4.6 Rates and Related Rates

1. If \( \frac{dy}{dx} = 5 \) and \( \frac{dx}{dt} = -2 \) then \( \frac{dy}{dt} = \)
   
   (a) 5
   
   (b) -2
   
   (c) -10
   
   (d) cannot be determined from the information given

2. If \( \frac{dz}{dx} = 12 \) and \( \frac{dy}{dx} = 2 \) then \( \frac{dz}{dy} = \)
   
   (a) 24
   
   (b) 6
   
   (c) 1/6
   
   (d) cannot be determined from the information given

3. If \( y = 5x^2 \) and \( \frac{dx}{dt} = 3 \), then when \( x = 4 \), \( \frac{dy}{dt} = \)
   
   (a) 30
   
   (b) 80
   
   (c) 120
   
   (d) 15x^2
   
   (e) cannot be determined from the information given

4. The radius of a snowball changes as the snow melts. The instantaneous rate of change in radius with respect to volume is
   
   (a) \( \frac{dv}{dr} \)
   
   (b) \( \frac{dr}{dv} \)
   
   (c) \( \frac{dv}{dr} + \frac{dr}{dv} \)
   
   (d) None of the above

5. Gravel is poured into a conical pile. The rate at which gravel is added to the pile is
   
   (a) \( \frac{dv}{dt} \)
6. Suppose a deli clerk can slice a stick of pepperoni so that its length $L$ changes at a rate of 2 inches per minute and the total weight $W$ of pepperoni that has been cut increases at a rate of 0.2 pounds per minute. The pepperoni weighs:

(a) 0.4 pounds per inch
(b) 0.1 pounds per inch
(c) 10 pounds per inch
(d) 2.2 pounds per inch
(e) None of the above

7. The area of a circle, $A = \pi r^2$, changes as its radius changes. If the radius changes with respect to time, the change in area with respect to time is:

(a) $\frac{dA}{dt} = 2\pi r$
(b) $\frac{dA}{dt} = 2\pi r + \frac{dr}{dt}$
(c) $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$
(d) Not enough information

8. As gravel is being poured into a conical pile, its volume $V$ changes with time. As a result, the height $h$ and radius $r$ also change with time. Knowing that at any moment $V = \frac{1}{3}\pi r^2 h$, the relationship between the changes in the volume, radius and height, with respect to time, is:

(a) $\frac{dV}{dt} = \frac{1}{3}\pi \left( 2r \frac{dr}{dt} h + r^2 \frac{dh}{dt} \right)$
(b) $\frac{dV}{dt} = \frac{1}{3}\pi \left( 2r^2 \frac{dr}{dt} \cdot \frac{dh}{dr} \right)$
(c) $\frac{dV}{dt} = \frac{1}{3}\pi \left( 2rh + r^2 \frac{dh}{dt} \right)$
(d) $\frac{dV}{dt} = \frac{1}{3}\pi \left( r^2 (1) + 2r \frac{dr}{dt} h \right)$

9. A boat is drawn close to a dock by pulling in a rope as shown. How is the rate at which the rope is pulled in related to the rate at which the boat approaches the dock?
(a) One is a constant multiple of the other.
(b) They are equal.
(c) It depends on how close the boat is to the dock.

10. A boat is drawn close to a dock by pulling in the rope at a constant rate.
   **True or False:** The closer the boat gets to the dock, the faster it is moving.
   (a) True, and I am very confident
   (b) True, but I am not very confident
   (c) False, but I am not very confident
   (d) False, and I am very confident

11. A streetlight is mounted at the top of a pole. A man walks away from the pole. How are the rate at which he walks away from the pole and the rate at which his shadow grows related?

   (a) One is a constant multiple of the other.
   (b) They are equal.
   (c) It depends also on how close the man is to the pole.

12. A spotlight installed in the ground shines on a wall. A woman stands between the light and the wall casting a shadow on the wall. How are the rate at which she walks away from the light and rate at which her shadow grows related?
(a) One is a constant multiple of the other.
(b) They are equal.
(c) It depends also on how close the woman is to the pole.