Differential Equations and Fishing Permits
MA 232 Project

Dear Mathematics Student,

The Montana Department of Fish and Wildlife is asking your advice in the management of the trout population in beautiful Lake Theoretical (in South Hypothetical County). We want to allow as much fishing as possible, but we need to make sure that there is no danger of over-fishing and wiping out the whole population. Past studies of this trout population have shown that it is accurately modeled by the logistic differential equation:

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{M}\right).$$

In this equation P is a function describing how the trout population changes over time, k is the exponential growth rate which is estimated to be roughly 20% per year (k = 0.2 per year), and M is the maximum carrying capacity of the lake, estimated to be 30,000 trout. If we allow fishing in the lake, then we must modify the differential equation by subtracting off F the number of trout taken from the lake per year, so the equation becomes:

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{M}\right) - F .$$

By setting the number of days each year that the lake is open for fishing, the Department thinks that it can control the number of trout taken fairly accurately. We need you to predict how the trout population will develop depending on how much fishing we choose to allow each year. One strategy is to allow some constant number of trout to be taken each year. Another idea is to change the number of trout taken depending on the current population, allowing more fishing if the population is higher and less if it is smaller. We want to see your projections of the future trout populations for many different constant fishing rates, and many different varying fishing rates.

Unfortunately due to budget cuts, our data on the lake is very rough, and we do not know the current population of trout in the lake. This means that you will need to explore the results of each fishing strategy for many different initial trout populations. Remember, we're biologists, so we want to see lots of graphs of your predictions in addition to all of your equations.

After explaining your mathematical models and exploring all the different fishing strategies with all the different possible initial populations, we want you to make a recommendation: What do you think is the best way to set the number of fish to be taken from the lake each year?

Sincerely yours,

Lake Hypothetical Management Division
Montana Department of Fish and Wildlife