A Question Library for Classroom Voting

Voting in the classroom can engage students and promote discussion. All that is needed is a good set of questions

Take a minute and imagine the ideal classroom learning environment. What would it be like? How would students learn? What would they be doing? Certainly, each student would be actively engaged in the lesson, exploring and discovering the key points. Perhaps students would work collaboratively, discussing various concepts and figuring out central ideas for themselves. The teacher would be responsive to each student’s ideas and reactions.

This type of learning environment is what we work to create using classroom voting. In this pedagogy, the teacher poses a multiple-choice question to the class, allows a few minutes for consideration and small-group discussion, calls on each student to vote on the correct answer, often with an electronic handheld clicker, and then leads a class discussion of the results. The vote itself allows the teacher to hear quickly from every member of the class. Perhaps more important, the act of voting motivates students to participate in discussions about the question both before and after the vote.

The research on this method has produced positive results in a wide range of disciplines. In particular, researchers have found the following to be true:

- Peer discussions that occur in tandem with classroom voting enhance students’ conceptual understanding (Smith et al. 2009).
- Classroom voting can produce measurable increases in student learning (MacArthur and Jones 2008).
- This method is particularly effective for students with less background knowledge of the subject (Lasry et al. 2008).
- Classroom voting allows every student in the class to engage in the material, form opinions, and participate in discussions; thus, it can be a positive way to enliven a mathematics lesson (see, e.g., Cline 2006; Rode et al. 2009; Lucas 2009).
- Classroom voting has been shown to improve student exam scores (Miller et al. 2006).
- Large majorities of students report that they enjoy this interactive teaching method and prefer it to more traditional instruction (Zullo et al. 2011).

For teachers, the challenge in using classroom voting lies in writing good multiple-choice questions that address key concepts. Fortunately, with the generous support of the National Science Foundation and a network of collaborators across the country, we have assembled a growing Web-based library of more than two thousand multiple-choice questions designed for classroom voting. These questions cover a wide range of topics common to high school and college mathematics courses and are free for interested teachers. Many questions are accompanied by teachers’ comments and a summary of votes on the question in previous classes (the percentage of students voting for each option). This information can be useful when selecting questions for an upcoming lesson.

A BRIEF HISTORY OF PROJECTS MATHQUEST AND MATHVOTE

In fall 2003, we began compiling questions to meet our own needs of engaging students in the courses we were teaching. The effectiveness of classroom voting in stimulating student discussions was immediately apparent, receiving instant feedback from every student in the class was extremely useful, and students enjoyed the process. We decided to use this technique several times in almost every class period.

Initially, we drew questions from a wide variety of sources, including the Cornell GoodQuestions project (Miller et al. 2006), questions written for the Hughes-Hallett et al. calculus textbook (2002,
Suppose that a teacher is preparing to teach the logarithm function to solve equations. The question library contains twenty-five questions currently available, broken down by topic and course needs of our courses that were not addressed in other sets of questions. By sharing our questions and results on the Internet, we found more colleagues excited about collaborating and contributing to this project. Table 1 shows the number of questions currently available, broken down by topic. (go to http://mathquest.carroll.edu). Table 1: Numbers of Questions by Course

<table>
<thead>
<tr>
<th>Course</th>
<th>Number of Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algebra</td>
<td>137</td>
</tr>
<tr>
<td>Statistics</td>
<td>107</td>
</tr>
<tr>
<td>Precalculus</td>
<td>226</td>
</tr>
<tr>
<td>Differential calculus</td>
<td>192</td>
</tr>
<tr>
<td>Integral calculus</td>
<td>151</td>
</tr>
<tr>
<td>Multivariable calculus</td>
<td>317</td>
</tr>
<tr>
<td>Linear algebra</td>
<td>311</td>
</tr>
<tr>
<td>Differential equations</td>
<td>350</td>
</tr>
<tr>
<td>Other topics</td>
<td>214</td>
</tr>
</tbody>
</table>

SELECTING QUESTIONS TO USE IN CLASS: A QUICK CHECK

By voting, students commit themselves in the question and contributing to the suspense as to its resolution. Further, students usually expect the majority to be correct. When a question produces only a slim majority or no majority at all, students are often surprised and intrigued, curious to find out how they reached an incorrect conclusion. Here is a question that can be used to uncover common student errors when considering division by zero in a prealgebra or algebra class:

\[ \frac{12}{2} = ? \]

(a) 12
(b) 0
(c) Undefined because there is no unique \( e \) to satisfy the equation \( 0 \cdot e = 0 \)
(d) Undefined because there is no integer \( e \), such that \( 0 \cdot e = 12 \)

When first introduced to division by zero, some students ignore the zero and report 12 as the answer (answer [a]). Other students believe that if zero appears anywhere in a division problem, the answer is zero (answer [b]). Still other students recognize that this problem is undefined but mistakenly conclude that there are infinitely many solutions to the corresponding multiplication problem (answer [c]). One author used a version of this question with her students and noted that votes were spread across all four options. In one follow-up discussion, a student defended her answer of (d) by relying on the definition of division: \( a = b \cdot c \) if and only if there is a unique \( c \) such that \( b \cdot c = a \). She argued mathematically as follows:

\[ \frac{12}{2} = 6 \] because \( 2 \cdot 6 = 12 \]

\[ \frac{12}{2} = 0 \] because \( 2 \cdot 0 = 12 \]

\[ \frac{12}{2} = 1 \] because \( 1 \cdot 12 = 12 \]

The teacher did not intervene once except to call on students to offer their explanations—the students were doing all the important mathematics!
VOTING IN CLASS FOR THE FIRST TIME
With classroom voting, as with any new pedagogy, teachers should lay the groundwork with their students. Students should be aware that they will be discussing multiple-choice questions with their classmates and that the quality of these discussions is as important as getting the correct answer.

Establishing the following simple rules helps ensure that all students engage in a discussion about the question:

- Everyone must vote.
- No one is allowed to vote until he or she has discussed the question with at least one other person.
- After the vote, several students will be called on to explain their votes.

With respect to this last point, students need to know that whether their vote is correct or not does not matter, as long as they can explain their thinking. The only unacceptable response is “I just guessed.”

After the vote, if a strong majority of students vote correctly, the discussion is usually short. However, when student responses are more evenly spread across the possibilities, there is a good opportunity for a rich class discussion. Asking specific students, “What did you vote for and why?” is a good way to get the conversation started. After the first student explains, we typically give no feedback, and we ask the same question of another student, encouraging students to figure out the correct answer for themselves. If students express contradictory arguments, ask them to respond to one another.

For example, let’s again consider the second question and its choices:

Solve for $x$ if $\ln x + e^x = 4$.

(a) $x = \frac{\ln \mu - 1}{\ln 2}$
(b) $x = \frac{\ln (\mu + 1)}{\ln 2}$
(c) $x = \frac{\ln y}{\ln 2}$
(d) $x = \frac{\ln (y + e)}{\ln 2}$

After a student explains why he voted for (a), the instructor could turn to a student who previously explained a vote for (d) and ask what she thinks about this reasoning. This approach will spur students to discuss whether distributing a natural logarithm function across two terms in a sum is permitted. We encourage students to reason through the problem themselves, and we confirm the correct answer after a majority of them have already reached the correct answer.

Class votes can be conducted in several ways. If no technology is available, students can simply raise their hands. To allow students to vote simultaneously, the teacher can ask students to hold up fingers. Another method is to hand out a set of four colored index cards for students to hold up (the red card stands for A, the blue card stands for B, etc.).

Using clickers (a classroom response system), if a set is available, can be the best way to conduct a class vote, for several reasons. First, clickers make individual votes more anonymous or at least less public; no one student can see how others vote. This anonymity encourages participation from students with less self-confidence than others.

Second, clickers help with time management; students can vote as soon as they are ready. As the votes trickle in, the teacher can judge when a sufficient number have been registered. Usually, when about 60 or 75 percent of a class has voted, it is time to call for the rest of the class to finish up and then close the vote.

Third, clickers make the results more precise. If not everyone has voted, this discrepancy is obvious, and the teacher can announce to the class: “We have thirty people in the room, but only twenty-eight votes. Everyone must vote, so please click in now!” After the vote, the clicker software can instantly present a precise bar graph of the results, indicating the exact number of students who voted for each option.

Questions can be presented to the students in a variety of ways. The questions on our website are numbered and can be downloaded in PDF form, to be printed, copied, and handed out to students. During class, the teacher can simply announce the number of the question to work on, and students can annotate their copy, work in the margins, and note the correct answer for later studying. Many classroom response systems are integrated with PowerPoint®, so another option is to present the questions in this way. This process may be a bit more time-consuming to set up; teachers must retype each question into PowerPoint or download a large-font version of the questions to cut and paste images of this into PowerPoint.

A TOOL TO ENGAGE ALL STUDENTS
Our library of questions has grown into a substantial resource, providing a starting place for anyone who wants to try classroom voting for the first time.

Table 3 Voting Statistics for Question about Normal Deviation

<table>
<thead>
<tr>
<th>Class</th>
<th>% for (a)</th>
<th>% for (b)</th>
<th>% for (c)</th>
<th>% for (d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class 1</td>
<td>52%</td>
<td>43%</td>
<td>0%</td>
<td>5%</td>
</tr>
<tr>
<td>Class 2</td>
<td>39%</td>
<td>26%</td>
<td>6%</td>
<td>29%</td>
</tr>
</tbody>
</table>

Next, let’s look at a statistics question that can be used to stimulate discussions about normal distribution (see fig. 1 and table 3). The intent of this question is to assess students’ understanding of the empirical rule and of how the mean and the standard deviation affect the shape of the normal curve. Students need to understand that the mean is the center of the distribution and that the standard deviation determines the spread of the graph. In this case, if students understand the role of the mean, they can quickly eliminate (c) because its center is less than 20; the voting statistics indicate that most students can do so.

Next, students have to apply what they know about the standard deviation and the empirical rule. Specifically, they can use the fact that approximately 68 percent of the area under the curve should fall within one standard deviation of the mean to conclude that the standard deviation in (a) is too large and in (c) is too small, leaving (b) as the correct answer. The voting statistics show that eliminating (a) and (d) is more challenging. Because significant numbers of students voted for these options, this question can be useful for discussion, so we ask students to explain their differing points of view.

Questions that involve the use of graphs or pictures, such as this one, can engage students in discussion more quickly. Students can rely more on their intuition and understanding about fundamental ideas to get started thinking about the question.
Open the Door and Keep It Open

Mathematics Teacher is eager to publish articles illustrating ways in which teachers of entry-level courses open the door for students to learn mathematics. These courses are critical to fostering students’ pursuit of and love for learning mathematics through the high school years and beyond.

What instructional methods do you find effective when teaching prealgebra, algebra, geometry, or first- and second-year integrated courses? What strategies are successful in addressing the needs of all students in our classrooms?

The MT Editorial Panel is soliciting manuscripts that address any of the following topics:

• Successful strategies for setting, communicating, and supporting high expectations for mathematical learning, especially for students who struggle
• Innovative ways to teach concepts in entry-level courses that work with students of all academic levels within the classroom
• Sample lessons or lesson sequences that spur students’ mathematical thinking, even after the bell rings
• Approaches that build on students’ interests or community heritage
• Examples of noteworthy lessons based on Common Core State Standards that have supported student learning
• Tips for encouraging students’ multiple inquiry approaches built on a foundation of reasoning and sense making
• Classroom ideas that inspire students to take mathematics courses beyond minimum graduation requirements
• Use of manipulatives, multimedia, or technology that make mathematics accessible, hands-on, or interactive for students

Share your best lessons or strategies so that we can open the door and keep it open for all students to learn mathematics.

You may submit your completed manuscript for review by accessing mt.submit.net. Indicate that the manuscript is being submitted in response to the call Open the Door. Be sure to enter the call’s title in the Department/ Calls field. No author identification should appear in the text of the manuscript. Additional guidelines for the preparation of manuscripts can be found at www.nctm.org/publications/content.aspx?id=22602.