MA233 – Updated Project
Minimum Center of Mass

A Sample Problem

Solve the following problem with your group members. This problem will give you the basic tools necessary to complete the project. When you're done with the problem you can check the answer with either MA233 professor. It would be beneficial to build MATLAB code to supplement your solution.

Problem: A cylindrical soda can has a radius of 4cm and a height of 12cm. When the can is full of soda, the center of mass of the contents of the can is 6cm above the base on the center axis of the can. As the can is drained, the center of mass descends for a while. However, when the can is empty (filled only with air), the center of mass is once again 6cm above the base on the axis of the can. Find the depth of the soda in the can for which the center of mass is at its lowest point. Neglect the mass of the can, and assume the density of the soda is 1g/cm^3 and the density of air is 0.001g/cm^3.

Definition: Let $\rho(x, y, z)$ be an integrable density function on a closed and bounded region in three spatial dimensions. The coordinates of the **center of mass** of the region are

$$\begin{aligned}
\text{mass} = m &= \iiint_D \rho(x, y, z) dV \\
x \text{ center} = \overline{x} &= \frac{1}{m} \iiint_D (x\rho(x, y, z))\, dV \\
y \text{ center} = \overline{y} &= \frac{1}{m} \iiint_D (y\rho(x, y, z))\, dV \\
z \text{ center} = \overline{z} &= \frac{1}{m} \iiint_D (z\rho(x, y, z))\, dV.
\end{aligned}$$

The center of mass can be thought of as the point where the mass distribution of the object is centered. On a 1- or 2-dimensional object it is the balancing point of the object.

The Problem & The Constraints

Your job is to design a mathematical object for which we will run the same experiment as in the sample problem: fill the object with soda then drain and measure how the center of mass changes. The goals and constraints of this project are as follows:

1. The depth which gives the lowest center of mass should be as close to zero as possible without actually reaching zero.
2. The drain is located on the xy-plane, and all of the fluid must drain (no traps).
3. The group with the smallest nonzero depth wins bonus points on the project.
4. The object must be centered at $(0, 0)$ on the xy-plane.
5. There should be no point in your object that is more than 4cm from the z-axis, and there must be at least one point that is exactly 4cm from the z-axis.
6. The maximum distance from the xy-plane to the top of your object must be 12cm, and the minimum distance from the xy-plane to the top of your object must be greater than 0cm. Note: your object does not need to be symmetric.
7. The shape of your object must be described by a mathematical function (or a set of mathematical functions). Piecewise defined objects are allowed, but the object must be completely connected.
8. You must produce at least **two** such objects (with your best model indicated clearly).

Mad Props & Respect Points

- Animate the draining of your object.
- Show a 3D animation of how your asymmetric center of mass moves in time.
- Have the center of mass be outside your object at all times. Super Duper Mad Props if you can do this for the asymmetric object!
- Design an object where the center of mass always stays ABOVE the 11cm line. Remember that all of the fluid needs to drain.
- Design and object where the center of mass traces a 3D helix in time. More rotations = more respect.
- Write a parameterization for the location of your center of mass in time.
- **Create an additional object that is asymmetric where the center of mass is not on the z-axis.**

Important Dates

There are several important dates for this project:

1. Receive the project on Thursday, Oct 23. Begin choosing groups of 3, exchange contact information, and discuss meeting times.
2. On Thursday, November 6, each group **must**
   - identify group members
   - turn in a 1 paragraph description of their ideas with all group member's names. Failure to do so will result in a loss of 10 points on the project.
3. On Tuesday, November 18 before 9:29am each group must upload their **finalized presentation** to Moodle. Failure to so will result in a grade of zero for the presentation.
4. **Presentations** will take place Tuesday, November 18 - Thursday, November 20 during class time. Time slots will be published on Moodle ahead of time.
5. On Tuesday, December 2 each group must upload the **final version** of their written report to Moodle by 9:29am. Late work (even by 1 second) will not be accepted.

The Writing Process

You will turn in a paper written in a technical report format that includes the following elements:

Abstract: A very brief explanation of the project, written for someone that doesn't know the assignment, along with the highlights of your solutions. This is intended to explain to your classmates what your shapes were and why they are the best. This should be no more than one or two paragraphs with minimal technical details.

Your Solutions: A detailed description of your shapes with figures and graphs demonstrating the 3D object's shape and the behavior of the center of mass while the shape is being drained. Clearly explain why these particular shapes were chosen. Since there isn't a single right approach you must explain what you have done so that anyone from from your class can understand it. Part of your grade depends upon the clarity with which you can explain your approach.

Verification that Requirements are Satisfied: Go through each of the requirements and present a series of calculations which demonstrate that your shapes satisfy each one. Remember that in a technical paper you should not include every detail of every calculation. The reader should be able to work through the details of the calculations given only what is in the text. In other words, only include that much detail which is necessary for the reader.

Conclusion: A small conclusion section at the end should tie together what you have done on the project, and leave the reader with a lasting impression.

Appendices: Any necessary MATLAB code or extra figures should be included in an appendix at the end of the paper. The reader should not be required to read your code in order to understand your solution.

LaTeX: Your project should be written in LaTeX using the `\section` and `\subsection` commands to separate each of the individual sections of the paper. A template file has been included to get you started.

The Presentation

You must prepare a **five** minute presentation using Power Point, Beamer (a LaTeX version of Power Point), or any other appropriate presentation software. Your presentation should essentially be a **report on your progress with the project at the current time**. It should include appropriate graphics **as well as a discussion of possible issues / hangups**. Each person needs to be equally involved with the presentation. Given that the presentation is two weeks before the written project is due, expect that your professors will be VERY hard on your presentation. Our goal is to make your written reports as strong as possible. We will not pull any punches and it may seem like we are publicly humiliating you if you have mistakes.

Contributions, References, Plagiarism, & Integrity

Each team must submit a page, signed by all team members, either stating that all team members contributed equally to the project or offering an explanation and relative efforts if contributions were not equal. All papers must contain a "References" section that lists our textbook and any other source used. This is very easily done in LaTeX. See the commented portion of the `*.tex` template file for examples on how to do this. Plagiarism and cheating will be dealt with according to the student handbook.

Grading Rubric

Project Requirement	**Score**	**Possible Points**
November 6 Deadline: One paragraph description of initial ideas with a list of group members.		10
Abstract: The abstract is clearly stated; written to someone who doesn't know what the project is.		10
Your Solutions: Give a detailed description of your shapes and your centers of mass. Include details on how you came up with your shapes, which shapes did you try (and why did some of them fail), and any other details that may be helpful for the reader to understand your thought process.		15
Flow: The paper flows well from section to section; tables and figures are numbered, captioned, and discussed in the paper.		5
Conclusion: The main results are stated and tied together at the end of the paper.		5
Spelling and Grammar: The paper uses correct grammar, spelling, and punctuation.		5
Creativity: Your solutions are creative.		10
Constraints: Your shapes all satisfy all of the constraints, are adequately described, and any computations necessary to check the constraints are discussed. Your computations are all correct.		35
References: The textbook and any other references used are documented. Any "borrowed" images are clearly credited.		5
Presentation		20
TOTAL		120

Presentation Rubric:

- Mathematics
  - 6 pts. All mathematics is correct. Proper mathematical notation is used. All unknown terminology is explained. Presentation demonstrates an understanding of the mathematics.
  - 4 pts. The above is mostly done well, but there are occasional minor problems with one or more areas.
  - 2 pts. One of the above is done badly, or there are many minor problems.
- Communication
  - 6 pts. Presenters communicate material effectively. Demonstrate enthusiasm, interest, and creativity. All team members contribute to the presentation.
  - 4 pts. The above is mostly done well, but there are occasional minor problems with one or more areas.
  - 2 pts. One of the above is done badly, or there are many minor problems.
- Organization
  - 6 pts. The presentation is organized and the method of presentation is well chosen. The problem is clearly described. Ideas are connected to the topic and each other. The timing and pace is appropriate for a 5-10 minute talk.
  - 4 pts. The above is mostly done well, but there are occasional minor problems with one or more areas.
  - 2 pts. One of the above is done badly, or there are many minor problems.
- Attendance (This is an individual grade, not a group grade.)
  - 2 pts. On time and present for all talks.
  - -2 pts. Late on one day.
  - -6 pts. Late on multiple days, or missed more than one full talk on any day.

Total presentation points: ______________ (20 possible)

Fill out this page on the hard copy that you submit for grading.
Either fill out A or B below:

A: We (group member's names) ______________________________, ______________________________,
and ______________________________ worked together on this project and shared the workload equally.

B: We (group member's names) ______________________________, ______________________________,
and ______________________________ worked together on this project but did not share the load equally.

Group Member's Name	Did this percent of the work
Total:	100%