections voted on the same set of classroom voting questions in differential calculus, finding that voting patterns can be used to identify some of the questions that have the most pedagogic value. We use statistics to identify three types of especially useful questions: (1) To identify good discussion questions, we look for those which produce the greatest diversity of responses, indicating that several answers are regularly plausible to students. (2) We identify questions that regularly provoke a common misconception, causing a majority of students to vote for one particular incorrect answer. When this is revealed to the students, they are usually quite surprised that the majority is wrong, and they are very curious to learn what they missed, resulting in a powerful teachable moment. (3) By looking for questions where the percentage of correct votes varies the most between classes, we can find checkpoint questions that provide effective formative assessment as to whether a class has mastered a particular concept.

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1. INTRODUCTION

Classroom voting is a teaching method where the instructor poses a multiple choice question to the class, then allows a few minutes for individual work and small group discussions, before asking all students to individually vote on the right answer, using either a hand-held “clicker” or nontechnologically by raising hands or holding up index cards of various colors (A = red, B = blue, etc.). After the vote the instructor can select students and ask those students to explain what they voted for and why, making sure that the selected students represent the various answers. The vote gives the instructor immediate feedback as to the state of the students’ understanding from each individual in the class. More importantly, the vote requires every single student to play an active role, to grapple with some mathematical issue, to discuss it in a small group, and to register an opinion. By “talking math” on a regular basis, students learn to articulate mathematical ideas, and to evaluate their peers’ mathematical thought processes.

There have been numerous studies about the use of this teaching method in various mathematics courses including college algebra [1, 2], calculus [3, 7, 8, 10, 15, 16], multivariable calculus [7, 14], linear algebra and differential equations [4]. All of these studies report how much students enjoy this teaching method and how it can create a positive and engaging learning environment. Further, many report increased student attendance, as well as increased enthusiasm for mathematics and the development of more mathematics majors. The most dramatic evidence of potential of this method comes from a study conducted by the Cornell GoodQuestions Project which not only demonstrates the effectiveness of classroom voting, but also provides important insight into why this teaching method works, finding that classroom voting had a significant effect on student exam scores, but only if it was used to motivate students to participate in small-group discussions of each question [9, 13].

Two of us (Zullo and Cline) began using classroom voting in calculus in the fall of 2004, at Carroll College, a small liberal arts institution in Helena, Montana. We drew questions from the Cornell GoodQuestions Project [5], from the questions that accompany the Hughes-Hallett *et al.* calculus text [11], and from questions that we authored ourselves. As we became more experienced, we used classroom voting regularly, integrating a few questions into almost every class period. We learned to not use them at the end of class for assessment, but interspersed throughout the lesson, so that we could teach the key points through the resulting discussions. Further, we became very Socratic in the post-vote classroom discussions, calling on students by name to explain their thinking, and then going on to another student without confirming or denying what was said. Instead, we tried to get the class to work together to figure out the issues for themselves, while we would act to focus the discussion on key points, and then clarify and generalize results after the class had developed a consensus. We were consistently impressed with the power of this teaching method to engage students and create a more active learning environment, while at the same time students reported that voting made calculus class more enjoyable.

2. STUDYING CLASSROOM VOTING STATISTICS

The research we discuss here was prompted when we began comparing how two sections of calculus (taught by Cline and Zullo) performed on the same questions. We were using an infrared clicker system that instantly tabulated the results of each vote, listing the percentages of students who had voted for each option. (Although the software can identify individual student's votes, this is not a feature that we used.) When we compared notes after class, we found that sometimes the results were very consistent, with similar percentages of students voting for the same options in both classes. However, other times, the two classes would perform very differently, leading us to discuss more specifically the reasons why this might have occurred.

In general we found that for many questions, strong majorities of students voted correctly, and the questions prompted relatively little discussion. Often the questions that created the most fruitful discussions resulted in more complex voting patterns. We realized that past voting statistics could be used to identify some of the more productive voting questions, and thus could be a useful tool when selecting questions to incorporate into a lesson plan. As a result, we began recording the voting statistics from each question, with data collection starting in the fall of 2005 and continuing to the present. After hearing a presentation about this project, VonEpps joined our collaboration and used this collection of questions in a summer 2007 session of applied calculus at the University of Montana, contributing the resulting statistics to the project. All questions involved in this project are freely available through our website [12], and all voting statistics are incorporated into the teacher's edition of these questions, which is available upon e-mail request.

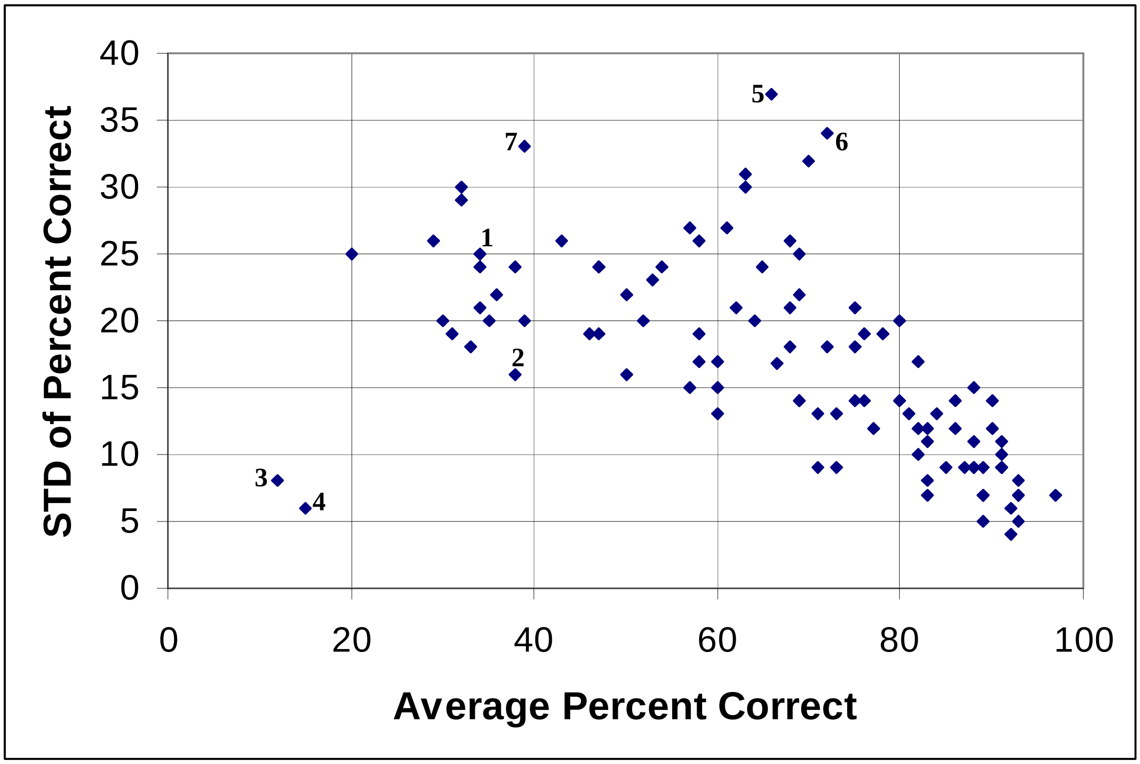

Figure 1: Each point represents one of the 101 questions on differential calculus for which we have data from at least five classes. On the horizontal axis we plot the average percentage of students who voted correctly, and on the vertical axis we plot the standard deviation between the percentages of students who voted correctly in the different classes. We have numbered the questions that are presented in this paper.

We currently have a set of 192 questions on differential calculus, and we have past voting statistics to accompany most of these. In this analysis, we choose to study only questions where we have statistics from at least five classes, which limits the data set to 101 questions. For each question we calculated the average percentage of correct votes across all classes, as well as the standard deviation of correct vote percentages across all classes. Figure 1 shows the standard deviation of the percentage of students who vote correctly on each question versus the average of the percentage of students who voted correctly. If we consider the entire set of questions as a whole, the average percentage of correct votes for each question is 67%, with an average standard deviation of 17%. Further, 37% of questions had an average of more than 80% of students vote correctly. This indicates that overall, students tend to do fairly well on most of these questions. We also see that in general, the easier questions tend to have lower standard deviations, while the more difficult questions tend to have higher standard deviations. When we look at the right side of Figure 1, where on average the largest percentage of students voted correctly, we find questions that tend to be quick and easy practice exercises. They may require only a minute before the students have voted, and they concern an essential subject, so they are often selected by the instructor, with the rationale that it is better for students to work examples for themselves, rather than to merely watch the instructor do them. However, these questions contain little subtlety and do not provoke substantial discussions, which is where the real value lies. Thus we will focus this paper on questions which we have found to be of substantial pedagogical value, which we have numbered in Figure 1. We have grouped the questions into three categories: questions producing the most diverse responses (Questions 1 and 2, left central in Figure 1), questions which attract large majorities of students to vote for a particular incorrect answer which we call *misconception magnets* (Questions 3 and 4, lower left in Figure 1), and questions producing large standard deviations (Questions 5, 6, and 7, upper left to upper right in Figure 1).

3. QUESTIONS PRODUCING THE MOST DIVERSE RESPONSES

The GoodQuestions study [9] indicates that the pre-vote peer-to-peer discussions are essential for the success of classroom voting. If all students agree on the answer, even incorrectly, then we would expect that there would be less pre-vote discussion. Instead, we would prefer that there be several plausible options that different students can defend. Thus, one way that we can use voting statistics to look for the most useful questions is to look for questions with the most diverse responses, where no option usually receives a majority. We want to find questions where the winner of the vote receives as small a percentage as possible, where we define the winner as the option receiving the most votes, regardless of whether or not it is the correct answer. Thus for each class that voted on a question, we select the percentage that voted for the winner, and take the average of these over all the classes. Then we look for the questions where this average is as small as possible. These appear in Figure 1 as Questions 1 and 2.

The question with the smallest average winner is from the beginning of the term, the first class period where we consider average and instantaneous velocity. We begin the class with a quick warm-up question, asking what information you would need in order to calculate average velocity. After the first question is resolved, we ask this follow-up, based on a question that accompanies the Hughes-Hallett *et al.* calculus text [11]:

Question 1:

The speedometer in my car is broken. In order to find my velocity at the instant I hit a speed trap, I need

i. the total distance of the trip

ii. the time spent traveling

iii. the number of stops I made during the trip

iv. a friend with a stopwatch

v. a working odometer

vi. none of the above

Select the best combination:

a. i, ii, & iii only

b. i & ii only

c. iv & v only

d. vi

e. a combination that is not listed here

	a)	**b)**	**c)**	**d)**	**e)**
Class 1	5%	20%	**5%**	20%	35%
Class 2	4%	40%	**36%**	20%	0%
Class 3	5%	20%	**30%**	15%	20%
Class 4	0%	41%	**7%**	38%	14%
Class 5	0%	23%	**62%**	0%	8%
Class 6	5%	5%	**26%**	32%	32%
Class 7	0%	15%	**70%**	15%	0%
Avg.	3%	23%	**34%**	20%	16%

Table 1: Here we present the voting results from Question 1 which was used in seven different classes. The column in bold indicates the correct answer.

These results are graphed and labeled as data point 1 in Figure 1. We see that on average only 34% of students vote correctly, so this is a regularly challenging question. However for this analysis we take the average of the winning percentages from each class (35%, 40%, 30%, 41%, 62%, 32%, and 70%) to find that the average winner gets 43% of the vote, which is smaller than any other question in our collection. This statistic identifies it as a good discussion question, and our experience confirms this.

It typically takes about 2 minutes for the students to read, work through, and discuss this question, before roughly 3/4 of the class has registered a vote. We call for the remaining students to finish up, and when the votes are in, we display a graph showing the class distribution. We begin the discussion by selecting a student by name, asking this person what they voted for and why, then going on to two or three other students, posing the same question. At this point we try to provide no feedback to the students as to whether their answers are correct or incorrect. Substantial numbers of students vote for all options except (a), so with this question we will hear some contrasting perspectives by surveying just a few students. It is often useful to take notes of the key ideas, either correct ones or incorrect ones, which are presented by students, in order to clarify the discussion. Later, when the discussion is resolved, we go back and explicitly cross out anything written on the board which is incorrect. When a student defends a vote for (e), we ask what combination they would need to calculate the speed, and write this on the board.

Usually a student who voted for (c), (d), or (e) will explain why (b) is incorrect in the course of justifying their own vote, stating that this information could only give the average velocity over the entire trip, rather than the instantaneous velocity. If this doesn't come up naturally, we can specifically ask one of these students why (b) wouldn't work. Occasionally, a student voting for (b) will volunteer to defend this answer, pointing out that it would work if you were driving at a constant speed for the whole trip. At this point we confirm for the class that (b) is not a good answer, because we want a method that would work no matter how you were driving.

The discussion between students voting for (c) and those voting for (d) and (e) is more interesting. To bring the issues out most fully, it can be useful to ask students to respond to each other's arguments: One student explains why they voted for (c), the next student explains why they voted for (d), and then we go back to the first student to ask them something like "Do you disagree with that? What makes you think differently?"

This discussion does not usually resolve itself, because these three answers are all reasonable. After students have articulated the key points, we resolve the discussion by stating that (c) is probably the best answer, because with an odometer and a stopwatch you could measure the period of time that it took to travel a short distance and calculate the velocity. However we acknowledge that (d) and (e) can also be defended, because even with option (c) we are still calculating an average velocity, rather than a truly instantaneous one. This ambiguity in the question means that we would never use it in this form on an exam for assessment, but it makes the question a very effective tool for stimulating in-class discussion.

To build on the discussion, we point out that if we measure the average velocity over smaller intervals, it will become a better approximation of the true instantaneous velocity, which brings us to the key idea behind the derivative as a limit.

The question with the next lowest average winner comes from the lesson on second derivatives, also from [11]. This is a question that we ask at the end of the class period, after we have already discussed the relationship between the second derivative and concavity, acceleration, and worked through several other graphical questions.

Question 2:

In *Star Trek: First Contact*, Worf almost gets knocked into space by the Borg. Assume he was knocked into space and his space suit was equipped with thrusters. Worf fires his thrusters for 1 second, which produces a constant acceleration in the positive direction. In the next second he turns off his thrusters. In the third second he fires his thruster producing a constant negative acceleration. The acceleration as a function of time is given in Figure 2.31. Which of the following graphs represent his position as a function of time?

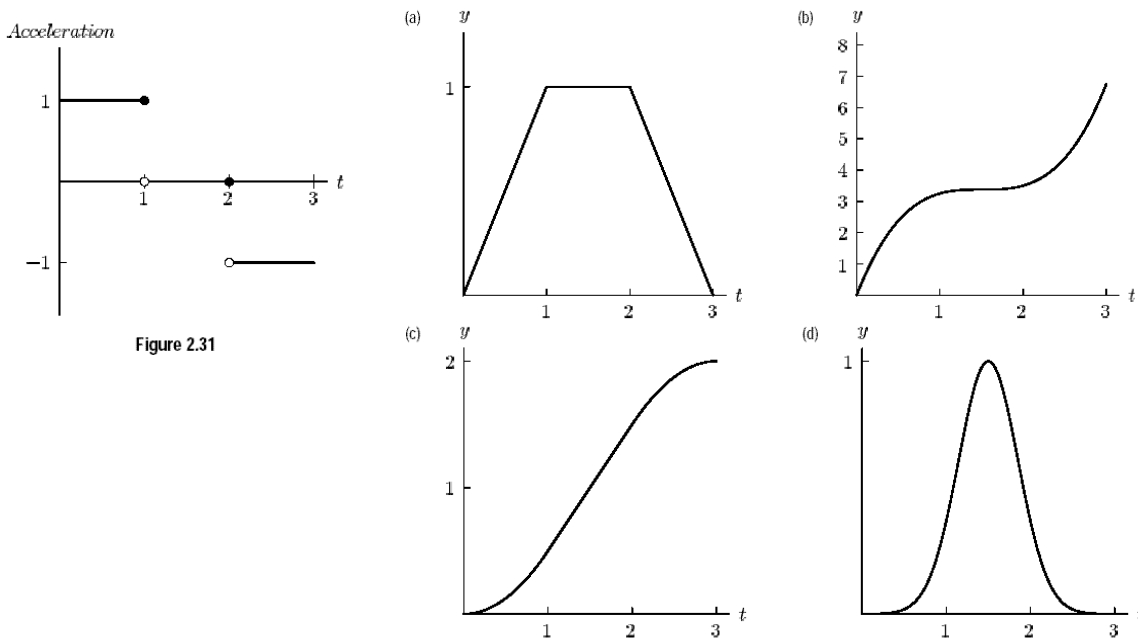

Figure 2: These graphs accompany Question 2, which is part of the collection published in [11]. The figures show a graph of acceleration versus time, and four possible corresponding graphs of velocity versus time.

	a)	**b)**	**c)**	**d)**
Class 1	45%	0%	**35%**	20%
Class 2	13%	0%	**48%**	39%
Class 3	40%	5%	**40%**	15%
Class 4	48%	7%	**7%**	37%
Class 5	35%	0%	**41%**	18%
Class 6	23%	0%	**54%**	23%
Avg.	34%	2%	**38%**	25%

Table 2: Here we present the voting results from Question 2. The column in bold indicates the correct answer.

This question takes between 2 and 4 minutes for most students to register a vote. Here we see that in only one of these classes did any option receive a majority. Significant numbers of students regularly vote for (a), (c), and (d), with the average winner receiving 44% (averaging the largest percentage from each of the six classes, 45%, 48%, 40%, 48%, 41%, and 54%). Again, this broad distribution means that when we call on three or four students, we will almost certainly hear some contrasting perspectives. If we have a substantial number of students voting for a particular option, but we don't happen to call on any students who voted this way, we might ask for volunteers: "Could someone who voted for (d) explain this choice?"

Students who vote for (c) or (d) will usually explain why answer (a) indicates confusion between the first derivative and the second, without any prompting. This is often the first issue addressed in the discussion, and we can confirm that (a) is incorrect. This leaves us with students willing to defend (c) and (d). This discussion often resolves naturally, when a student voting for (c) points out that a zero second derivative between $t = 1$ and $t = 2$, means that the function should have no concavity and be linear on this interval. Similarly, students may point out that (d) shows positive concavity for $t > 2$, when the acceleration should be negative. If these do not come up clearly, we can directly ask a student who voted for (c) why they think option (d) is wrong, and usually these points will come out.

It is interesting that few if any students ever vote for (b), which shows zero velocity for $1 < t < 2$. This is probably because the concavities are reversed with this graph showing negative acceleration in the first interval, and positive acceleration in the third. It might be interesting to modify this option, giving it the right concavities, and then add option (e) "More than one of the above are possible" as the correct answer, to emphasize that we can have a nonzero first derivative while the second derivative is zero.

4. MISCONCEPTION MAGNETS

Another way in which classroom voting can produce a very effective learning environment is when the question provokes a majority of students to vote for one particular incorrect answer. When this is revealed to the students, they are usually quite surprised that the majority is wrong, and they are very curious to learn what they missed, resulting in a powerful teachable moment. We can identify these misconception magnets by taking each question, identifying the most popular incorrect answer, and calculating the average percentage of students voting for this option. We can then look for the questions where the largest average percentage of students voted for the most popular incorrect answer. When we search our data in this way, we find that the two best misconception magnets are also quite apparent in Figure 1: These are the two questions in the lower left of the plot, questions 3 and 4, which have the smallest average percentage of correct votes as well as very low standard deviations.

Question 3 is from the class period on derivatives of trigonometric functions, which follows the period where students learned the product rule. This particular question is generally asked late in the period, after the students have worked through other voting questions, requiring them to take derivatives of simpler trigonometrically based functions, such as $f(x) = -3\sin x$, or asking for the 30th derivative of $\cos x$.

Question 3:

If $f(x) = \sin x \cos x$, then $f'(x) =$

a. $1 - 2\sin^2 x$

b. $2\cos^2 x - 1$

c. $\cos 2x$

d. All of the above

e. None of the above

We have statistics from seven classes which voted on this question, ranging from fall 2005, to fall 2007, which show that on average only 12% of students voted correctly, with a standard deviation of 8.3%. Also, the incorrect votes overwhelmingly favor option (e).

	a)	b)	c)	d)	e)
Class 1	10%	0%	10%	**10%**	70%
Class 2	40%	32%	8%	**4%**	16%
Class 3	3%	6%	6%	**15%**	70%
Class 4	4%	0%	11%	**0%**	85%
Class 5	0%	22%	0%	**17%**	61%
Class 6	0%	20%	0%	**25%**	55%
Class 7	0%	43%	0%	**13%**	44%
Avg.	8%	18%	5%	**12%**	57%

Table 3: Here we present the voting results from Question 3. The column in bold indicates the correct answer.

If this question was being used for assessment, these results would be a disaster. However, when used for teaching, this is an excellent question. Most students apply the product rule in a straight-forward manner, getting $f'(x) = \cos^2 x - \sin^2 x$, which is not among the options, and so a majority usually votes for (e). The post-vote discussion often begins with a student explaining this reasoning, but then another student objects that a calculator has indicated option (a) or (b). At this point the instructor may ask other students to try this out on their calculators and confirm whether this is right. This often produces considerable confusion among the students, as they tend to think that both expressions can't be right. The instructor may then graph the functions or help the students to algebraically work out the equivalence of these functions. (a) and (b) are not difficult, as they both follow from the identity $\sin^2 x + \cos^2 x = 1$. As a whole, this question serves as an important reminder that there may be many ways to analytically represent a function, especially if it is composed of trigonometric functions.

The other of these misconception magnets is from the Cornell GoodQuestions project, and is on the topic of the Mean Value Theorem (MVT), which says that if *f* is a continuous and differentiable function on some interval, then at some point within that interval the derivative of the function must be equal to the average slope over the entire interval. A theorem like this can often be very abstract and difficult for students to interpret, so we give them a tangible scenario, where they already have substantial intuition:

Question 4:

Two racers start a race at the same moment and finish in a tie. Which of the following must be true?

a. At some point during the race the two racers were not tied.

b. The racers' speeds at the end of the race must have been exactly the same.

c. The racers must have had the same speed at exactly the same time at some point in the race.

d. The racers had to have the same speed at some moment, but not necessarily at exactly the same time.

The statistics on this question show only an average of 15% voting correctly with a standard deviation of 6.0%, while on average 81% chose answer (d). Students often learn to be cautious about our use of definitive language in mathematics, which is generally a good thing, but as a result they tend to vote for weaker options like (d), afraid that there may be unseen counterexamples to more strongly worded options like (c).

	a)	**b)**	**c)**	**d)**
Class 1	0%	10%	**25%**	65%
Class 2	0%	0%	**8%**	92%
Class 3	5%	5%	**10%**	80%
Class 4	4%	0%	**14%**	82%
Class 5	0%	0%	**17%**	83%
Class 6	0%	0%	**14%**	86%
Avg.	2%	3%	**15%**	81%

Table 4: Here we present the voting results from Question 4. The column in bold indicates the correct answer.

In the post-vote discussions of this question it is usually clear that while most students have some intuition, they cannot see how to apply the MVT. Thus, after a few opinions have been shared, it can be helpful to directly ask the students how we can use the MVT to analyze this scenario. This is usually quite a puzzle, because the MVT concerns a single function, its average slope and its derivative, while here we have two functions, each giving the position of one of the racers, and we're interested in when these two derivatives are equal, not when they are equal to an average slope. It may be necessary for the instructor to suggest that our function could be the difference between the two runner's positions. We can then sketch out several possible graphs for this scenario, showing what happens if the racers stay tied the whole time, or if different racers lead at different times. Next, we may ask students to consider the meaning of the derivative of this difference function. What does it mean when the slope of this function is positive, negative, or zero? From this point, it is a little more straight-forward, and students can see that the average slope of the difference function must be zero, because the racers begin and end in a tie, and further that if the derivative of the difference function is zero, this means that the racers are going at the same speed, and so the MVT tells us that (c) must be true.

This question may require fifteen minutes of classroom discussion or more, but in the process we can work through the implications of the MVT in great detail. This question can be a powerful tool to motivate the class, to pique student interest, and to focus the resulting discussion. Questions like this are very potent, because they connect theoretical ideas with tangible situations, drawing out the students' intuitive ideas and linking them to the general mathematical principles.

5. LARGE STANDARD DEVIATIONS

The questions along the top of Figure 1 also stand out. Questions with large standard deviations sometimes indicate good checkpoint questions, where some classes do very well, while others do very poorly, allowing the instructor to quickly assess whether the class has mastered a particular concept. However, other questions with large standard deviations instead may indicate that a question has been used in very different ways by different instructors.

For example, the following question comes from the beginning of the lesson where we show that the derivative of $\ln(x)$ is $\frac{1}{x}$:

Question 5:

$\frac{d}{dt}\ln\left(t^2+1\right)$ is

a. $2t\ln\left(t^2+1\right)$

b. $\frac{2t}{t^2+1}$

c. $\frac{dt}{\ln\left(t^2+1\right)}$

d. $\frac{1}{t^2+1}$

	a)	**b)**	**c)**	**d)**
Class 1	0%	**84%**	0%	16%
Class 2	10%	**80%**	0%	10%
Class 3	4%	**80%**	0%	16%
Class 4	0%	**88%**	0%	12%
Class 5	0%	**0%**	50%	50%
Avg.	3%	**66%**	10%	21%

Table 5: Here we present the voting results from Question 5. The column in bold indicates the correct answer.

We see that the question is remarkably consistent, with the exception of the results in Class 5, which is what gives these results the largest standard deviation in our data set (STD = 37%). The first four classes used this as a simple practice question, to apply the chain rule and the natural log rule in a straight-forward context. However, in class 5, the instructor used the question in a very different way: This question was asked to open the period, before the natural log rule had been presented in class, with the expectation that students should have read this section in the text before coming to class. The students referred to their text in class, worked together, and settled for options (c) and (d). No one voted correctly, but the post-vote discussion revealed that this was a very effective teaching moment and, when prompted, the students quickly realized that they needed to apply the chain rule in conjunction with the natural log rule.

The question with the next largest standard deviation (from [11]) does indeed appear to serve as an effective checkpoint question:

Question 6:

Let $f(x) = ax + b/x$. Suppose *a* and *b* are positive. What happens to $f(x)$ as *a* increases?

a. The critical *points* move further apart.

b. The critical *points* move closer together.

	a)	**b)**
Class 1	4%	**96%**
Class 2	0%	**100%**
Class 3	82%	**18%**
Class 4	13%	**87%**
Class 5	40%	**60%**
Avg.	28%	**72%**

Table 6: Here we present the voting results from Question 6. The column in bold indicates the correct answer.

Although an average of 72% of students vote correctly, we see that the percentage of students voting correctly varies from 18% to 100% with a standard deviation of 34%. Most of our students attempt this question with optimization and the use of parameters, which are two of the most challenging topics in differential calculus (although it could also be analyzed geometrically). Students must take a derivative, set it equal to zero, solve for the critical points, and interpret how this expression will be affected by changes in a parameter, thus there are several places where errors can be made which would cause students to get the incorrect result. Thus the question gives the instructor feedback as to whether students can perform and synthesize these steps.

Another good checkpoint question is from the beginning of the class period where we first learn to use derivatives for optimization:

Question 7:

True or False: A local maximum of *f* only occurs at a point where $f'(x) = 0$.

	T)	**F)**
Class 1	90%	**10%**
Class 2	40%	**60%**
Class 3	88%	**12%**
Class 4	80%	**20%**
Class 5	75%	**25%**
Class 6	53%	**47%**
Class 7	0%	**100%**
Avg.	61%	**39%**

Table 7: Here we present the voting results from Question 7. The column in bold indicates the correct answer.

On average only 39% of students get this one right, with a standard deviation of 33%. The votes on this question are probably so diverse because the question hinges on whether students recognize that we may have a maximum at a point where the derivative is undefined. Either this has already come up in class discussions and been absorbed by the students or it hasn't.

6. CONCLUSIONS

Classroom voting requires a substantial amount of class time, with the votes and discussions often occupying more than half of a class period. However, this is not time lost. For the past several years we have covered exactly the same material in our calculus classes and given the same types of exams that we did before we used classroom voting. We did not eliminate any topics from our courses to make room for voting. While in the past we covered the topics through traditional lecture, by working examples on the board, and by asking questions of the class as a whole, more recently we have learned to use the voting to bring up these same issues. By posing a few carefully chosen multiple-choice questions and giving students the opportunity to work them out individually and then discuss them in small groups, the students become more invested in the lesson, and more curious to see how the particular issues are resolved. Thus voting tends to make the class more focused, making class time more productive. Further, we tend to do fewer examples on the board, using voting to make the students to work the examples out for themselves. As a result, the course can go at the same pace, teaching the same topics in a more student-focused way.

We have found that studying past voting statistics can be a useful method for identifying productive voting questions when preparing a lesson plan. Class time is precious, and each voting question usually takes between 5 and 10 minutes from beginning to end. Thus it is particularly important to choose questions that will provoke the richest discussions and raise significant issues, seriously engaging the students before the vote, and result in a post-vote discussion that will address key goals of the class period. Our experience is that generally, the questions where majorities of students regularly vote incorrectly are more likely to produce the most valuable classroom discussions. When preparing a lesson plan, we review all of the relevant questions available, selecting those that focus on the central topics and giving preference to questions which regularly produce voting patterns that indicate a particularly diverse set of responses, a question which effectively provokes a common misconception, or one that may serve as an effective checkpoint question for this material.

As a whole, we have found that when used thoughtfully, classroom voting can have several dramatic and positive effects. It prevents the students from passively observing the class, and instead requires each person to engage in the material and to actively participate in small group discussions. It provides immediate feedback to both the instructor and the students as to the students' level of understanding. Voting is a powerful springboard for creating class-wide discussions where students are highly involved, because they have formed an opinion about the current issue and they have voted their opinion. This gives students a real stake in the discussion, which increases their level of engagement enormously. Further, when classroom voting is used in this way, we find that students have more fun, they enjoy class more, and strongly prefer this teaching method, with our post-course surveys indicating that 74% would choose to take a section of a mathematics course using voting rather than one without [17]. By using classroom voting, we create an environment where our students find enjoyment and take pleasure in working through substantial mathematics, as a result of which our students learn in a deep and effective way.

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BIOGRAPHICAL SKETCHES

Originally from Homer, Alaska, Kelly Cline is an associate professor of mathematics at Carroll College in Helena, Montana. His interests include innovative teaching methods and technologies, including classroom voting and online homework systems. He is PI on the NSF-Funded Project MathVote to study the use of classroom voting in collegiate mathematics.

Holly Zullo is an Associate Professor of Mathematics at Carroll College, in Helena, Montana. She has been using classroom voting in calculus and other courses for several years, and she was PI on the NSF-funded Project MathQuest to develop voting questions for linear algebra and differential equations.

Lahna VonEpps has returned to her *alma mater* as mathematics instructor at Columbia College in Sonora, CA. Her pedagogical goals are to reduce mathematics anxiety while actively engaging students in their learning using classroom voting and other fun techniques.