MathQuest: Differential Equations

What is a Differential Equation?

1. Which of the following is not a differential equation?
   (a) \( y' = 3y \)
   (b) \( 2x^2y + y^2 = 6 \)
   (c) \( tx \frac{dx}{dt} = 2 \)
   (d) \( \frac{d^2y}{dx^2} + 4 \frac{dy}{dx} + 7y + 8x = 0 \)
   (e) All are differential equations.

2. Which of the following is not a differential equation?
   (a) \( 6 \frac{dy}{dx} + 3xy \)
   (b) \( 8 = \frac{y'}{y} \)
   (c) \( 2 \frac{d^2f}{dt^2} + 7 \frac{df}{dt} = f \)
   (d) \( h(x) + 2h'(x) = g(x) \)
   (e) All are differential equations.

3. Which of the following couldn’t be the solution of a differential equation?
   (a) \( z(t) = 6 \)
   (b) \( y = 3x^2 + 7 \)
   (c) \( x = 0 \)
   (d) \( y = 3x + y' \)
   (e) All could be solutions of a differential equation.

4. Which of the following could not be a solution of a differential equation?
5. Which of the following could not be a solution of a differential equation?

(a) \( f = 2y + 7 \)
(b) \( q(d) = 2d^2 - 6e^d \)
(c) \( 6y^2 + 2yx = \sqrt{x} \)
(d) \( y = 4\sin 8\pi z \)
(e) All could be a solution of a differential equation.

6. True or False? A differential equation is a type of function.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

7. Suppose \( \frac{dx}{dt} = 0.5x \) and \( x(0) = 8 \). Then the value of \( x(2) \) is approximately

(a) 4
(b) 8
8. Which of the following is a solution to the differential equation \( \frac{dy}{dt} = 72 - y \)?

(a) \( y(t) = 72t - \frac{1}{2}t^2 \)
(b) \( y(t) = 72 + e^{-t} \)
(c) \( y(t) = e^{-72t} \)
(d) \( y(t) = e^{-t} \)

9. The amount of a chemical in a lake is decreasing at a rate of 30% per year. If \( p(t) \) is the total amount of the chemical in the lake as a function of time \( t \) (in years), which differential equation models this situation?

(a) \( p'(t) = -30 \)
(b) \( p'(t) = -0.30 \)
(c) \( p'(t) = p - 30 \)
(d) \( p'(t) = -0.3p \)
(e) \( p'(t) = 0.7p \)

10. The evolution of the temperature of a hot cup of coffee cooling off in a room is described by \( \frac{dT}{dt} = -0.01T + 0.6 \), where \( T \) is in °F and \( t \) is in hours. What are the units of the numbers -0.01 and 0.6?

(a) -0.01 °F, and 0.6 °F
(b) -0.01 per hour, and 0.6 °F per hour
(c) -0.01 °F per hour, and 0.6 °F
(d) neither number has units

11. We want to test the function \( z(x) = 4 \sin 3x \) to see if it solves \( z'' + 2z' + 4z = 0 \), by substituting the function into the differential equation. What is the resulting equation before simplification?

(a) \( -36 \sin 3x + 24 \cos 3x + 16 \sin 3x = 0 \)
(b) \( 4 \sin 3x + 8 \sin 3x + 16 \sin 3x = 0 \)
(c) \( -36 \sin 3x + 12 \cos 3x + 4 \sin 3x = 0 \).
(d) \[ 4 \sin 3x + 8 \cos 3x + 4 \sin 3x = 0 \]
(e) none of the above

12. If we test the function \( f(x) = ae^{bx} \) to see if it could solve \( \frac{df}{dx} = cf^2 \), which equation is the result?

(a) \( \frac{df}{dx} = ca^2 e^{2bx} \)
(b) \( abe^{bx} = cf^2 \)
(c) \( ae^{bx} = ca^2 e^{(bx)^2} \)
(d) \( abe^{bx} = ca^2 e^{2bx} \)
(e) \( abe^{bx} = cae^{bx} \)
(f) None of the above

13. We want to test the function \( f(x) = 3e^{2x} + 6x \) to see if it solves the differential equation \( \frac{df}{dx} = 2f + 3x \), so we insert the function and its derivative, getting \( 6e^{2x} + 6 = 2(3e^{2x} + 6x) + 3x \). This means that:

(a) This function is a solution.
(b) This function is a solution if \( x = 2/5 \).
(c) This function is not a solution.
(d) Not enough information is given.

14. A bookstore is constantly discarding a certain percentage of its unsold inventory and also receiving new books from its supplier so that the rate of change of the number of books in inventory is \( B'(t) = -0.02B + 400 + 0.05t \), where \( B \) is the number of books and \( t \) is in months. If the store begins with 10,000 books in inventory, at what rate is it receiving books from its supplier at \( t = 0 \)?

(a) 200 books per month
(b) 400 books per month
(c) 200 books per month
(d) 900 books per month