The Hypothetical Metallurgical Company 123 Main Street Theoretical, MT 12345

Dear Calculus Student:

We need your help designing a new thermal cycle for our heat treatment system. We use heat treatments of metals and alloys, such as annealing, quenching, and tempering processes, to achieve a desired level of hardness or toughness for different components that we manufacture. Our lab has produced a new alloy, and they would like us to create a new heat treatment for it. This treatment will last for 15 hours, the oven must start at a temperature of 300 degrees C, and it must end at a temperature of 100 degrees C. At the beginning of the process, our metallurgists need the temperature to increase gradually for the first few hours, then vary dramatically up and down, and at the end of the process, after reaching a large temperature to gradually cool for several hours and smoothly approach 100 degrees C. More dramatic and intense heating and cooling cycles will produce a harder alloy, which is an important goal of the process. Our heat treatment system cannot produce temperatures above 500 degrees C, and cannot be easily cooled to temperatures below 50 degrees C. Our system can only increase in temperature at a rate of 5 degrees C per minute, but it can cool a metal more rapidly, at a rate of 7 degrees C per minute. Because of stress to the heating/cooling system, the rate at which the temperature is changing should be not be adjusted by more than 4 degrees C per minute over the course of 10 minutes.

Our heat treatment system requires you to describe the temperatures it will produce with a mathematical function where the input *t* is the time in hours since the beginning of the cycle, and the output is temperature in degrees C. Thus, in order to have the right initial and final temperatures, this function will need to have T(0) = 300 and T(15) = 100. This mathematical function should be described by a single formula, so please, no piecewise functions. Your report must contain a graphical plot of this function for verification. We would further like a full report describing your rationale for why this will be a particularly good heat treatment process and mathematically verifying that all our requirements are met by your design. Further since we may be designing other heat treatment processes for similar alloys in the future, please describe how you created your process.

Thank you so much for your help!

Sincerely yours,

The Hypothetical Metallurgical Company