A Question Library for Classroom Voting

Voting in the classroom can engage students and promote discussion. All that is needed is a good set of questions

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Take a minute and imagine the ideal classroom learning environment. What would it be like? How would students learn? What would they be doing? Certainly, each student would be actively engaged in the lesson, exploring and discovering the key points. Perhaps students would work collaboratively, discussing various concepts and figuring out central ideas for themselves. The teacher would be responsive to each student's ideas and reactions.

This type of learning environment is what we work to create using classroom voting. In this pedagogy, the teacher poses a multiple-choice question to the class, allows a few minutes for consideration and small-group discussion, calls on each student to vote on the correct answer, often with an electronic handheld clicker, and then leads a class discussion of the results. The vote itself allows the teacher to hear quickly from every member of the class. Perhaps more important, the act of voting motivates students to participate in discussions about the question both before and after the vote.

The research on this method has produced positive results in a wide range of disciplines. In particular, researchers have found the following to be true:

- Peer discussions that occur in tandem with classroom voting enhance students' conceptual understanding (Smith et al. 2009).
- Classroom voting can produce measurable increases in student learning (MacArthur and Jones 2008).
- This method is particularly effective for students with less background knowledge of the subject (Lasry et al. 2008).
- Classroom voting allows every student in the class to engage in the material, form opinions, and participate in discussions; thus, it can be a positive way to enliven a mathematics lesson (see, e.g., Cline 2006; Bode et al. 2009; Lucas 2009).
- Classroom voting has been shown to improve student exam scores (Miller et al. 2006).
- Large majorities of students report that they enjoy this interactive teaching method and prefer it to more traditional instruction (Zullo et al. 2011).

For teachers, the challenge in using classroom voting lies in writing good multiple-choice questions that address key concepts. Fortunately, with the generous support of the National Science Foundation and a network of collaborators across the country, we have assembled a growing Web-based library of more than two thousand multiple-choice questions designed for classroom voting. These questions cover a wide range of topics common to high school and college mathematics courses and are free for interested teachers. Many questions are accompanied by teachers' comments and a summary of votes on the question in previous classes (the percentage of students voting for each option). This information can be useful when selecting questions for an upcoming lesson.

A BRIEF HISTORY OF PROJECTS MATHQUEST AND MATHVOTE

In fall 2003, we began compiling questions to meet our own needs of engaging students in the courses we were teaching. The effectiveness of classroom voting in stimulating student discussions was immediately apparent, receiving instant feedback from every student in the class was extremely useful, and students enjoyed the process. We decided to use this technique several times in almost every class period.

Initially, we drew questions from a wide variety of sources, including the Cornell GoodQuestions project (Miller et al. 2006), questions written for the Hughes-Hallett et al. calculus textbook (2002;

see also Pilzer et al. 2001), and a collection shared by Mark Schlatter of Centenary College (2002). Supported by two grants from the National Science Foundation, for Project MathQuest (2006–9) and Project MathVote (2010–12), we wrote a large number of questions to address content-specific needs of our courses that were not addressed in other sets of questions. By sharing our questions and results on the Internet, we found more colleagues excited about collaborating and contributing to this project. **Table 1** shows the number of questions currently available, broken down by topic (go to http://mathquest.carroll.edu).

SELECTING QUESTIONS TO USE IN CLASS: A QUICK CHECK

How can teachers use this vast collection of questions to make voting work in their classrooms? First, the questions posed must be directly relevant to the material being covered. Also, the time spent on these questions must replace time spent covering that material in other ways. This approach is necessary both to maintain coverage of material and to gain student buy-in. If students see that classroom voting and related discussions help them work through the assignments that follow, they will take this approach more seriously. Conversely, if they perceive voting as an optional detour, they will not invest themselves in the discussion.

Suppose that a teacher is preparing to teach the use of the natural logarithm function to solve equations. The question library contains twenty-five voting questions on the topic of logarithms, including this one (Pilzer et al. 2003):

Solve for x if $8y = 3e^x$.
(*a*) $x = \ln 8 + \ln 3 + \ln y$
(*b*) $x = \ln 3 - \ln 8 + \ln y$
(*c*) $x = \ln 8 + \ln y - \ln 3$
(*d*) $x = \ln 3 - \ln 8 - \ln y$

This question is fairly straightforward. We take the natural logarithm of both sides, simplify, and solve for x. The distracters (i.e., wrong answers that reflect common student mistakes) probe for sign errors in the algebra. Teacher comments on this question include records of two previous votes, which indicate that more than 90 percent of both classes voted correctly. These results clearly identify this as a "quick check" question. This question will most likely give students some practice with the new ideas, build confidence, and not consume too much class time. Whenever possible, we ask students to work through the basic examples themselves, and we choose a smaller set of questions to work fully on the board. However, sometimes a basic question is not what we are looking for, and the voting statistics can help us distinguish between easy and difficult questions.

Table 1 Numbers of Questions by Course

Course	Number of Questions
Algebra	137
Statistics	107
Precalculus	226
Differential calculus	192
Integral calculus	151
Multivariable calculus	317
Linear algebra	311
Differential equations	350
Other topics	214

COMMON-ERROR QUESTIONS

If we want a more challenging question on the same topic, the voting statistics help us recognize that the following one (Pilzer et al. 2003) is a bit more interesting:

Solve for x if $y = e + 2^x$.

(*a*) $x = \dfrac{\ln y - 1}{\ln 2}$

(*b*) $x = \dfrac{\ln(y-1)}{\ln 2}$

(*c*) $x = \dfrac{\ln y}{\ln 2} - 1$

(*d*) $x = \dfrac{\ln(y-e)}{\ln 2}$

Table 2 shows voting statistics from two classes; in each case, the majority voted incorrectly.

Table 2 Voting Statistics for Question 2

Class	% for (*a*)	% for (*b*)	% for (*c*)	% for (*d*) (correct answer)
Class 1	54%	0%	0%	**46%**
Class 2	27%	0%	54%	**18%**

This question requires several algebraic steps. First, we subtract e, and then we apply the natural logarithm function to both sides of the equation. Then we must recall that $\ln(2^x) = x \cdot \ln 2$ before dividing both sides by $\ln 2$; thus, we find that (*d*) is the correct answer.

Often, the best voting questions are those for which many students choose incorrect answers. Note that in both classes substantial numbers of students voted for (*a*), indicating that they erroneously distributed the natural logarithm function over the two terms in the sum, thinking that $\ln(y - e) = \ln y - \ln e$. The question provokes this common mistake, and we can deal with this misconception clearly and definitively in class, before students make this error on a graded assignment.

By voting, students commit to a particular answer, investing themselves in the question and contributing to the suspense as to its resolution. Further, students usually expect the majority to be correct. When a question produces only a slim majority or no majority at all, students are often surprised and intrigued, curious to find out how they reached an incorrect conclusion.

Here is a question that can be used to uncover common student errors when considering division by zero in a prealgebra or algebra class:

$12 \div 0 = ?$
(*a*) 12
(*b*) 0
(*c*) Undefined because there is no unique c to satisfy the equation $0 \cdot c = 0$
(*d*) Undefined because there is no integer, c, such that $0 \cdot c = 12$

When first introduced to division by zero, some students ignore the zero and report 12 as the answer (answer [*a*]). Other students believe that if zero appears anywhere in a division problem, the answer is zero (answer [*b*]). Still other students recognize that this problem is undefined but mistakenly conclude that there are infinitely many solutions to the corresponding multiplication problem (answer [*c*]). One author used a version of this question with her students and noted that votes were spread across all four options. In one follow-up discussion, a student defended her answer of (*d*) by relying on the definition of *division*: $a \div b = c$ if and only if there is a unique c such that $b \cdot c = a$. She argued mathematically as follows:

$12 \div 3 = 4$ because $3 \cdot 4 = 12$
$12 \div 2 = 6$ because $2 \cdot 6 = 12$
$12 \div 1 = 12$ because $1 \cdot 12 = 12$

The teacher did not intervene once except to call on students to offer their explanations—the students were doing all the important mathematics!

So $12 \div 0$ is not possible because there is no number that we can multiply 0 by to get 12. The answer has to be (*d*).

The discussion also included a few other less convincing or incorrect arguments, such as "It's like, you have 12 pens and 0 students, and you want to know how many pens each student should get. Well, there are no students, so they don't have any pens, so the answer is 0." Then the students were asked to revote, and the majority voted for the correct answer. Note that the teacher did not intervene once except to call on students to offer their explanations—the students were doing all the important mathematics! This question also opened up a discussion about answer (*c*). An interesting discussion about the problem $0 \div 0$ ensued.

QUESTIONS TO PROVOKE DISCUSSIONS

Our collection has a great diversity of questions. Some are more procedural, like the ones above, whereas others deal with deeper conceptual issues, such as the following question from the Cornell GoodQuestions project (Miller et al. 2006). This question is designed to be presented after the topics of continuity and the intermediate value theorem have been introduced:

True or false? At some time since you were born, your weight in pounds equaled your height in inches.

This question regularly confounds students. We used this question nine times, and a majority voted "true" only three times. Perhaps even more important, both options regularly attract a substantial percentage of votes. These starkly contrasting views permit a substantive discussion to take place. Further, because this question concerns an applied, everyday scenario, it helps students connect mathematical ideas to the real world.

When asked to defend a "false" vote, students often argue that, whereas at some point one's height and weight may have been close, they were probably never exactly equal. Students might not be able to resolve this question themselves, so the teacher might directly ask: "Right now, which is greater, your weight in pounds or your height in inches? Which was greater when you were born?" Once students see how the intermediate value theorem can apply to this scenario, they usually realize that this assertion is true.

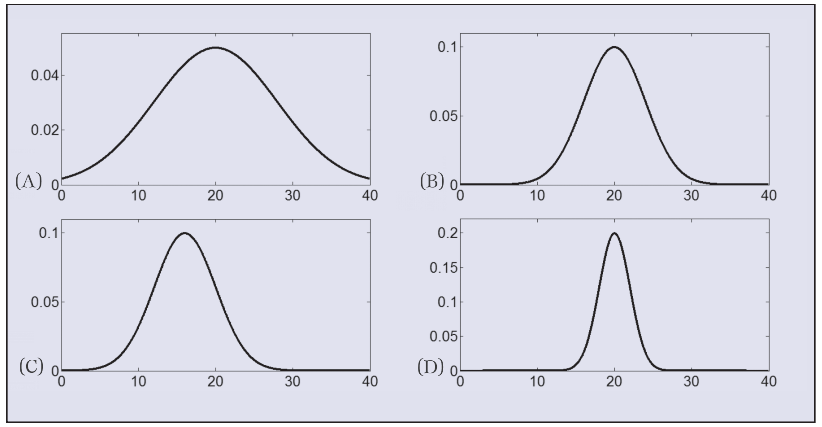

Fig. 1 If X is a normal random variable with $\mu = 20$ and standard deviation $\sigma = 4$, which of the graphs shown could be the graph of the probability density function of X?

Table 3 Voting Statistics for Question about Normal Deviation

Class	% for (*a*)	% for (*b*) (correct answer)	% for (*c*)	% for (*d*)
Class 1	52%	**43%**	0%	5%
Class 2	39%	**26%**	6%	29%

Next, let's look at a statistics question that can be used to stimulate discussions about normal distribution (see **fig. 1** and **table 3**). The intent of this question is to assess students' understanding of the empirical rule and of how the mean and the standard deviation affect the shape of the normal curve. Students need to understand that the mean is the center of the distribution and that the standard deviation determines the spread of the graph. In this case, if students understand the role of the mean, they can quickly eliminate (*c*) because its center is less than 20; the voting statistics indicate that most students can do so.

Next, students have to apply what they know about the standard deviation and the empirical rule. Specifically, they can use the fact that approximately 68 percent of the area under the curve should fall within one standard deviation of the mean to conclude that the standard deviation in (*a*) is too large and in (*d*) is too small, leaving (*b*) as the correct answer. The voting statistics show that eliminating (*a*) and (*d*) is more challenging. Because significant numbers of students vote for these options, this question can be useful for discussion, so we ask students to explain their differing points of view.

Questions that involve the use of graphs or pictures, such as this one, can engage students in discussion more quickly. Students can rely more on their intuition and understanding about fundamental ideas to get started thinking about the question.

VOTING IN CLASS FOR THE FIRST TIME

With classroom voting, as with any new pedagogy, teachers should lay the groundwork with their students. Students should be aware that they will be discussing multiple-choice questions with their classmates and that the quality of these discussions is as important as getting the correct answer.

Establishing the following simple rules helps ensure that all students engage in a discussion about the question:

- Everyone must vote.
- No one is allowed to vote until he or she has discussed the question with at least one other person.
- After the vote, several students will be called on to explain their votes.

With respect to this last point, students need to know that whether their vote is correct or not does not matter, as long as they can explain their thinking. The only unacceptable response is "I just guessed."

After the vote, if a strong majority of students vote correctly, the discussion is usually short. However, when student responses are more evenly spread across the possibilities, there is a good opportunity for a rich class discussion. Asking specific students, "What did you vote for and why?" is a good way to get the conversation started. After the first student explains, we typically give no feedback, and we ask the same question of another student, encouraging students to figure out the correct answer for themselves. If students express contradictory arguments, ask them to respond to one another.

For example, let's again consider the second question and its choices:

Solve for x if $y = e + 2^x$.

(*a*) $x = \dfrac{\ln y - 1}{\ln 2}$

(*b*) $x = \dfrac{\ln(y-1)}{\ln 2}$

(*c*) $x = \dfrac{\ln y}{\ln 2} - 1$

(*d*) $x = \dfrac{\ln(y-e)}{\ln 2}$

After a student explains why he voted for (*a*), the instructor could turn to a student who previously explained a vote for (*d*) and ask what she thinks about this reasoning. This approach will spur students to discuss whether distributing a natural logarithm function across two terms in a sum is permitted. We encourage students to reason through the problem themselves, and we confirm the correct answer after a majority of them have already reached the correct answer.

Class votes can be conducted in several ways. If no technology is available, students can simply raise their hands. To allow students to vote simultaneously, the teacher can ask students to hold up fingers. Another method is to hand out a set of four colored index cards for students to hold up (the red card stands for A, the blue card stands for B, etc.).

Using clickers (a classroom response system), if a set is available, can be the best way to conduct a class vote, for several reasons. First, clickers make individual votes more anonymous or at least less public; no one student can see how others vote. This anonymity encourages participation from students with less self-confidence than others.

Second, clickers help with time management; students can vote as soon as they are ready. As the votes trickle in, the teacher can judge when a sufficient number have been registered. Usually, when about 60 or 75 percent of a class has voted, it is time to call for the rest of the class to finish up and then close the vote.

Third, clickers make the results more precise. If not everyone has voted, this discrepancy is obvious, and the teacher can announce to the class: "We have thirty people in the room, but only twenty-eight votes. Everyone must vote, so please click in now!" After the vote, the clicker software can instantly present a precise bar graph of the results, indicating the exact number of students who voted for each option.

Questions can be presented to the students in a variety of ways. The questions on our website are numbered and can be downloaded in PDF form, to be printed, copied, and handed out to students. During class, the teacher can simply announce the number of the question to work on, and students can annotate their copy, work in the margins, and note the correct answer for later studying. Many classroom response systems are integrated with PowerPoint®, so another option is to present the questions in this way. This process may be a bit more time-consuming to set up; teachers must retype each question into PowerPoint or download a large-font version of the questions to cut and paste images of this into PowerPoint.

A TOOL TO ENGAGE ALL STUDENTS

Our library of questions has grown into a substantial resource, providing a starting place for anyone who wants to try classroom voting for the first time. In addition to assessing students' understanding of particular skills or processes, classroom voting can be a tool to engage all students in mathematical conversations. But when considering a particular question to include in a lesson plan, teachers cannot know for sure whether it will provoke discussion and get students to express contrasting opinions. This is why having records of previous votes available can be so useful.

Of course, every class is unique, and students in different classes may respond to the same question in different ways. However, the results from previous votes usually give teachers a reasonable indication of what to expect and thus can help them find questions that are worth the time and effort to vote on and discuss carefully.

Classroom voting motivates students to play a more active role in a lesson and thus helps them learn.

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BIBLIOGRAPHY

Bode, Martina, Denise Drane, Yifat Ben-David Kolikant, and Mary Schuller. 2009. "A Clicker Approach to Teaching Calculus." *Notices of the American Mathematical Society* 56: 253–56.

Cline, Kelly. 2006. "Sharing Teaching Ideas: Classroom Voting in Mathematics." *Mathematics Teacher* 100 (2): 100–104.

Hughes-Hallett, Deborah, Andrew M. Gleason, Daniel E. Flath, Sheldon P. Gordon, Patti F. Lock, David O. Lomen, David Lovelock, David Mumford, William G. McCallum, Brad G. Osgood, Andrew Pasquale, Douglas Quinney, Wayne Raskind, Karen Rhea, and Jeff Tecosky-Feldman. 2002. *Calculus, Single and Multivariable: Third Edition.* New York: John Wiley and Sons.

Lasry, Nathaniel, Eric Mazur, and Jessica Watkins. 2008. "Peer Instruction: From Harvard to the Two-Year College." *American Journal of Physics* 76: 1066–69.

Lucas, Adam. 2009. "Using Peer Instruction and i-Clickers to Enhance Student Participation in Calculus." *PRIMUS* 19: 219–31.

MacArthur, James R., and Loretta Jones. 2008. "A Review of Literature Reports of Clickers Applicable to College Chemistry Classrooms." *Chemistry Education Research and Practice* 9: 187–95. Doi: 10-1039/B812407H.

Miller, Robyn L., Everilis Santana-Vega, and Maria S. Terrell. 2006. "Can Good Questions and Peer Discussion Improve Calculus Instruction?" *PRIMUS* 16: 193–203. http://www.tandf.co.uk/journals/titles/10511970.

National Council of Teachers of Mathematics (NCTM). 2000. *Principles and Standards for School Mathematics*. Reston, VA: NCTM.

Pilzer, Scott. 2001. "Peer Instruction in Physics and Mathematics." *PRIMUS* 11: 185–92.

Pilzer, Scott, M. Robinson, D. Lomen, D. Flath, Deborah Hughes-Hallet, B. Lahme, J. Morris, W. McCallum, and J. Thrash. 2003. *ConcepTests to Accompany Calculus Third Edition.* New York: John Wiley and Sons.

Schlatter, Mark. 2002. "Writing ConcepTests for a Multivariable Calculus Course." *PRIMUS* 12: 305–14.

Smith, Michelle K., William B. Wood, Wendy K. Adams, Carl Wieman, Jenny K. Knight, Nancy Gulid, and Tin T. Su. 2009. "Why Peer Discussion Improves Student Performance on In-Class Concept Questions." *Science* 323: 122–24.

Zullo, Holly, Kelly Cline, Mark Parker, Ron Buckmire, John George, Katharine Gurski, Jacob Larsen, Blake Mellor, Jack Oberweiser, Dennis Peterson, Richard Spindler, Ann Stewart, and John Scharf. 2011. "Student Surveys: What Do They Think?" In *MAA Notes: Teaching Mathematics with Classroom Voting*, edited by Kelly Cline and Holly Zullo. Washington, DC: Mathematical Association of America.



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Editor's note: Student editions of these collections of questions can be downloaded at no cost from a PDF file that contains a numbered list of questions. For each collection, a teacher's edition is available, along with the questions, solutions, commentary, and the percentage of students voting for each option. This teacher's edition is available by e-mail request to the authors.