3.9 Linear Approximation and the Derivative

1. If $e^{0.5}$ is approximated by using the tangent line to the graph of $f(x) = e^x$ at (0,1), and we know $f'(0) = 1$, the approximation is

(a) 0.5
(b) $1 + e^{0.5}$
(c) $1 + 0.5$

2. The line tangent to the graph of $f(x) = \sin x$ at (0,0) is $y = x$. This implies that

(a) $\sin(0.0005) \approx 0.0005$
(b) The line $y = x$ touches the graph of $f(x) = \sin x$ at exactly one point, (0,0).
(c) $y = x$ is the best straight line approximation to the graph of $f$ for all $x$.

3. The line $y = 1$ is tangent to the graph of $f(x) = \cos x$ at (0,1). This means that

(a) the only $x$-values for which $y = 1$ is a good estimate for $y = \cos x$ are those that are close enough to 0.
(b) tangent lines can intersect the graph of $f$ infinitely many times.
(c) the farther $x$ is from 0, the worse the linear approximation is.
(d) All of the above

4. Suppose that $f''(x) < 0$ for $x$ near a point $a$. Then the linearization of $f$ at $a$ is

(a) an over approximation
(b) an under approximation
(c) unknown without more information.

5. Peeling an orange changes its volume $V$. What does $\Delta V$ represent?

(a) the volume of the rind
(b) the surface area of the orange
(c) the volume of the “edible part” of the orange
(d) $-1 \times$ (the volume of the rind)

6. You wish to approximate $\sqrt{9.3}$. You know the equation of the line tangent to the graph of $f(x) = \sqrt{x}$ where $x = 9$. What value do you put into the tangent line equation to approximate $\sqrt{9.3}$?

(a) $\sqrt{9.3}$
(b) 9
(c) 9.3
(d) 0.3

7. We can use a tangent line approximation to $\sqrt{x}$ to approximate square roots of numbers. If we do that for each of the square roots below, for which one would we get the smallest error?

(a) $\sqrt{4.2}$
(b) $\sqrt{4.5}$
(c) $\sqrt{9.2}$
(d) $\sqrt{9.5}$
(e) $\sqrt{16.2}$
(f) $\sqrt{16.5}$