Integrating Factors

1. If \( y \) is a function of \( t \), which of the following is equivalent to \( t \left( y' + \frac{1}{t} y \right) \)?
   (a) \([ty]'\)
   (b) \([\frac{1}{t} y]'\)
   (c) \(ty'\)
   (d) \(\frac{1}{t} y'\)
   (e) None of the above

2. Which of the following is an integrating factor for \( y' + 3ty = \sin t \)?
   (a) \(e^{\frac{3}{2}t^2}\)
   (b) \(e^3\)
   (c) \(e^{\sin t}\)
   (d) \(e^{-\cos t}\)
   (e) \(e^{3y}\)
   (f) All of the above

3. Which of the following is an integrating factor for \( y' + 2y = 3t \)?
   (a) \(e^{2t}\)
   (b) \(e^{2t+5}\)
   (c) \(e^2e^{2t}\)
   (d) \(7e^{2t}\)
   (e) All of the above
   (f) None of the above

4. Which of the following is an integrating factor for \( 3y' + 6ty = 8t \)?
   (a) \(e^{3t^2}\)
   (b) \(e^{t^2}\)
(c) $e^6$
(d) All of the above
(e) None of the above
(f) This problem cannot be solved with integrating factors.

5. Can integrating factors be used to solve $y' + 2ty = 1$?
   (a) Yes. The solution is $y = e^{-t^2} \int e^{t^2} dt$.
   (b) No - we cannot evaluate $\int e^{t^2} dt$.

6. The differential equation $y' + 2y = 3t$ is solved using integrating factors. The solution is $y = \frac{3}{2}t - \frac{3}{4} + Ce^{-2t}$. Which of the following statements describes the long-term behavior as $t \to \infty$?
   (a) The solutions will approach zero, because $e^{-2t} \to 0$ as $t \to \infty$.
   (b) The solutions will grow without bound because $\frac{3}{2}t \to \infty$ as $t \to \infty$.
   (c) The long-term behavior depends on the initial condition; we need to know the value of $C$ before we can answer this.

7. Which of the following is $e^{3t}(y' + 2y)$ equivalent to?
   (a) $[e^{2t}y]'$
   (b) $[e^{3t}y]'$
   (c) $e^{3t}y'$
   (d) $e^{2t}y'$
   (e) None of the above