3.9 Linear Approximation and the Derivative

1. If \( e^{0.5} \) is approximated by using the tangent line to the graph of \( f(x) = e^x \) at (0,1), and we know \( f'(0) = 1 \), the approximation is
   
   (a) 0.5  
   (b) 1 + e^{0.5}  
   (c) 1 + 0.5

2. The line tangent to the graph of \( f(x) = \sin x \) at (0,0) is \( y = x \). This implies that
   
   (a) \( \sin(0.0005) \approx 0.0005 \)  
   (b) The line \( y = x \) touches the graph of \( f(x) = \sin x \) at exactly one point, (0,0).  
   (c) \( y = x \) is the best straight line approximation to the graph of \( f \) for all \( x \).

3. The line \( y = 1 \) is tangent to the graph of \( f(x) = \cos x \) at (0,1). This means that
   
   (a) the only \( x \)-values for which \( y = 1 \) is a good estimate for \( y = \cos x \) are those that are close enough to 0.  
   (b) tangent lines can intersect the graph of \( f \) infinitely many times.  
   (c) the farther \( x \) is from 0, the worse the linear approximation is.  
   (d) All of the above

4. Suppose that \( f''(x) < 0 \) for \( x \) near a point \( a \). Then the linearization of \( f \) at \( a \) is
   
   (a) an over approximation  
   (b) an under approximation  
   (c) unknown without more information.

5. Peeling an orange changes its volume \( V \). What does \( \Delta V \) represent?
   
   (a) the volume of the rind  
   (b) the surface area of the orange  
   (c) the volume of the “edible part” of the orange
(d) \(-1\times \) (the volume of the rind)

6. You wish to approximate \(\sqrt{9.3}\). You know the equation of the line tangent to the graph of \(f(x) = \sqrt{x}\) where \(x = 9\). What value do you put into the tangent line equation to approximate \(\sqrt{9.3}\)?

(a) \(\sqrt{9.3}\)
(b) 9
(c) 9.3
(d) 0.3

7. We can use a tangent line approximation to \(\sqrt{x}\) to approximate square roots of numbers. If we do that for each of the square roots below, for which one would we get the smallest error?

(a) \(\sqrt{4.2}\)
(b) \(\sqrt{4.5}\)
(c) \(\sqrt{9.2}\)
(d) \(\sqrt{9.5}\)
(e) \(\sqrt{16.2}\)
(f) \(\sqrt{16.5}\)